The Quality of Argumentation in an Euclidean Geometry Context: A Case Study

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Abstract

The research reported in this study examined the quality of argumentation of South African Grade 11 learners through the lens of Toulmin's argument pattern (TAP). Very little research has quantified the argumentation of learners in mathematics across the school grades. The focus was on measuring the mathematical knowledge and quality of arguments formulated by learners as they engage in a reasoning task set in a Euclidean geometry investigative context. Mathematics education reform efforts have highlighted the importance of argumentation in the acquisition of mathematical knowledge. To describe these participants' quality of arguments, a sample of 135 Grade 11 learners was drawn from a target population of high schools located in one large South African province. Using an analytical framework modified from Osborne et al. (2004), the findings suggested that although learners' knowledge of properties of parallel lines was encouragingly satisfactory, the level of their argumentation quality was low. The implication of this finding is that mathematics initial teacher education programs need to design investigations that feature the TAP (core) in their courses. It is recommended that future studies may need to design intervention strategies to address high school learners' lack of argumentation skills.

Introduction

Recent mathematics curriculum reform statements have framed investigations as a key feature in the learning of mathematics in high schools (e.g., National Council of Teachers of Mathematics [NCTM], 2000; Department of Basic Education [DBE], 2011). These efforts are supported by research suggesting that conducting investigations in mathematics requires the formulation arguments (Jahnke, 2008; Rumsey & Langrall, 2016). In addition, current research in learning, teaching, and assessment has repeatedly pointed to the importance of using investigations to elicit learners' preconceptions (Ausubel, 2012; National Research Council [NRC], 1993). I argue that these approaches, which differ from the more dominant knowledge transmission, are appropriate as they seek to create classroom environments that resemble the practice of mathematicians; they engage in arguments during and after the proving activity.

Thus, argumentation enculturates learners into the disciplinary practices, a term used to "refer to the ways in which mathematicians go about their profession" (Rasmussen, Wawro, & Zandieh, 2015, p. 264). In short, arguments—over mathematical ideas and opinions—are a way to highlight the rational and scientific character of the mathematics discipline (Alcolea Banegas, 2003). Perhaps such approaches may help in addressing the decline in the number of learners obtaining a post-secondary science, technology, engineering, and mathematics (STEM) degree (Younes et al., 2020). Knowledge transmission relates to the notion that the "expert" (teacher) is required to fill learners' minds with information to be memorised and regurgitated when required (Thomas & Pedersen, 2003). In contrast, the methods advocated by curriculum reform efforts underscore investigations as a mathematical activity.

Returning to investigations in the context of argumentation, Schwarz and Linchevski (2007) assert that a natural approach that incorporates investigations entails fostering argumentation in classrooms. I follow Blair (2012) here in distinguishing between an "argument" and "argumentation" by pointing out that the former is a "set of one or more reasons for doing something" and the latter the "activity of making or giving arguments" (p. 72). Perhaps it is important to note from this distinction that an argument is the product of the process of argumentation. Further, argumentation is not used to refer to a debate, although that is one form of argumentation, but rather, to a process of thinking and dialogue in which learners construct and critique each other's arguments (Nussbaum, 2011). According to DeJarnette and González (2017), making and justifying a *claim*, defined in this study as a statement which an interlocutor makes to convince their audience, is a fundamental aspect of doing mathematics. However, the fact that argumentation is not explicitly stated in the *Curriculum and Assessment Policy Statement (*CAPS) (Department of Basic Education [DBE], 2011) document as a teaching and learning tool, is an indictment of the discipline that portrays itself as the epitome of reasoning. Ricks (2010) bemoans the character of school mathematics by pointing out that it deprives learners of the natural socialising appeal of mathematical activity.

Several studies on argumentation have focused on studying opportunities in mathematics classrooms focusing on identifying, creating, and evaluating argument structures (e.g., Aberdein, 2012; Mariotti, 2006; Pedemonte, 2007; Stylianides & Stylianides, 2017). The underlying theme of the findings of these studies is that the argumentation process enables the shifting of mathematical authority and ownership from the textbook or teacher, to the community of learners who become producers of mathematical understanding and knowledge (Bay-Williams, McGatha, Kobett, & Wray, 2013; Rumsey & Langrall, 2016). In addition, the power of argumentation is that it bears resemblance to how mathematical knowledge is constructed. Rumsey (2013) conducted a snapshot of learners' early attempts at argumentation when it is emphasised in classroom lessons. She found that learners consider all of the components of an argument when exposed to argumentation for a longer period of time.

According to Abdullah and Mohamed (2008), learners' inability to argue made it difficult for them to achieve the higher levels of geometric thinking as proposed by the van Hiele (1986) model. Justification of mathematical claims is the hallmark of the mathematics discipline. This viewpoint is also reflected in the widespread interest in the analysis of arguments in mathematics education literature (Inglis, Mejia-Ramos, & Simpson, 2007; Krummheuer, 2000). These statements suggest that learners must not only engage in argumentation practices but also make argumentation a habit of mind. There is a fairly vast body of literature focusing on analysing mathematical argumentation discourse through TAP (e.g., Aberdein, 2005, 2012; Knipping, 2003; Krummheuer, 1995; Mariotti, 2006; Pedemonte, 2007). Mathematics curriculum reform efforts have emphasised the importance of argumentation in the construction of mathematical knowledge in the classroom. Abroad, the *Standards for Mathematical Practices* (SMP) in the Common Core State Standards for Mathematics (CCSSM) prescribes that learners need to 'construct viable arguments and critique the reasoning of others' (Common Core State Standards Initiative [CCSSI], 2010, p. 6). Locally, the *Curriculum and Assessment Policy Statement* (CAPS) stipulates that learners in high school "should be exposed to mathematical experiences that give them many opportunities to develop their mathematical reasoning" (Department of Basic Education [DBE], 2011, p. 10).

Whereas some scholars, in their characterisation of classroom discussions, retained all six of TAP's components, others used its reduced versions. For instance, Aberdein (2005), Alcolea Banegas (2003), Forman, Larreamendy-Joerns, Stein, & Brown (1998), Fukawa-Connely and Silverman (2015), Inglis, Mejia-Ramos, and Simpson (2007), and Knipping (2003) used Toulmin's (2003) argumentation scheme in its entirety. In contrast, DeJarnette and González (2017), Evens and Houssart (2004), Krummheuer (1995), Pedemonte (2007), Weber and Alcock (2005) used Toulmin's scheme in which rebuttals and qualifiers were omitted. I contend that in relating the two approaches to TAP, the latter approach need not be seen as if it were a subordinate of the former; the determining factor is context.

Using a complete version of Toulmin's argument pattern (TAP), Knipping (2003) describes argumentation structures of learners engaged in a geometry context. She conducted a comparative study of French and German learners involved in proving discourses relating to the theorem of Pythagoras. In those lessons, teachers encouraged their learners to formulate conjectures which were interrogated by the class. She referred to refuted conjectures as objections rather than rebuttals. Unlike Pedemonte (2007) whose argumentation structures were devoid of rebuttals, in Knipping's (2003) study, refuting a conjecture received significant attention in the argumentation discourse. Knipping (2003) found that the learners in both countries were equally able to formulate arguments in all components of TAP and that the scheme helped in particularly singling out distinct arguments in proving discourses in ordinary classroom situations.

Fukawa-Connely and Silverman (2015) conducted online research with nearly 100 participants organised into 34 teams. These teams were working on tasks that progressed from learning how to use the GeoGebra tools to creating and making explicit claims about angle relationships and figures such as triangles and perpendicular bisectors, and other complex polygons. They explored how argumentation developed in an online environment that allowed small groups to synchronously create, manipulate, conjecture, and discuss dynamic geometry sketches. Demonstrating the efficacy of TAP, they provided a detailed analysis showing that learners made detailed and mathematical descriptions of their data and developed abstract warrants. However, inconsistent with other studies, the major finding of Fukawa-Connely and Silverman (2015) was that providing a warrant for claims was normative in the discussions.

In relation to gender differences, Healy and Hoyles (1998) conducted a study of mathematics classes in high schools across England and Wales to investigate, among other variables, factors shaping learners' argumentation in proof. They sought to explain these understanding by reference to a landscape of Level 1 variables (learner factors such as individual competency in arguments) as well as Level 2 variables (class, school, curriculum such as hours dedicated to mathematics per week, and teacher factors), using statistical and interview methods. Their finding, among others, was that learners' argumentation was shaped by their gender. However, in a study conducted by Blackwell, Trzesniewski, and Dweck (2007), it was found that gender differences in mathematics performance only existed among learners who held fixed rather than growth mindset about mathematical knowledge. By mindset, according to Dweck (2014), is meant assumptions and expectations individuals have for themselves and others that guide their behaviour and influence responses to daily events.

Considered as a whole, the studies cited above make a case for argumentation in mathematics classrooms. This paper takes a step toward addressing this scarcity by examining not only learners' ability to construct a mathematical argument but also the quality of their arguments. Put another way, this study contributes to the field by broadening our understanding of argumentation quality in high school classrooms. In addition, this paper needs to be seen as an attempt to develop a case for embedding argumentation in mathematics curriculum statements as a heuristic tool, at least. The findings in this study add to the research literature by describing the argumentation of high school learners tapping into learners' reasoning within the geometry context.

Aim and Research Question

Although argumentation is seen in mathematics education documents, and by researchers in mathematics alike, as vital for the learning of mathematics, there is very little attention given to the quantification of arguments in high school mathematics classrooms. The purpose of the present study was to bridge this gap in the literature on mathematical argumentation by quantifying and describing learners' written geometrical argumentation. More specifically, the following research question was posed: How is the quality of learners' argumentation as they engage in this task?

The rest of this paper is organised as follows. Having provided an overview of literature related to argumentation in mathematics classrooms, I describe the Toulmin's framework guiding this study. Next, I provide the methods and rationale for their choices. Then, I present the results and discuss the coding and categorising of data based on the framework of the study. On the basis of those findings, I end the paper with implications for classroom practice and make recommendations for future research in this area.

Theoretical Framework

In 1958 Stephen Toulmin presented a model, generally referred to as Toulmin's argument pattern (TAP), to describe the structure of an argument and how its components were related. TAP is a framework that has been extensively used in instructional practices as a tool to construct mathematically sound arguments (Osborne, Erduran, & Simon, 2004; Venville & Dawson, 2010). In the context of mathematics lessons, the use of TAP has mainly concentrated on the description of small group discussions among learners (see, for example, Knipping, 2003; Krummheuer, 1995, 2000; Pedemonte, 2007). For the present purpose, the assessment of learners' written argumentation was performed through a modified TAP structure.

The TAP model consists of six interdependent components: claim, data, warrant, backings, qualifiers, and rebuttals. In brief, the basic idea of this model is that a statement, claim or conclusion is justified by providing a ground (as shared by the mathematical community). For this study, "ground" refers to datum, warrant or backing provided by the interlocutor in justifying their claim. A w*arrant* is a proposition that connects datum and claim. Rebuttals are taken to mean statements that sought to show the weakness in a conclusion. However, not every one of these components is used in every argument. For instance, given the tentative nature of mathematical knowledge and the fact that for learners the knowledge being constructed is new, qualifying phrases such as "most probably" or "presumably" are omitted and therefore implied in a claim.

Whereas in this study, akin to that of Erduran, Simon and Osborne (2007), I encountered little difficulty in distinguishing claims from rebuttals, other studies, Toulmin (2003) himself points out that a statement that appears as a claim in one context may serve as a warrant in another. To circumvent this problem, Osborne and colleagues (2004) grouped the components of an argument into first and second orders. First-order components comprises claims, grounds, and rebuttals. Second-order components are constituted by components of the grounds; the data, warrants, and backings. In this study, the terms used were claim, ground, and rebuttal.

As a consequence, Figure 1 visually illustrates how the components of an argument are linked together in the structure of the modified TAP that underpins this study. The figure relates to a reasoning task involving a diagram relating to the theorem about the “sum of interior angles of a triangle add up to 180 degrees.” Paralinguistic cues such as “Given”, “Since”, “or “Because,” indicate specific components of an argument (González & Herbst, 2013). For instance, the rebuttal that “Unless lines lay on a spherical surface” indicates the circumstance under which the claim would be rebutted.

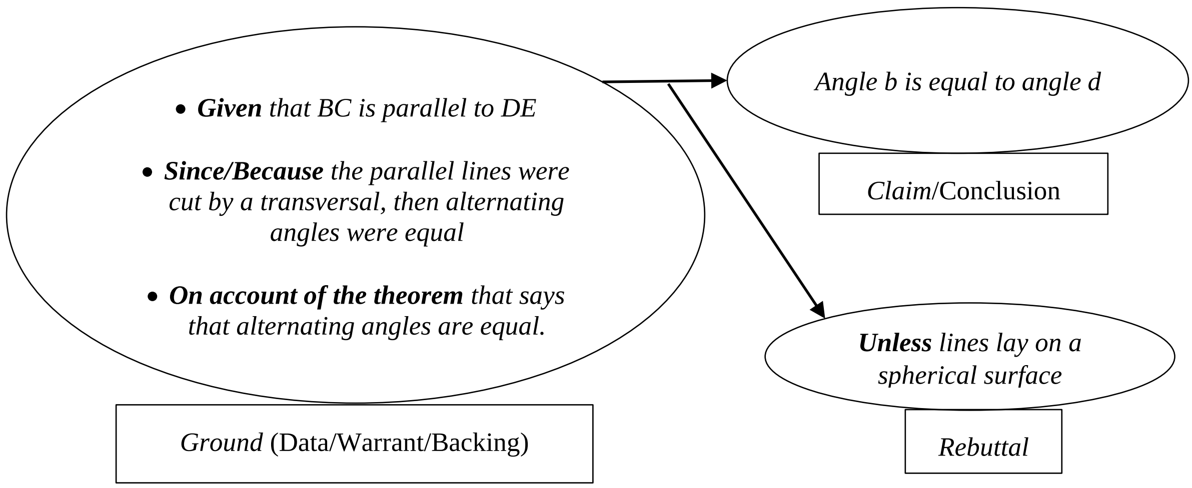


Figure 1: The modified argumentation model

Methods

The present study employed a descriptive design to characterise Grade 11 learners’ achievement and skill in a cognitive task embedded in argumentation at selected Dinaledi schools. To this end, a written argumentation frame for Euclidean geometry (AFEG) was employed to collect data. To analyse the data, a specific rubric comprising rating scales and descriptors to assess content and skill was designed and an existing tool adapted to respectively measure learners’ achievement and skill in the task.

Participants and context

Participants were 135 grade 11 learners in three classes at three randomly surveyed Dinaledi schools in a South African province, of which two were a subsample of extreme cases purposively selected to describe the distinction between a low-level argumentation and a high-level argumentation. These schools were part of a population of 10 Dinaledi School Project (Department of Basic Education [DBE], 2009) dedicated to increasing participation and performance in mathematics and physical sciences by historically disadvantaged learners, and were located in suburban, township, and rural areas spread across the province. The sample consisted of learners with low, medium and high socio-economic background, of which 57.8% were female, and with an age range of 17 to 20 years and a mean age of 17.4 years (SD=0.93), as shown in Table 1. The educational attainment of both parents was used as an indicator of learners’ socio-economic background. In each of the schools, all of these participants were studying mathematics, physical sciences, life orientation, and at least four other subjects, including two compulsory official South African languages at first- and second-language level.

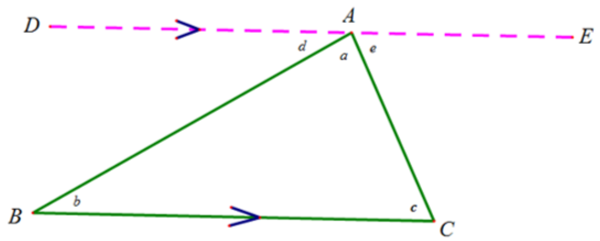
Table 1: Summary of demographic characteristics of the three schools and participants

School	Gender		Home Language		Level (location)	Total
	Female	Male	IsiZulu	English		
South End High	22	16	10	28	1 (suburban)	38
Zakheni High	29	21	46	4	2 (township)	50
Fundisani High	27	20	36	11	3 (rural)	47

Instrumentation

The data were collected in September 2017, by the author and a research assistant with a bachelor's degree in mathematics education. The relevant teachers also assisted in controlling the learners. The test was in English and the author and his research assistant administered the Test individually in one session in a relatively quiet classroom. It must be noted that Zakheni and Fundisani High consisted of very few learners whose home language was English. Written informed consent of parents was obtained before the start of the study for 96.2% of the learners. The study was approved by the Ethical Committee of the School of Education at a university in South-eastern South Africa (Protocol number 2/4/8/1126).

Specifically, argumentation data were drawn from the Argumentation Frame in Euclidean Geometry (AFEG) as shown in Appendix A (page 71). The AFEG instrument is a modification of Wray and Lewis' (1997) instrument. In the instrument, the mathematical statement that *The interior angles of a triangle sum up to* 180^0 was part of the Euclidean geometry content covered in Grade 11 mathematics. The answer to the research question was sourced from the AFEG instrument (Figure 2). In short, AFEG is a written survey tool designed to test and elicit learners' ideas embedded in argumentation. It is phrased in a generic way and thus not meant to refer to any specific theorem in the curriculum. However, the tool is important in that measurement of learners' success is through the assessment (e.g., homework, test, or examination) of their ability to recall and use the information learnt as they make connections between pieces of mathematical knowledge (Kulp-Brach, 2004). Thus, the aim of this AFEG instrument was to gain insight into the quality of learners' argumentation. Thus, participants had to conjecture from a labelled geometric figure.



In Δ ABC, DE is constructed such that it is parallel to BC.

Please, make ANY statement or claim from the diagram and justify it. Please, think carefully as you argue your points using the guide provided below.

1. My statement is that …………………………………………………………. (Claim)
2. My reason for making this statement is that ....…………………. (Ground)
3. Arguments against my idea might be that ……….. (Rebuttal)

Figure 2. The argumentation task frame (AFEG)

Given that constructing an argument in Toulmin's terms is not a simple task in the sense that learners require guidance and support for what constitutes a quality argument (Osborne, et al., 2004), I employed Wray and Lewis' (1997) notion of writing frames. The frame is, therefore, meant to support the process of argumentation as it provides vital support and clues as to what is needed in the absence of implicit instruction on argumentation as a learning and teaching tool. The task in which the participants engage may be used by teachers in designing pedagogical strategies to meet the individual needs of their learners, rather than spending their time marking assessments (Larrain, Navarro, Buraschi, Torres, & Muñoz, 2018).

Analysis

Responses in the AFEG instrument were coded independently by the author and an outside rater experienced in the teaching of high school geometry. In addition, efforts were made to ensure a coherent link between the task and the inferences drawn from participants' responses. Given that TAP merely focuses on the structure of an argument, the quality of an argument was judged through a revised version of Osborne, Erduran, and Simon's (2004), a discussion on the categories, coding, and scoring system of learners' data took place to develop a scoring tool. A participant's argument was labelled as "Low" if it was devoid of a rebuttal and "High" if it consisted of a rebuttal. Some examples for each of the codes are provided in Table 2.

Table 2. Definition and coding of argument components employed in the modified analytic framework

Argument	Definition	Code description	Quality
My statement is that …	A *claim* (C) is a conclusion – made on the basis of available data – put forward publicly for general acceptance (Toulmin, 2003).	No reply; uncodifiable; nonargumentative	Low
My *statement is* that …		C (Claim; conclusion)	Low
My reason is that …	A *warrant* is ground (G) provided in justifying the claim.	C+G (Providing reason for claim)	Low
Arguments against my idea might be that …	A *rebuttal* (R) meant statement that sought to diminish the strength of a conclusion (Pollock, 2001).	C+G+R (Refutation of claim/ground)	High

The scoring to a large extent, was guided by the principle that whether a mathematics assessment is an examination, test or only a single task, it should be evaluated against the educational principles of content, learning, and equity (National Research Council [NRC], 1993). The term "reliability" is used to indicate the degree to which the researchers tend to classify a given response into the same predefined category.

Results and Discussion

The present study investigated the quality of learners' written geometrical argumentation, using Osborne, Erduran, and Simon's (2004) analytical tool. The various instances in which components of TAP were used are labelled C, C+G, and C+CG+R to indicate a claim, claim with ground, and claim (which not only includes a ground but a rebuttal, exception or counter-example), respectively. The analysis of the learners' writing frames revealed several noteworthy findings. Although the data were analysed by two researchers, I used Cohen's (1968) kappa coefficient (κ) to determine the reliability of the analysis. In addition, this coefficient was appropriate to use on the basis that I adopted a multicategory rubric comprising an ordinal scale in which responses were classified into 1 of 5 types of categories. Cohen's interrater agreements (κ) were calculated for each of the five responses using *STATA*, a statistical software that enables analysis, management and graphical visualisation of data. The very few unanticipated responses received were fitted into the rubric such that the following kappa (κ) coefficients were obtained: content = 0.947 and argumentation = 0.971. As Altman (1991) suggested, these values indicated very good agreement between raters. A few examples of each category, Unclassifiable, Claim, Claim+Ground and Claim+Ground+Rebuttal from the responses gathered are provided in Table 3.

Table 3. Examples of learners' data

Category	**Examples**	
	Incorrect	**Correct**
Claim	$\angle b = \angle d$	$\angle a + \angle b + \angle c = 180^0$
Claim+Ground	$BC \parallel DE$	$\angle b = \angle d \quad ...BC \parallel DE$ and $\angle c = \angle e \quad ...BC \parallel DE$ *But,* $\angle a + \angle d + \angle e = 180^0$. $\therefore \angle a + \angle b + \angle c = 180^0$
Claim+Ground+Rebuttal	*Given*	*Only on a plane surface*
Unclassifiable	ΔABC *is acute angled*	

First, the majority of arguments emerging from the data was of a low-level quality (70%). Second, only a small minority (18%) of these arguments (claims, claims + grounds) included claims that were substantiated. Third, particularly discouraging was that only approximately 2% of arguments developed by learners were characterised as being of high quality because they consisted of rebuttals. What was important about these findings was that they provided deeper insights into learners' difficulties with constructing and sustaining a mathematical argument. The other notable feature of these results was that learners in School A (a fee-paying school located in a suburban area) provided the least number of arguments developed by its learners that could not be classified.

As shown in Figure 3, in each of the schools, the weak quality of arguments notwithstanding, female learners seemed to provide better arguments than their male counterparts. This result supports Healy and Hoyles' (1998) findings that gender plays a role in argumentation. In my view, the gender differences observed here might be accounted for by cultural influences rather than they being innate. In a study conducted by Blackwell, Trzesniewski, and Dweck (2007), it was found that gender differences in mathematics performance only existed among learners who held fixed rather than growth mindset about mathematical knowledge. The coded data as constructed by participants is summarised further in Figure 3.

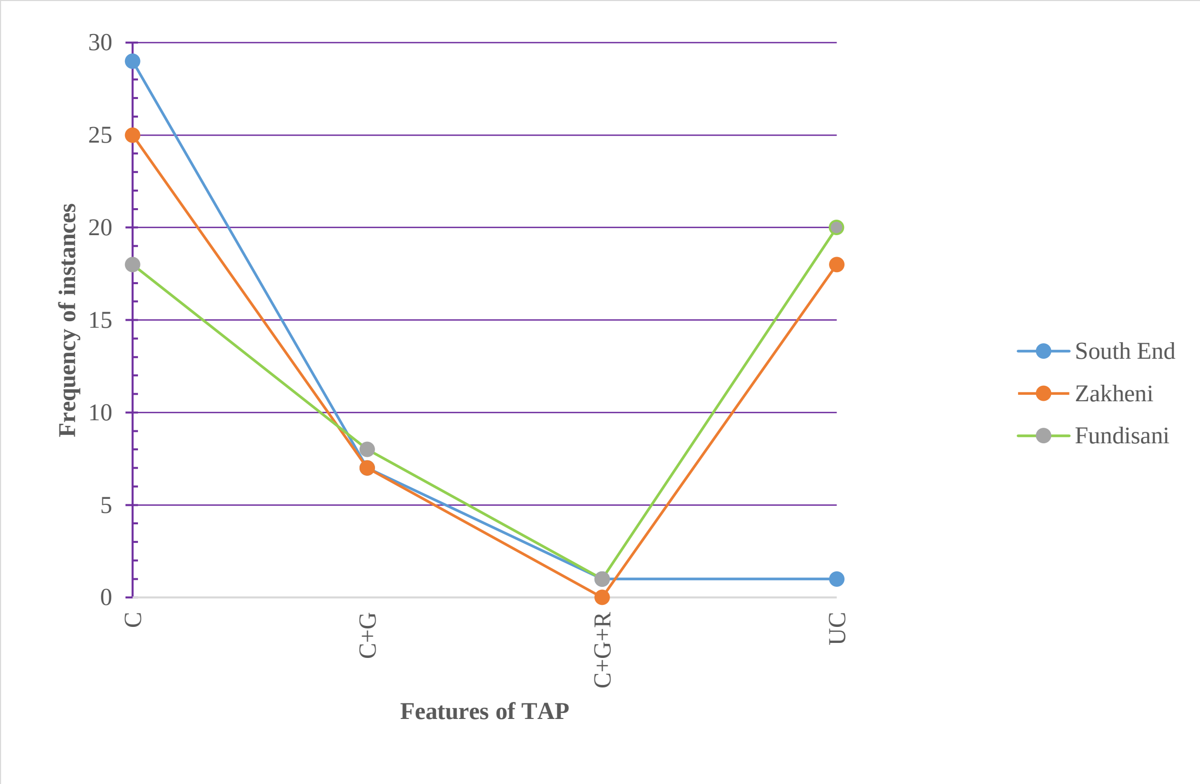
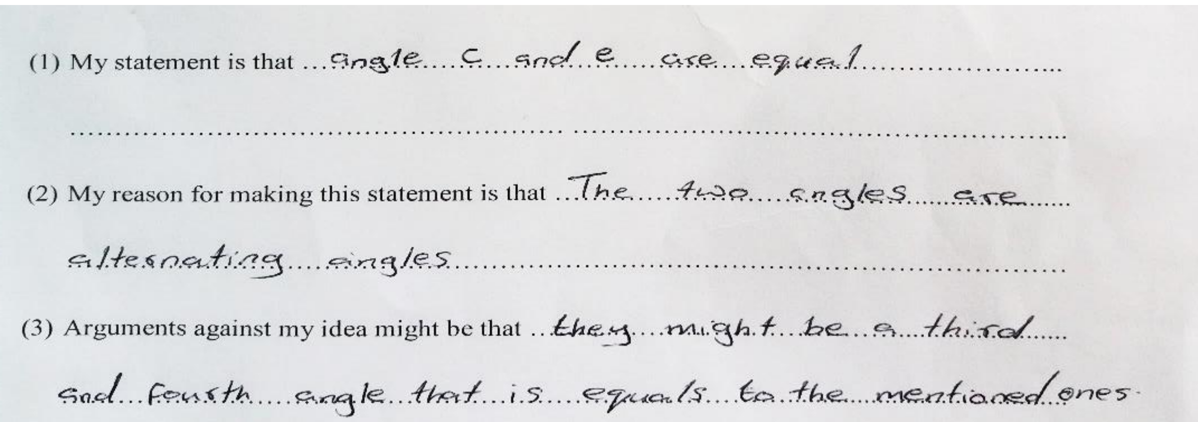


Figure 3: Distribution of TAP components across the three schools

Next, I present and discuss two sample participants' written argumentation frames, labelled Learner A and Learner B. The sample frames provided an example of application of the coding system adopted in this study. In the frames, Learner A's argumentation (Figure 4) was judged to be of low quality given that the statement provided for rebuttal did not constitute a rebuttal. Thus, Learner A provided a statement that could not be categorised as a condition under which his claim or ground cannot hold. He suggested that an argument against his claim that "angle c and e are equal" may be that "they might be a third and fourth angle that is equals to the mentioned ones". This statement seems to point to the learners' inability to understand the question; perhaps another example of language interference with learning, a phenomenon common among many in the sample for whom English is not their home language.



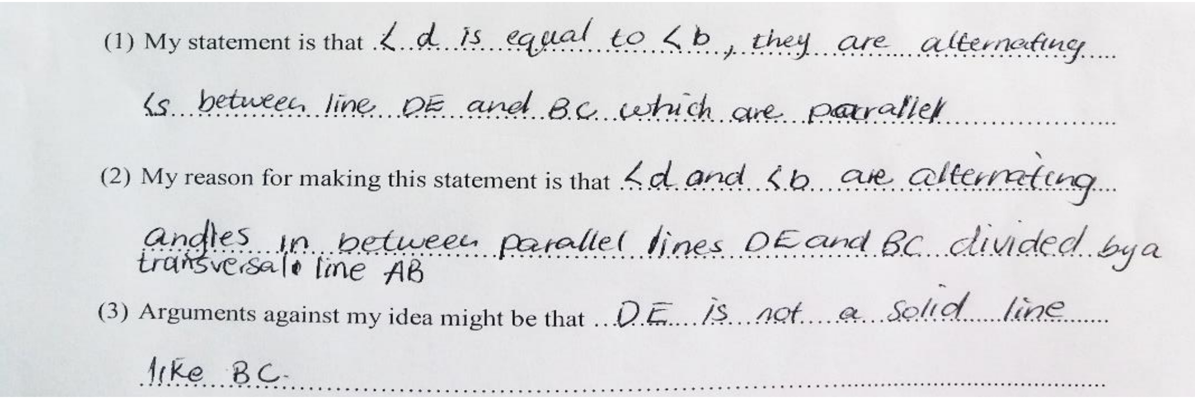
(1) My statement is that angle c and e are equal

(2) My reason for making this statement is that The two angles are alternating angles

(3) Arguments against my idea might be that they might be a third and fourth angle that is equals to the mentioned ones.

Figure 4: Learner A's argument

In contrast, Learner B's frame (Figure 5) represented a high-quality argumentation. In her rebuttal, she indicated that the claim would not hold if "DE is not a solid line like BC." Indeed this naïve observation might arise particularly from learners who demonstrated lack of understanding that the auxiliary line represents a construction for the purpose of proving the theorem.



(1) My statement is that ∠d is equal to ∠b, they are alternating ∠s between line DE and BC which are parallel

(2) My reason for making this statement is that ∠d and ∠b are alternating angles in between parallel lines DE and BC divided by a transversal line AB

(3) Arguments against my idea might be that DE is not a solid line like BC.

Figure 5. Learner B's argument

Both Learner A and Learner B used their prior knowledge to justify their statements both in terms of accuracy and language. On these two aspects of the content embedded in the task, they scored a total of 6 points thus reflecting a highly competent achievement in the task. The writing frames offered learners a place to demonstrate both their content knowledge and the quality of their argumentation.

The analysis of the learners' writing frames revealed several noteworthy findings. First, the majority of arguments emerging from the data was at a low level (70%). Second, though only a small minority, 18% of these arguments (claims, claims+grounds) included claims that were substantiated. Third, particularly discouraging was that only 2% of arguments developed by learners were characterised as being of high quality because they consisted of rebuttals. What was important about these findings was that it provided deeper insights into learners' difficulties with constructing and sustaining a mathematical argument. The other notable feature of these results was that learners in School A (a fee-paying school located in a suburban area) provided the least number of arguments that could not be classified.

The results in this study were similar to those obtained by Fukawa-Connely and Silverman (2015) in that the learners were not only able to make claims, they were also able to justify them as shown in the abstract warrants they had developed. However, the results were in contrast to Knipping's (2003), who found that learners were able to formulate arguments in all components of TAP, including providing rebuttals for their claims. The challenge for teachers lay in designing learning experiences and assessment tasks that take an *investigative approach* to "working mathematically". By extension, teachers needing to understand how to conduct instruction on mathematical argumentation in the classroom are faced with two challenges. First, the majority of teachers need to change their instructional practices in order to allow their learners to effectively learn argumentation as a skill essential for learning mathematically. Second, as a consequence of adopting an investigative approach to teaching mathematics, teachers may have to abandon some of the authority in the classroom (Osborne, Erduran, & Simon, 2004).

The study has several limitations. First, because the sample was selected from Dinaledi schools, the findings cannot be generalised to the larger population of high schools, and second was that I believe that an intervention programme should have been integrated into the study, funding permitting. Throughout the study, however, I attempted to devise a methodology that future research can use to quantify arguments in mathematics and thus report not only on statistically significant findings but also on their practical significance. It is hoped that such reporting can serve as a stimulus for placing argumentation at the heart of mathematics education. Finally, the results in this study may have been affected by the fact that Zakheni and Fundisani High consisted of very few learners whose home language was English.

These limitations notwithstanding, the recommendation emanating from these findings is that attempts need to be made to use mathematical investigations as a platform for explicitly teaching with a view to developing argumentation skills in learners. Future research on argumentation writing frames should also explore the use of open-ended interviews to probe learners' responses and thus provide insights into English language proficiency as a perceived barrier to constructing high quality arguments. Although gender differences were not a focus of this study, the emergence of the finding that females argued better than males, requires further investigation. Specifically, a *t*-test may be conducted to determine whether this difference is indeed statistically significant.

Conclusions

The purpose of this study was to investigate the quality of Grade 11 learners' written geometrical argumentation. The results revealed that argumentation is a centre of struggles for them; their arguments are of low quality; they lack rebuttals. The discussion highlighted the need to focus on explicit instruction on argumentation as a heuristic in mathematics classrooms given the scant attention to argumentation in mathematics classrooms. The finding that learners' argumentation ability is poor in the Euclidean geometry domain has implications for practice. Practicing teachers have a responsibility to create a classroom environment that supports argumentation, given that argumentation is a skill that learners cannot develop on their own; it must be taught explicitly. Further implication for this result is that mathematics initial teacher education programmes need to make investigations through TAP (core) a common feature in their methods courses.

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Appendix A

Argumentation Frame in Euclidean Geometry (AFEG)

This questionnaire consists of two Sections. The first one is intended to make analysis simple. The second section asks you to use your previous knowledge of Euclidean geometry axioms.

I am interested in the claim that you can make from the data in the diagram, below. This questionnaire is not part of your regular geometry activity and so it will NOT affect your marks. Your name will not be linked to your responses. Please, use the dotted lines to respond to each prompt.

SECTION A: Demographic information

Code: ……………….

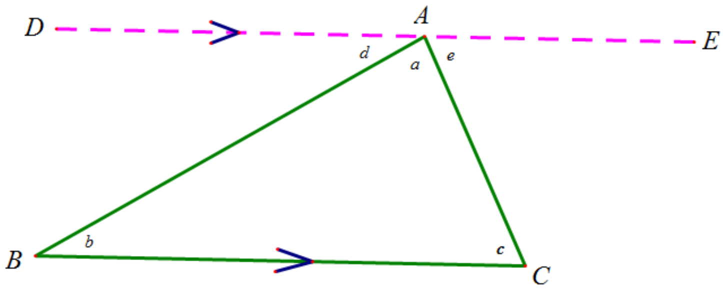
Please, circle/tick one answer for each of the following.

<table>
<tr><th colspan="5">Personal particulars</th></tr>
<tr><td>Gender:</td><td>Female</td><td>Male</td><td>Class (e.g. 11A)</td><td></td></tr>
<tr><td>Home language:</td><td>IsiZulu</td><td>English</td><td>Afrikaans</td><td>Other:</td></tr>
</table>

SECTION B: Geometry Task

Instructions

- *This questionnaire will **NOT** affect your marks. Please, do not spend a long time on any one statement – **your first thoughts are usually your best**.*
- *Write your responses on the spaces provided after the statement. Please respond to every statement – it's important that you respond to each statement honestly.*
- *All the information will be used for research purposes only. Your responses will be treated **confidentially**. Your responses will not reveal any information that could identify you.*
- *This survey should take you about **20 minutes** to complete.*



In the Figure 2, line DE is parallel to line BC on triangle ABC. Please, make ANY statement or claim from the diagram and justify it. Please, think carefully as you argue your points using the guide provided below.

1. My statement is that

   ……………………………………………………..…………………………….
   (Claim)
2. My reason for making this statement is that

   …………………………………......................................…………………
   (Ground)
3. Arguments against my idea might be that

   ……....…………………………………...………..
   (Rebuttal)

End. Thank you.