Is the Bridge Really So Far Away? Elementary Mathematics Teachers' Competencies to Implement Neuroscience Theory into Their Teaching Practices

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Abstract

This research investigates elementary mathematics teachers' competencies to implement neuroscience theory (NS) of mathematical cognition into their classroom practices during and after, participation in a professional development (PD) course. We examine the teachers' familiarity with Approximate Number System (ANS), relevant to elementary school level, and build their competencies to implement this NS concept into teaching. Data for this qualitative study were collected from the reflective reports written by nineteen female elementary mathematics teachers (aged 25–40) and were analysed using two complementary methodologies: content and linguistic analyses. Our findings indicate that the teachers acquired NS knowledge, and their competencies evolved from listing the facts and analysing their meanings to weighing possible applications and attempts to integrate newly acquired knowledge into their teaching. This study seeks to contribute to the field of professional teacher development by demonstrating how teachers can lend meaning to theoretical knowledge they obtain in a PD course, in meaningful ways by connecting it with their classroom teaching practices.

Introduction

Neuroscience (NS) has become a topic of extensive, rich, and diverse research around the world in recent years. Scholars' opinions about the application of NS findings vary from claims that it is a distant vision (Goswami, 2005) to the idea that interdisciplinary teams tasked with seeking ways to apply NS findings to education and teaching, can already be established (Schumacher, 2007). Proponents of the former idea, stressing the dearth of cases in which NS would inspire novice and effective teaching approaches, suggest that NS findings have little influence on education (Bowers, 2016). The latter approach has manifested itself in the numerous attempts to translate NS research into educational practice since 1998, when Byrnes and Fox proposed that educational practice could benefit from NS research findings. Masson and Foisy (2014) see neuropedagogy as playing a central role in several areas of education, affecting educational theories and policies, as well as teaching methods and ways in which pedagogies are delivered. A cohort of scientists supports this idea by pointing at substantial progress in educational NS in collaboration with educators and by suggesting that educational NS provides original insights into the prospects of individualised education for struggling students (Ansari & Lyons, 2016; Howard-Jones, Varma, Ansari, Butterworth, De Smedt, Goswami & Thomas, 2016; Gabrieli, 2016; Ansari, König, Leask, & Tokuhama-Espinosa, 2017). Willingham (2009) states that neuroscience findings, applied in the context of strong behavioural theory, may promote better understanding of learning processes. These trends are evident in the professional literature and in scientific and popular publications that mull the possibility of applying information from NS research to pedagogical activity and education research (Geake, 2009). These publications point to several possible ways of applying such findings to education. One such tactic, suggested by Stern (2005) and de Bruin (2016), endorses the establishment of interdisciplinary teams that would work on 'evidence-based' reform. Another approach, recommended by Gabrieli (2016) and other scholars, suggests that educational neuroscience contributes to ideas about education practices and policies, beyond classroom curricula, that are important for helping vulnerable students. An additional approach is neuropedagogy, which, relying on recent NS insights, is expected to impact teaching experiences and practices by addressing needs that arise in working with a wide range of students (Patten, 2011). There is, however, a lacuna in the research literature on potentially effective ways of applying NS insights to teaching practices. In this study, we wish to build 'small bridges' between NS and mathematics education with the help of 'boundary research', in accordance with a worldwide trend of initiatives that have this goal in mind (Edelenbosch, Kupper, Krabbendam, & Broerse, 2015). We believe that one way to apply NS discoveries to teaching is to make relevant aspects of NS theory accessible and known to teachers and then, in collaboration with teachers as the experts in their field, seek strategies to convert this theoretical knowledge into viable teaching practices (Fridman, Teichman-Weinberg, & Grobgeld, 2016).

Teacher knowledge and professional development

Knowledge acquisition among teachers, like knowledge acquisition in general, may be viewed through the lenses of various theories of learning. This is a complex learning process that involves many aspects of a teacher's work; study of this process must be based in diverse epistemological perspectives on the complexity and the multiple sources of teachers' knowledge (Kennedy, 2002). Following Biggs and Tang (2011) and Nielsen and Lund (2020), in this paper, we conceptualise teacher knowledge by drawing a distinction between declarative and functional knowledge, where the former refers to theoretical knowledge whereas the latter refers to "knowing to do" (Nielsen & Lund, 2020: p. 12), which, in some contexts, is synonymous to skills and competencies. The focal point of this perspective is translating knowledge into practice, a subject that has been explored by various authors and referred to using a variety of terms. Teachers may appropriate both conceptual, that is theoretical, and practical tools in a developmental process involving the internalization of specific culturally defined ways of thinking through active participation in social practices (Leont'ev, 1981; Wertsch, 1991).

A constant quest for more efficient teaching methods is integral to teachers' professional growth and development. Teachers' beliefs and concepts can be seen as a crucial factor in shaping and reshaping their behaviour in the classroom (Lavicza, 2010; Sadaf, Newby, & Ertmer, 2012; Umit, 2018) and can be formed by teachers' personal experiences, their academic education (Philipp, 2007), relevant professional development (PD) courses, conferences, seminars, and working groups, and by reflective self-learning after being exposed to issues that arise in class or any other situation involving their teaching work (Borko, 2004; Desimone, 2009). Extensive research suggests that reflection and collaboration are crucial strategies for teachers' development and improving their professional knowledge (Lee & Son, 2015; Santos & Cai, 2016). PD courses expose participants to a meaningful learning process comprising active learning, collective participation, coherence, and duration (Desimone, 2009). By implication, PD should focus on changes in teaching practice that will enhance students' learning (Lumpe et al., 2012; Taylor & Govan, 2017). In this study, a course addressing several NS theory issues relevant to elementary math teachers was planned and delivered. One of its key aims was imparting familiarity with the Approximate Number System (ANS).

Approximate Number System (ANS)

ANS is a cognitive system that supports the estimation of the size of a group of items without relying on language or graphic symbols (Dehaene, 2011). For instance, an adult using the ANS may correctly estimate that a container with 105 berries has more than a container with 65 berries; but may fail to determine that a container with 105 berries has more than one with 95 berries – these quantities are too close to reliably distinguish without counting (Mazzocco, Feigenson, & Halberda, 2011). Piazza (2010) lists ANS as one of the 'neurocognitive start-up tools.' Halberda et al. (2008) examined 64 normally developed 14-year-old children and tested the correlation between their achievements and ANS acuity; they found that success in elementary-level arithmetic can be predicted by ANS acuity measured by means of dot arrays. That is, performance with nonsymbolic comparison of quantities was found to be related to mathematics achievement.

Additionally, some researchers in this domain see ANS as a core system that supports more sophisticated mathematics (Albarracín & Gorgorió, 2019; Feigenson, Deheane, & Spelke, 2004). Therefore, to enhance mathematical achievements, methods of enhancing ANS acuity should be developed (Clayton & Gilmore, 2015). This concept was examined by several researchers who suggested ways to improve ANS acuity. Hyde, Khanum & Spelke (2014) examined children ages 6-7 and found that nonsymbolic assessment tasks improved students' achievements in regular arithmetic tests (Hyde et al., 2014). Another study, conducted by Park & Brannon (2013) on adult subjects, showed that practicing approximate addition and subtraction of arrays of dots lead to improvement in ANS and, in turn, in the ability to solve standard addition and subtraction problems. Researchers hypothesise that learning is most effective when the learner knows how to activate two pathways in the brain, visuospatial processing and mathematical skills (Park & Brannon, 2013). The findings and conclusions of all these studies give the reason for educational interventions in elementary schools (De Smedt & Grabner, 2015, 2016; Cargnelutti, Tomasetto, & Passolunghi, 2017; Tomasetto, Morsanyi, Guardabassi, & O'Connor, 2021). At the same time, it is important to address the studies that found no relation between children's performance of a nonsymbolic comparison task and their mathematics achievements in school (Holloway & Ansari, 2009). So far, however, most of the existing studies have explored the connection between ANS and students' ability to solve standard mathematical problems. Only a few recent studies addressed professional development for mathematics teachers based on the brain training approach (Shahsavani, Baratali, & Keshtiaray, 2020). Our present work aims to fill the gap in the field by suggesting how NS theory can contribute to creating innovative mathematics teaching practices.

Our underlying hypothesis in this study is that teachers' familiarity with NS, specifically ANS, will create opportunities to integrate it into the education system. Given the frequently found correlation between ANS acuity and the ability to perform basic arithmetic tasks, we consider familiarity with this system important for mathematics teachers. Consequently, observing the process by which, on having been exposed to NS theory, elementary school math teachers start weighing possible applications of this newly-acquired theoretical knowledge and cautiously piloting teaching practices that implement NS knowledge, can indicate how PD courses positively impact teaching. More specifically, this article aims to answer the following question: How does elementary school math teachers' theoretical knowledge of NS theory, obtained during the PD course and relevant to teaching in elementary school, contribute to their readiness and skills to design classroom practices implementing the NS knowledge? Notably, the PD course at issue aimed at having teachers propose and design new pedagogical methods themselves.

Methodology

Participants and context of the study

To answer the research question, we had to design a suitable course that would include theoretical aspects of NS research, examples of application, and workshops where participants could discuss possible ways of applying NS findings. Courses associated with the relationship between mind, brain, and education are not included in the training of elementary teachers in Israel; nor did we encounter such courses at PD centres. Consequently, a team composed of math teaching experts and NS specialists was assembled to develop the course, which was meant to pilot the possibility of applying NS research to math teaching in elementary schools. Examples of the topics of the meetings: ANS, the effect of the various representations, multiplication, spatial vision etc.

This course was given within the framework of the 'Academia-to-Classroom' project, launched by the Israeli Ministry of Education as an initiative aiming to strengthen the relations between academia and schools. The team at the college, composed of the authors of this article, chose an intervention that would acquaint teachers with NS findings and examples of their application in schools around the world so that through this acquaintance, teachers would seek a way to apply the findings in their schools and their classes.

During the school year, we offered teachers a thirty-hour PD that gave participants a general background of NS studies relating to the relevance of NS to elementary mathematics teaching, with most of the course devoted to ANS. The activity included lectures meant to familiarise the teachers with the findings of contemporary ANS research. There were also several workshops to discuss possible ways of applying what they had learned. In addition, the teachers prepared lesson plans attempting to implement the insights from the ANS research into their teaching. The course was attended by nineteen elementary school math teachers (females aged 25–40). They are all teachers who teach mathematics and have 5-20 years of experience. The teachers who have chosen to attend the course teach in the1-6 grades in elementary school. In addition, the teachers prepared lesson plans attempting to implement the insights from the ANS research into their teaching. It should be noted that the participants had no prior learning of NS.

In an introductory conversation preceding the course, however, all the participants expressed keen interest in learning about NS research of relevance to math education and finding ways to apply relevant findings in their teaching. In addition, all the teachers signed the consent form to participate in the research. The researchers were involved in the planning of the teachers' PD program, gave the lectures, and lead the workshops. These two researchers serve as the lecturers in the Department of Mathematical Education; they underwent departmental training on neuropedagogy. One of the researchers' doctoral research deals with neuropedagogy. The researcher who performed linguistic analysis has a Ph.D. in Applied Linguistics. The study was approved by the Institutional Research Ethics Committee (Approval number 1613193297917).

Data collection and analysis

We asked the teachers to write two reflective reports: one when the theoretical lectures and the workshops were over, and a summative paper on the entire process at the end of the course. Overall, 26 reflective reports were submitted. In each reflection, they were asked to relate to the following question: *'How do you think you will be able to apply the knowledge you acquired?'* Twelve participants reported in pairs because they worked with pupils in the same age group. The remaining seven participants worked individually. Addressing this question, the participants created both retrospective and prospective reflections. The latter allowed us to examine their pedagogical intentions and commitments to specific pedagogical strategies among a broad range of possibilities (Urzua & Vasquez, 2008). Given the teachers' awareness that some answers may be considered more desirable than others, the two complementary methodologies (i.e. content and linguistic analyses) were integrated in order to understand both explicitly and implicitly conveyed messages (as per Muchnik-Rozanov & Tsybulsky, 2019)) addressing the teachers' willingness and self-reported competencies to implement NS knowledge in their teaching practice after the PD course. In addition, the lecturer's observations were documented and used as a supplementary source of data.

Content analysis

The content analysis involved several stages. First, we gathered all the data collected from 26 reflective reports into one file. Then we combed the reflections to extract main themes that represent knowledge acquisition regarding the material learned in the activity. After each of us proposed themes, we discussed them together and agreed on the main ones. This stage yielded five themes, where the themes of *familiarization with NS* and *the meaning of facts about NS* can be associated with the acquisition of theoretical knowledge, while the themes of *directions of thinking in view of the new information*, *directions of action pursuant to the new information*, and *practical application* can be viewed in the context of functional knowledge. Next, we chose only statements associated with ANS because familiarity with this system was central in the course, and most of the participants' statements related to it.

Third, three investigators analysed several participants' reflections in view of the primary themes. Then we compared our analyses and attained consensus by merging them. An investigators' triangulation was then performed, with each of us analysing the reflective reports conducted by the two other members of the team. This analysis narrowed the five themes found in the initial analysis into four: *learning about ANS*, *directions of thinking and planning*, *planning steps to be taken*, and *trying out activities*.

In the next stage, the above categories were merged into core categories, designed as proposed by De Smedt and Grabner (2015), for modes of NS application in math education: *neurounderstanding*, *neurointervention*, and *neuroprediction*. The third category was not relevant to this study because it concerns applying NS via use of brain imaging. Therefore, we referred to the first two. The concept of *neurounderstanding* (NU) denotes familiarity with and understanding of knowledge created in NS about how people acquire cognitive skills and how this learning is expressed biologically. *Neurointervention* (NI), in turn, concerns how NS findings may underpin teaching interventions and how teaching moulds the neural circuits that are responsible for complex learning skills such as mathematics. We also built a set of subcategories: learning about ANS, analysing the learned NS facts, planning steps to be taken, and implementing activities that encourage ANS proficiency. The array of categories and subcategories thus obtained, was tested by each of us across the full dataset.

Linguistic analysis

This linguistic inquiry, based on systemic functional linguistic theory (SFL) (Halliday, 1994; Eggins, 1994), focuses on the study of lexical cohesion (Fine, 2006; Smirnova et al., 2015) in teachers' language behaviour. Lexical cohesion is the element of lexis that connects stretches of language by selecting vocabulary items (Fine, 1994; Halliday & Matthiessen, 2004) and involves relations established between lexical items by lexical repetition (exact repetition of a lexical item), root cohesion (repetition of one root through different lexemes) and use of synonyms and hyponyms. Lexical items connected by the lexical cohesion and referential system form cohesive chains, the longest ones being sustained as the text unfolds and gradually emerging as major textual motifs.

Our linguistic analysis identified main motifs in the participants' reports which defined their principal topics of discussion. These motifs tended to reflect the process of transition from the theoretical knowledge of ANS that the teachers had acquired during the course, to "functional" knowledge (Nielsen & Lund, 2020) manifested in their attitudes and commitments toward the possible application of this knowledge to math teaching. In neuropedagogic terms, the linguistic analysis addressed two neuropedagogy-related thematic categories in math teaching: neurounderstanding and neurointervention.

We coded the reflective reports from each summative paper for lexical cohesion, limiting the analysis to lexical repetitions, root cohesion, synonyms, and hyponyms. References to items connected by lexical cohesion were coded as well (Table 1). The most-sustained (longest) cohesive chains were analysed to extract the major motifs.

Table 1. Three main categories of motifs based on Linguistic Analysis

Motif categories	Motifs (examples)	Cohesive chains (examples)
NS-related context	'ANS system'	'ANS system, ANS, the system, cognitive system, Approximate Number System, it, the basic system'
Math teaching class practice-related context	'multiplication'	'multiplication, the multiplication, multiplication table, multiple'
Implementing NS-based planned activities context	'develop'	'to develop, development, develops, evolution'

Findings

Findings from content analysis

The data analysis revealed four categories that described stages of knowledge transformation observed in the teachers. The first two categories are associated with theoretical knowledge: *facts about ANS* and *the meaning of facts about ANS*. The other two categories are related to functional knowledge (Nielsen & Lund, 2020): *planning* and *trying out activities* that cultivate ANS. The categories dealing with theoretical knowledge were merged into the *neurounderstanding* category (NU), containing teachers' insights about math studies from the NS research perspective. The two other categories dealing with functional knowledge were merged into the *neurointervention* (NI) category, holding possible applications of NS findings in math teaching and learning processes.

Neurounderstanding

The statements in this category related to knowledge about the structure of the brain and the functions of the brain areas involved in numerical cognition processes, that was covered by the PD course in reference to ANS. Our in-depth analysis of these statements revealed that they could be further divided into two subcategories: facts about ANS and the meaning of facts about ANS. An example of a statement assigned to the former category is:

> *"In this workshop we learned about the parietal lobe, where activities connected to numerical cognition occur. The lobe has a slit called the intraparietal sulcus, which is responsible for ANS."*

In this instance, the teacher reports knowing about the structure of the brain and its parts, as well as the functions of those parts: the teacher learned that the cerebral cortex is divided into lobes filled with slits that fulfil different functions of absorbing and processing information. Another teacher whose statement we placed in this category related to numerical cognition more specifically:

> *"We learned about ANS – a cognitive system that supports our ability to estimate the size of a group without relying on language or symbols. This system also allows a person to identify differences between two or more groups when these groups are presented as quantities and not as numbers or names of numbers."*

This quotation shows that the teacher simply repeats in her own words the accepted description of ANS as given to the course participants in the first lecture. It is important to emphasise that this was the student's first exposure to ANS knowledge; her ability to reproduce its content reflects serious consideration. Acquiring this theoretical knowledge paves the way to further development ideas related to implementing into their teaching.

The second subcategory includes statements in which the teachers ascribed meaning to facts they had learned about ANS. One statement that recurred in several variations was:

> *"Elementary school students with poor ANS struggle with math studies. Since this system constitutes the basis for meaningful learning of arithmetic, it is important to develop it among young children."*

By implication, the teacher realises that ANS is the cognitive foundation of normal development of mathematical ability; hence, its practice will improve the learning of arithmetic if we know how to develop it adequately. In addition to the evidence drawn from teachers' self-reports, the workshop lecturers described the participants' ability to propose various ways of implementing the NS in their teaching.

Neurointervention

Statements assigned to the category of *neurointervention* are those in which teachers spoke about using the knowledge they acquired in the course to enhance their work. They were sorted into two subcategories: 'planning steps to be taken' and 'trying out activities that cultivate ANS'. Here we present findings for the latter category.

Statements belonging to this category are few, relative to the other categories. Nevertheless, some teachers did try to implement what they had learned into their teaching and reported on this both in writing and orally. One teacher said:

> *"During the individual teaching hours, we chose to take two pupils once a day to the computer room for exercises designed to improve the ANS* [referring to exercises with blue and yellow dots] *and we actually saw that one of them improved in his classwork."*

This teacher chose two pupils with low-to-average scores in math and tried to strengthen their ANS through regular quantity estimation exercises that can easily be found on the internet. The success of one of the pupils and his progress in mathematics studies encouraged other teachers to try to implement NS findings into their work; it also gave this teacher the great satisfaction of knowing how to improve the achievements of one more pupil. Following this, another teacher wrote:

> *"I also tried.… at first, I used small quantities and then larger ones. In the first sessions, it was very hard for the pupils and it took them a long time to reach an answer, but after a while there was noticeable improvement. I think because the sessions were not intensive but only twice a week, I couldn't determine whether there was any improvement in achievements, but I could see an improvement in the confidence of those who struggle in math class."*

Findings from linguistic analysis

The linguistic analysis echoed the findings of the content analysis. Basing ourselves on the study of lexical cohesion, we identified 67 major motifs throughout the teachers' reflections. Seven of these motifs were directly related to the facts about the ANS that were taught in the course, that is, addressing theoretical knowledge of the taught NS concept. Central theoretical concepts discussed in the PD course were observed among the major motifs in the reflective reports (e.g., 'ANS system,' 'performance,' and 'insights,' etc.) These results characterise the *neurounderstanding* category when the teachers become familiar with ANS theory and its importance for their students' success but do not elaborate on these concepts. Thirty of the identified motifs were related to math teaching class practices. The teachers discussed these themes in planning broad-spectrum classroom practices and, to a lesser extent, activities meant to foster ANS. Although these motifs were not directly related to the NS concepts taught in the course, they reflect the teachers' profound understanding of math teaching and its complexity. The most recurrent motifs discussed in the context of math teaching classroom practices were 'pupil,' 'struggling,' 'lesson,' and so on.

Regarding the category of *neurointervention*, twenty-three of the major themes addressed possible applications of ANS knowledge and the teachers' experiences when implementing the suggested activities in their classrooms; these emerged as the major topics of the reflective reports. These themes developed from lexical items that belonged concurrently to the semantic fields of ANS knowledge and math teaching class practices (e.g., 'learning,' 'practice,' and 'reasoning'). Table 2 summarises the findings of linguistic analysis concerning content analysis.

Table 2. Content analysis themes in relation to linguistic analysis motifs

			Linguistic analysis motifs
Content analysis themes	*Neurounderstanding*	Teachers' insights about math studies from the NS research perspective	Motifs directly related to the facts about the ANS (NS-related context)
	Neurointervention	Possible applications of NS findings in math teaching and learning	Planning and implementing NS-based activities (teachers' experiences when implementing the suggested activities in their classrooms)
			Math-teaching classroom practices (broad-spectrum)

Discussion

The findings of the content and linguistic analyses indicate that mathematics teachers were able to transform their understanding of the PD course content into competencies to implement NS theory in classroom practice. It was observed that teachers write about ANS on several levels: learning about the brain, analysing the learned NS facts, planning steps, and trying out activities that encourage ANS proficiency. These levels may be seen in building teacher knowledge, with theoretical knowledge evolving into functional knowledge directly associated with teaching skills and competencies (Biggs & Tang, 2011; Nielsen & Lund, 2020). Lave and Wenger (1991) describe teacher learning as transitioning from peripheral to full participation in specific working practices and associated ways of knowing and thinking. The teachers not only articulate the theoretical knowledge that they obtained during the course, but rather transition from a theoretical foundation of activity (Kelly, 2006) toward a practical one, by suggesting specific working practices that, on the one hand, tend to rely on the newly acquired knowledge and, on the other hand, reflect these teachers' professional expertise and classroom experience. In sum, our findings align with the existing literature suggesting that NS research findings can be integrated into learning (Ansari & Lyons, 2016; Howard-Jones et al., 2016) and distance us from thinking that the bridge is too far (Goswami, 2005).

It is also noteworthy that the teachers expressed a desire to attend this kind of course:

> *"The main idea here is to get an explanation of the process of learning math: what happens to the struggling pupil and the more advanced pupil. From here, one can build a treatment process and plan accordingly."*

This quote indicates the direction that should be taken in working with teachers in this kind of course. Namely, teachers want to learn about relevant issues in NS, with all the difficulty this involves, and they intend to connect the new ideas to the reality of their teaching work. This finding strengthens a key idea: teachers can lend meaning to information that a PD course gives them in complex and significant ways by connecting it with their classroom experiences. By implication, it is worth our while to make teachers active partners in PD courses by having them develop learning materials on their own, thereby upgrading their professional development and expanding their knowledge about math teaching, by participating in the interaction with teacher-educators and research scholars. Importantly, however, despite the attempt to develop tools and strategies together with the teachers, applying such tools and their introduction into the teacher's everyday work requires time and further effort; one or two courses will not be sufficient.

Conclusion

In sum, our findings suggest that ANS instruction should focus on imparting relevant theoretical concepts and encouraging teachers to assimilate them into their teaching. It seems that there is no point in giving courses on NS or cognitive psychology to teachers as separate topics. Instead, it is recommended to link them directly to a subject matter as we did with mathematics and mathematics teaching. Courses should be designed to be delivered by two lecturers, one from the field of ANS and the other from the field of math teaching. In this manner, one lecturer would provide the background associated with the NS, while the other would look for and suggest ways of thinking about applying NS knowledge in the math classroom. The course on ANS can be seen as an example of how NS knowledge may be integrated into mathematics teaching school practices. This idea of bridging NS insights and mathematics teaching can be expanded to other topics, such as the employment of eye-tracking methodologies in research on algebra and geometry problem solving, that provided some insight into the mental processing involved (Obersteiner et.al, 2014; Muldner & Burleston, 2015; Schindler & Lilienthal, 2020). Another example of an Event-Related Potentials (ERP) research methodology was employed to examine behavioural and electrophysiological measures associated with insight-based and learning-based problem-solving (Waisman et al., 2014; Leikin et al., 2017). The above-presented study illustrates that it is necessary to know how learning processes occur in a student's brain to develop innovative and personalised approaches to teaching and learning.

It is important to note that the research has several limitations. First, because of the relatively small number of informants (19) and since the participants were all women teaching mathematics in elementary school, it is hardly possible to generalise the findings to the broad population of elementary mathematics teachers. Second, the data analysed above derive from the respondents' self-reports. In addition, all the data were obtained from the reflective reports collected at two time points and concise reports by the main lecturer. Additional studies are needed to address these limitations. As such, analysing additional data collected during such PD courses and exploring observations in a classroom may shed light on optimal PD course design.

We hope this study will serve as one of the initial attempts to bridge the gap between NS and classroom practices by adding NS content to teachers' PD, in the hope that this will lead to a much-needed change in the classroom. In addition, the PD course may contribute to improving mathematics education by exposing teachers to relevant aspects of NS theory and collaboratively designing innovative teaching practices.

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