Evaluation of Changes in Cognitive Structures after the Learning Process in Mathematics

Sofia Maria Veríssimo Catarreira[a], Vítor Godinho Lopes[a], Luis Manuel Casas García[a] and Ricardo Luengo González[a]

Corresponding author: Sofia Verissimo (sofiaverissimo@gmail.com)
[a]University of Extremadura. CiberDidact Research Group, Badajoz, Spain

Keywords: mathematics concepts, cognitive structure, theory of nuclear concepts, Pathfinder Associative Networks, GOLUCA software.

International Journal of Innovation in Science and Mathematics Education, *25*(2), 17–33, 2017.

Abstract

This article focuses on the study of the changes produced in the cognitive structures of students due to the learning process using the Pathfinder Associative Networks technique and the GOLUCA software to graphically represent these structures. This way it is possible to identify the concepts that stand out and the relationships established between them, and thus assess whether the student establishes correct or incorrect relationships. The research involved 188 students aged between 14 and 16 years, attending the 9th grade in several schools in Portugal. First, a network was built in order to represent the structure of relationships between concepts that could be considered correct in a matter of geometry. Afterwards the cognitive structure of the students participating in the pretest in that area was obtained. Subsequently the participating teachers came to teach a unit on geometry. In the post-test, data were collected on the cognitive structure of students, reassessing relations between concepts, and checking what was right or wrong, and the modifications to the pretest. The results indicate that, using this technique it is possible to evaluate the student learning in detail, not only globally, but checking in detail the type of relationships established between the concepts.

Introduction

Learning and cognitive structure

It has always been an immense challenge to understand how learning takes place in the human mind and this has been the subject of major research over the years. Knowing how a student thinks, how a new process of learning takes place in his/her mind has always represented one of the greatest puzzles for the teacher.

According to current theories, the construction of knowledge is a dynamic process in which the action of the subject influences the learning process, which is done through the organized storage of information in memory. There are investigations that describe the structure of mathematical knowledge as a network of relations between properties, objects, and procedures (Papert, 1993; Wilkerson-Jerde & Wilensky, 2011).

The introduction of new knowledge involves the reorganization of cognitive structures, relating concepts with others already existing (Norman, Gentner, & Stevens, 1976). We can say that in the field of education, understanding how the whole process of integrating new knowledge into the cognitive structure of the student happens is fundamental for educational success.

The most important factor that influences an individual's learning is the knowledge that he/she has acquired, so he/she should be taught according to that same knowledge (Ausubel, Novak, & Hanesian, 1983). For several authors (Ausubel, 1963; 1968; Ausubel, Novak, & Hanesian, 1983; Casas & Luengo, 2004) the cognitive structure of the individual plays an important role in learning. The knowledge of the cognitive structure is an important point for the construction of knowledge itself. The study of graphical representations of mental structures might give us relevant information as shown by Van den Heuvel-Panhuizen and Van den Boogaard (2008).

In our opinion detailed knowledge of a student's cognitive structure on a particular content allows for a complete and targeted intervention by the teacher. Wilkerson-Jerde and Wilensky, (2011,) argued that:

> *Often, researchers interested in the flexibility and adaptive nature of mathematical understanding describe the structure of mathematical knowledge as a network of relations between different properties, objects, and procedures that come to bear on a given mathematical idea* (p. 24).

We think it is essential to combine the study of the cognitive structure with the students' learning process, which allows for a more complete view of the entire process and a more effective intervention by the teacher or by the students themselves, thus leading to better outcomes. The characterization of a student's knowledge about a particular content enables the identification of wrong connections in his/her cognitive structure and also helps to identify the absence of other connections that are considered essential in a particular field of knowledge.

This characterization of the knowledge of the connections established between concepts enables the development of targeted tasks that alter the cognitive structure, especially those that eliminate erroneous relations, strengthen those that are considered correct or identify those which are non-existent, but which are fundamental. Similarly, the identification of the most important concepts and established connections enables the creation of tasks that strengthen and consolidate their presence in the cognitive structure of students.

Acquisition and representation of the cognitive structure

There are several methods that allow us to obtain data on the cognitive structure of the subject, revealing the organization of knowledge (Casas & Luengo, 2002). For example, in the technique "establishment by the subject", the student establishes directly, and explains, the type of relationships between the concepts. The student can also be asked to select the concepts he/she considers more important, in a certain field of knowledge, as occurs in the case of the construction of concept maps (Liu, 1994; Enger, 1996; Ruiz-Primo & Shavelson, 1997; Ruiz-Primo, Schult, Li, & Shavelson, 2001; Lavigne, 2005; Shavelson, Ruiz-Primo, & Wiley, 2005; Muller, Sharma, & Reimann, 2008; Lindstrøm & Sharma, 2011).

Alternatively, the technique "similarity score between the concepts" assumes that the relationships between concepts in memory can be graphically represented. The representation is obtained from numerical values indicating the semantic distance between concepts for a given individual. Graphically, the semantic distance corresponds to the geometric distance between concepts: semantically close concepts are graphically represented as closer and vice-versa (Godinho, 2007). Generally the common procedure in these methods consists in, after selecting the concepts, asking the student to score the similarity or difference between all possible pairs of randomly shown concepts. The values obtained are transformed into coefficients in a scale from 0 to 1, from the farthest to the nearest, and in this range the higher the value, the smaller the distance between them. Secondly, the data are transformed into points of a space with minimum size, using statistical techniques, such as Pathfinder Associative Networks. This way, according to many researchers, these representations are valid to define the cognitive structure (Casas, Luengo, & Godinho, 2011; Geeslin & Shavelson, 1975; Jonassen, Beissner, & Yacci, 1993; Fenker, 1975; Preece, 1976; Wainer & Kaye, 1974).

Pathfinder Associative Networks are determined by applying the Pathfinder algorithm (Schvaneveldt, 1989; Casas & Luengo, 2002; Clariana, 2005). The representation of the cognitive structure using the Pathfinder method involves the use of algorithms for graphical representation of graphs, in which all the work of collection and graphical representation of the networks is facilitated by using specific software like KNOT (Schvaneveldt, 1989) or GOLUCA (Godinho, 2007).

Pathfinder Associative Networks are used in many fields of research, particularly in education, teacher training, in applications for the design and evaluation of educational hypermedia products, among others. Some of the studies are related to the validation of the technique itself and others integrate this technique with others by comparing the results of its applications (Jonassen et al., 1993; Gonzalvo, Cañas, & Bajo, 1994; Bajo, & Cañas,1994; McGaghie, 1996; Eckert 1997; DiCerbo, 2007; Clariana, Wallace, & Godshalk, 2009; Lau & Yuen, 2009, 2010; Trumpower & Sarwar, 2010; Chen, 1999; Moya et al., 2004; Guerrero-Bote, Zapico-Alonso, Espinosa-Calvo, Crisóstomo, & Moya, 2006; Zhang, 2008; Schvaneveldt, Beringer, & Lamonica, 2001).

Theory of Nuclear Concepts and teaching units

Rooted in the theories of Ausubel (1968), Novak (1998), Novak and Gowin (1984), the Theory of Nuclear Concepts (Casas & Luengo, 2004; 2013) presents the following assumptions:

- Knowledge is organized from small elements we call concepts. If we take for example a mathematical concept such as “circle”, we must understand this concept as a structure that includes the word, the sound of the word, experiences of round objects, experiences of generic circles in mathematics lessons, moving along the circular paths, stories in which circles are important, etc. Each concept in mind equates to a relatively stable structure, the cognitive structure, with the elements interrelated.
- The elements forming these structures follow a functional correspondence with the neuronal circuits of the human brain and a mental correspondence with the representations in the form of diagrams.
- Prior knowledge can be represented by these structures.
- Learning corresponds to the modification of the cognitive structure, by assimilation and restructuring (accommodation).

The Theory of Nuclear Concepts (TNC) proposes as key elements the “geographical organization of knowledge”, the “nuclear concepts” and the “least cost paths” (Casas & Luengo, 2004; Luengo, 2013). According to this theory, knowledge is formed in a process analogous to the process by which an individual acquires geographical knowledge, in which certain points of the landscape (nuclear concepts) stand out and from them, multiple routes are set up. Despite the numerous paths between these points, the individual will choose the one which, for various reasons, corresponds to that which is more meaningful for him/her, therefore the one with less energy costs, the so-called minimum cost paths.

It is pertinent to note that the term “concept” must be understood as a mental representation, it can be an abstract notion of an object or a general idea and/or understanding that a subject has. It should be noted, as indicated in Tall and Vinner (1981), referring to “concept image” and “concept definition”, that a full cognitive structure refers to a concept including both its definition as images and associations that are inherent to the individual thereto. Most concepts that exist in cognitive structure are not only associated with formal definition, but also with a variety of personal mental images when the individual evokes the concept, among which are the examples used by teachers in teaching, as proposed below.

According to the TNC the most important concepts (nuclear concepts) in the student cognitive structure are not necessarily the most general, but those with a greater number of connections. These nodes can be comprehensive concepts, or simply associations that the student performs the theme, such as everyday objects, relationships, images and not properly mathematical concepts.

This theory and its associated technique, Pathfinder Associative Networks, identifies the most important concepts in the cognitive structure of students and the relationships between them, and thus allows to create learning sequences departing from these concepts, reinforcing the right connections and identifying and modifying the incorrect ones. That theory also proposes a new way of developing teaching units (Veríssimo, 2013).The aim of a Teaching Unit based on the assumptions of the TNC is to provide students with tasks that serve to reinforce the right relationships, create those that were not set or delete the incorrect ones and alter the students’ cognitive structures in an attempt to approximate their cognitive structure to the one considered correct by the teacher.

While in traditional teaching units mathematical concepts are presented in order to correspond to the course syllabus, and follow the structure from more general to more specific mathematical concepts, in our proposal, the order of presentation of activities is determined by the more or less important character of these concepts in the cognitive structure of students. In the development of this Teaching Unit the contents that are intended to be conveyed are taken into account and, simultaneously, tasks aiming to change the connections established between the concepts are drafted or selected, since they may be regarded as correct or incorrect. Thus, depending on the categorization of each connection, tasks with different objectives are developed, that is, if the connection is considered correct, the tasks proposed aim at strengthening it, and if it is found to be incorrect, the tasks intend its elimination.

Objectives and Research Questions

The main objective of our study was to investigate the changes produced in the cognitive structure of learners after the learning process, by examining whether changes do occur in the number and quality of connections established between concepts, namely reduction in relationships that are considered incorrect, and enhancement of those that are considered correct, through the learning of a Teaching Unit whose theme was: "Circumference, central angle and inscribed angle".

According to this objective, the following research questions were raised:

- Can we identify associations between concepts that are not present in the cognitive structure of students, but that are important?
- Can we identify incorrect associations between concepts in the cognitive structure of students?
- Can we, through a Teaching Unit based on the assumptions of the TNC, correct these wrong associations?
- Can we, through a Teaching Unit based on the assumptions of the TNC, reinforce correct associations between concepts?

Method

Considering the theme of Geometry "Circumference, central angle and inscribed angle", initially, a prior study allowed us to identify the nuclear concepts, the starting point for the elaboration of a Teaching Unit. It was necessary to determine which concepts stood out; for this we performed an exhaustive literature review, both in terms of research and education (textbooks) and interviews with several experienced teachers were held. This preliminary study allowed us also to know the cognitive structure of a group of students who already knew the mathematical concepts involved and the identification of the most important concepts and their relationships, which were categorized as correct, incorrect or not well established. After this a Teaching Unit was developed based on the Theory of Nuclear Concepts described above.

For the main study, we adopted a research design divided into three main stages: data collection (pretest); application of the mathematics unit, and data collection (postest) as described in Figure 1.

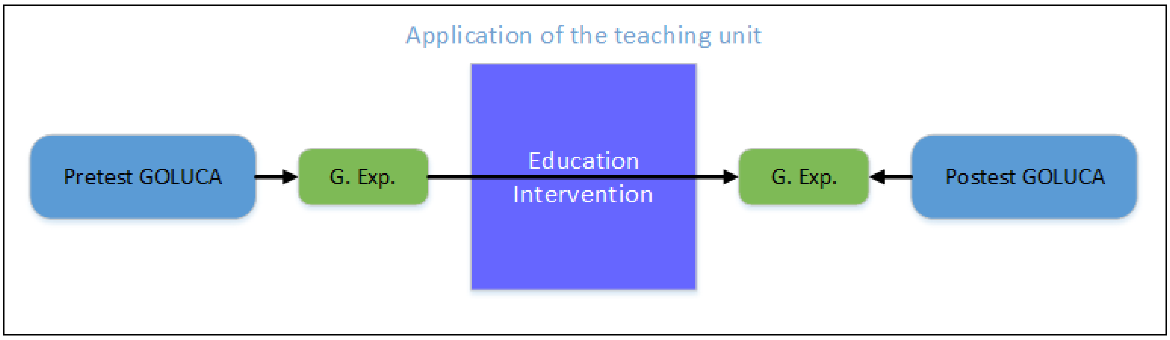

Figure 1: Diagram of the educational intervention

In the pretest, the students were subjected to a test using GOLUCA software, collecting the data as described below, and thus we were able to know their cognitive structures related to the topic. Following this, the implementation of the Teaching Unit took place, being the knowledge transmitted, and tasks performed, such as exercises, problems or research tasks, among others. Prior to the implementation stage of the teaching unit, the teachers involved in the research were given the teaching and scientific materials needed and the respective planning based on the principles of the Theory of Nuclear Concepts, in order to give all students equal learning conditions. The exercises and examples used in both didactic units were similar, changing only the order in which they were presented, and insisting on those intended to reinforce the correct connections between concepts or eliminate the incorrect ones, as described above. Several meetings took place between the researcher and the teachers, in order to assess implementation criteria in the different learning environments, since the several students involved in the investigation came from different schools and were taught by different teachers. Finally, the postest took place, wherein again all the students were subjected to a GOLUCA postest, thereby allowing the comparison with the results obtained in the initial phase.

Study sample

The prior study of our research involved 76 students attending the 9th grade (14-16 years old) and 6 experienced teachers. In the main study, eight schools of Alto Alentejo in Portugal participated and 188 students and 16 teachers were involved. The study took place in the 9th grade and the age of the individuals involved ranged between 14 and 16 years.

The collection of our sample was made up by geographical convenience after a first contact with several schools that had classes in the level of education that our research referred to. Later, with the schools that agreed to cooperate with us, a random sample was chosen from the several classes that each school had.

Data collection

For collecting the data that enabled the representation of students' cognitive structures the GOLUCA software was used, and this allows the collection, graphical representation and analysis of cognitive structures in the form of Pathfinder Associative Networks. This software is detailed in Godinho (2007).

The collection of data on GOLUCA software starts with the identification of the student and, when he/she is ready, he/she is presented with a proper interface, in which the student will evaluate the similarity between concepts. During data collection, the GOLUCA software presents a pair of concepts and a bar of weights (Figure 2). The program displays all possible combinations of pairs of concepts, selecting them randomly. The bar of weights corresponds to the values that can be assigned as weight between two concepts.

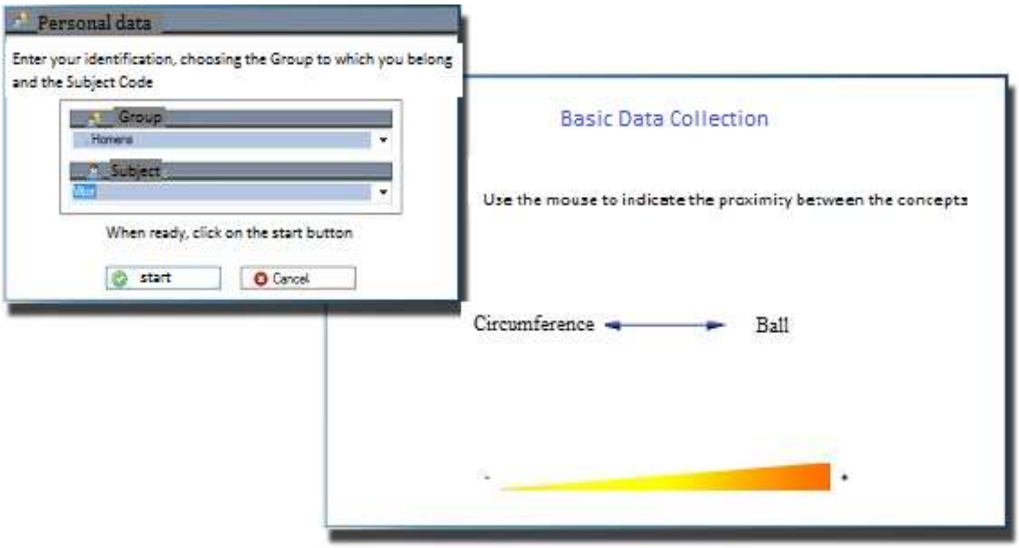

Figure 2: Interface of evaluation of the similarity between concepts

After collecting the data, the GOLUCA software stores them in the form of a similarity matrix for further analysis and graphical representation in the form of Pathfinder Associative Networks (Figure 3).

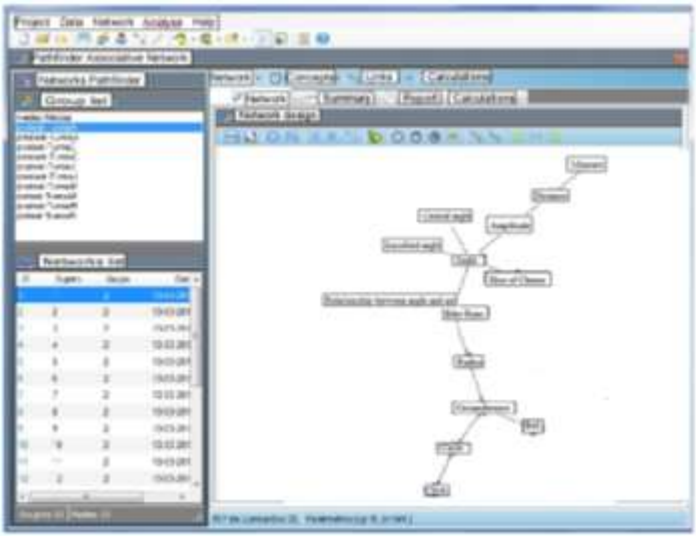

Figure 3: Interface of representation and analysis of Pathfinder Associative networks

Results

Results of the prior study

As a result of the prior study, the concepts considered most important were some like circle, radius, circumference, angle or inscribed angle. But for many students, concepts are represented by simple examples, such as ball, bicycle rims or slice of cheese (respectively representatives of concepts such as sphere, circumference or circular sector) occupying a prominent place in their cognitive structure.

Finally, we selected the following concepts to apply in the test of the GOLUCA software: circumference; circle; radius; ball; clock; bike rims; slice of cheese; relationship between angle and arc; angle; inscribed angle; central angle; amplitude; distance; and, measure. These so called concepts refer to the idea of "concept image" and "concept definition" (Tall & Vinner, 1981) and, as it can be seen, include not only abstract mathematical concepts, but also real objects or generic examples (ball, clock, bike rims or slice of cheese, that represent respectively spheres, angles, circumferences or circular sectors) as images used in instruction.

Using the GOLUCA software, we attained an average network of 76 students participating in the prior study, which is presented below:

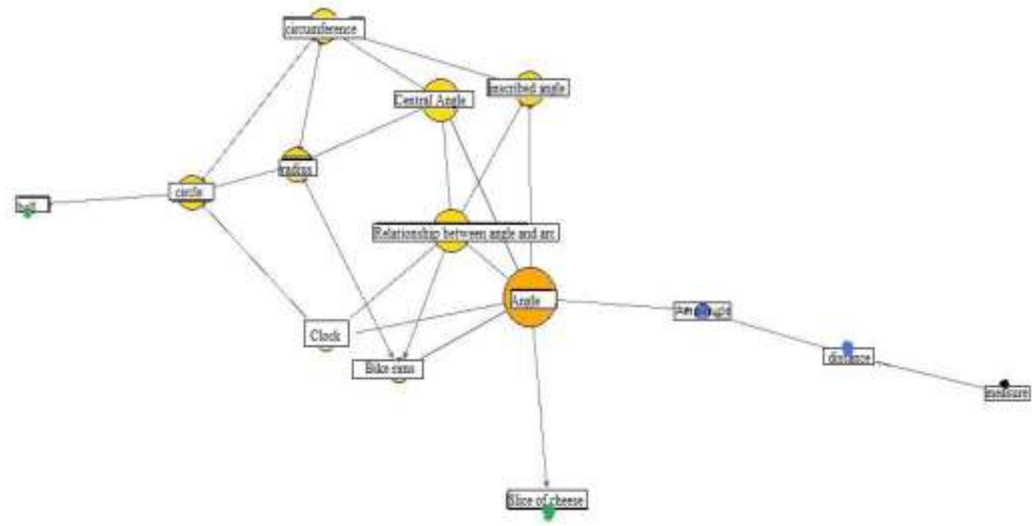

Figure 4: Average network of the prior study

As it can be seen, among these concepts, students establish relationships, which in some cases are correct and incorrect in others. As a sign of bad relationships we can consider, for example, the relationship between ball and circumference as students, in some cases, even confused the concepts of sphere and circumference.

The connections "circumference-inscribed angle", "circumference-central angle", "radius-central angle" were considered correct, because it was considered that there is a relation between the concepts, and its existence is critical to the understanding of the contents. By teaching the contents, we sought to reinforce the connection or to create it, in case it existed or was nonexistent in the associative network obtained by the student in the pretest, respectively.

The "circumference-ball" relationship was found to be incorrect, because many students confuse sphere with circumference, and think of the circumference as a three-dimensional concept. Thus, it was intended to reduce the incidence of this connection in students' cognitive structures, and ultimately, to eliminate it. Moreover, there are relationships between concepts that are not well established by the students, such as the relationship between circumference and inscribed angle.

For this study we selected a few connections as well as the associated objective of each one, namely:

Table 1: Connections in this study

Connection	Objective
Connection “circumference-inscribed angle”	Reinforce/Create
Connection “circumference-central angle”	Reinforce/Create
Connection “circumference-ball”	Eliminate
Connection “angle to radius-central angle”	Reinforce/Create

Results of the definitive pretest-postest study

To make the final study, the networks of 188 students participating were obtained before and after the application of the Mathematics Teaching Unit. The following figures present the average networks in the group before and after the experience.

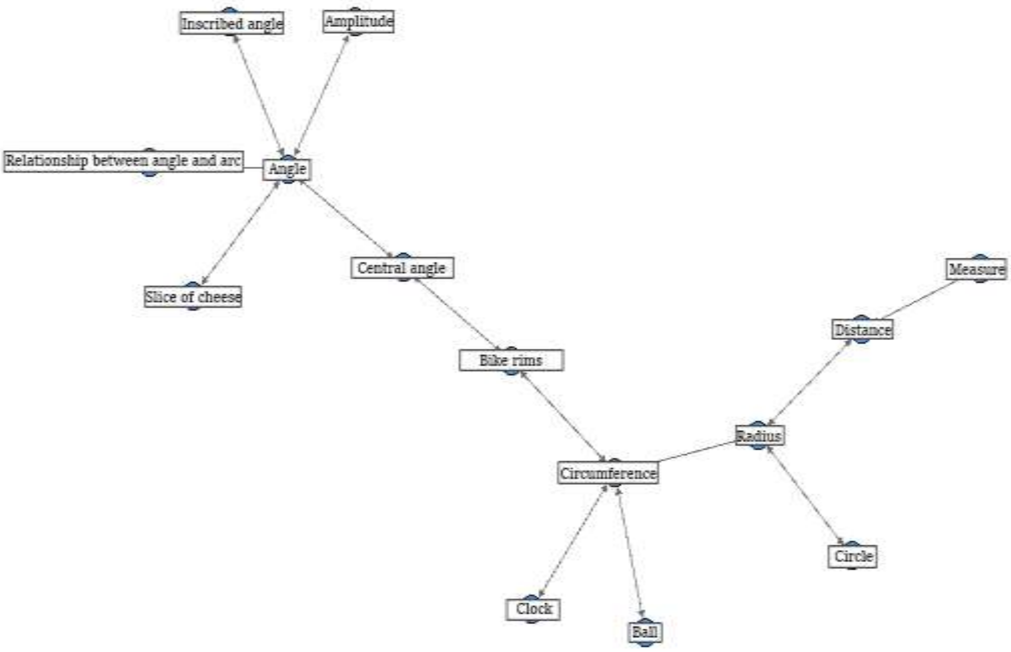

Figure 5: Pretest study average network

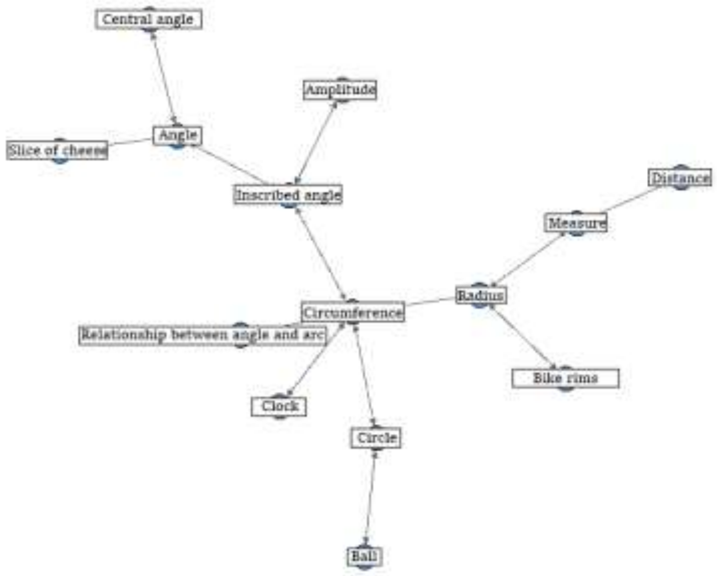

Figure 6: Postest study average network

For analysis, however, we will not use these average networks, but was made from the individual data of each student, as shown in Figures 7 and 8.

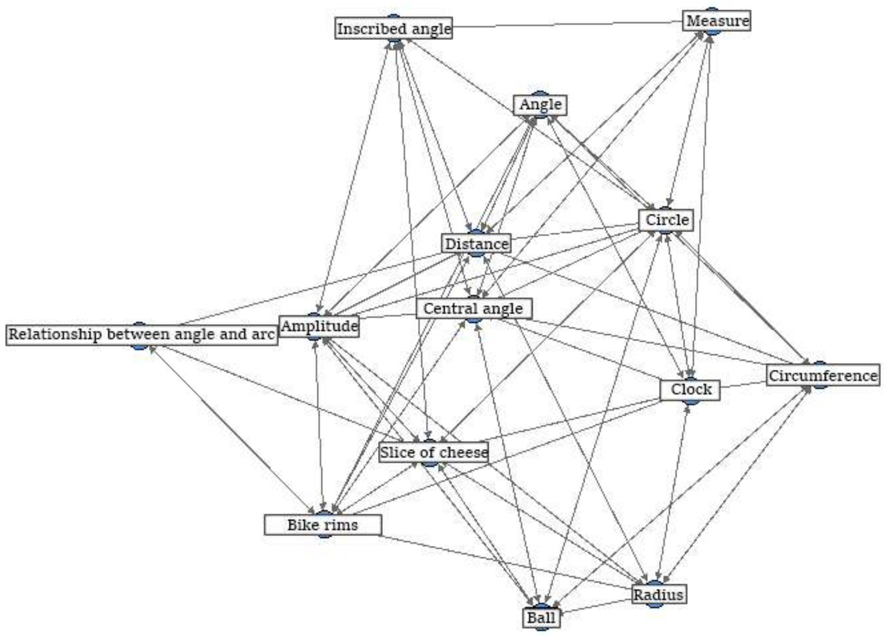

Figure 7: Pretest AR17 subject network

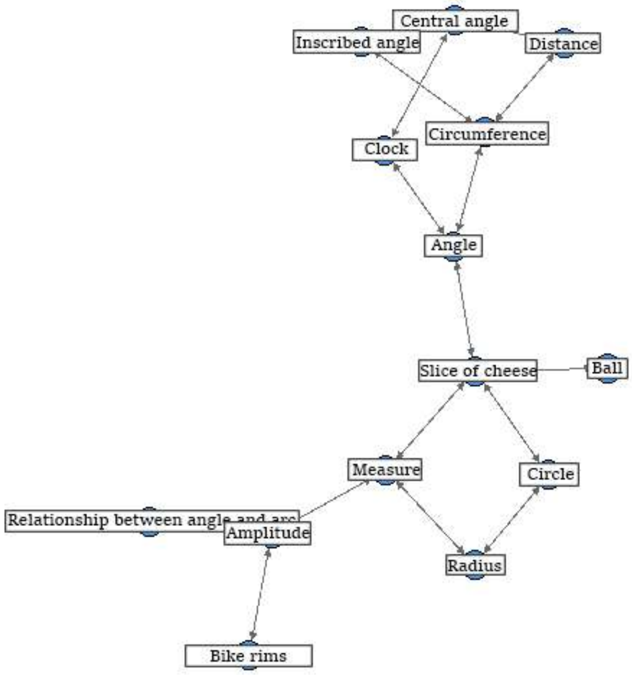

Figure 8: Postest AR17 subject network

Hereupon we present the results obtained in the Pathfinder Associative Networks in the students in the definitive study. The analysis focuses only on the connections that we highlighted in Table 1.

Analysis of the Connection "Circumference-Inscribed Angle"

In the analysis of this connection, we observed an increased number of connections established between the concepts "Circumference" and "Inscribed Angle" (Table 2) in the Pathfinder Associative networks from the pretest to the postest. At the percentage level it is observed that in the pretest 10.64% of the individuals present the connection, and in the postest there is an incidence of 25.53%.

Table 2: Average number of connections between "inscribed angle" and "circumference"

Connection "inscribed angle-circumference"	Mean	N	Standard deviation	Standard error of the mean
Pretest	,1064	188	,30915	,02255
Postest	,2553	188	,43720	,03189

To study the statistical significance of the results, we used the McNemar Test. The McNemar Test, also called the "change of opinion" test, is used to decide whether or not a given treatment induces a change in a dichotomous response, usually of the yes / no type. In this study, it tries to determine if there have been changes in the number of correct or incorrect links between concepts, from pretest to postest. Regarding the McNemar Test (Table 3) it appears that there was an increased connection between "Inscribed Angle" and "Circumference", specifically it appears that 13 individuals presented the connection in the pretest and 41 began to present it in the postest. Thus, an increase of the existence of this connection occurred in 28 individuals, being a statistically significant increase (p=0.000). In the following table we can see that 127 students do not present the connection and 7 students presented the connection in the two tests.

Table 3: McNemar Test of the connection "Inscribed Angle-Circumference"

	Postest	
Pretest	No relation	Relation
No relation	127	41
Relation	13	7
Chi-cuadrado(a)	13,500	
Asymptotic significance	,000	

a Corrected by continuity

Analysis of the connection "circumference – central angle"

In the analysis of the results there was an increase in the existing connections "Circumference" and "Central Angle" (Table 4). In the pretest an incidence of 17.55 % has been found and in the postest it was of 26.60%.

Table 4: Average number of connections between "central angle" and "circumference"

Connection "central angle-circumference"	Mean	N	Standard deviation	Standard error of the mean
Pretest	,1755	188	,38144	,02782
Postest	,2660	188	,44302	,03231

Using the McNemar Test (Table 5) we observed a significant increase in the number of connections from the pretest to the postest. As it can be seen in the following table, there was an increase in the number of connections between "Central Angle" and "Circumference", more specifically it was found that 20 individuals presented the connection in the pretest and 37 started presenting it in the postest. An increase of the existence of the connection occurred in 17 individuals, which is a statistically significant increase (p = 0.034).

Table 5: McNemar Test of the connection "central angle-circumference"

Pretest	Postest	
	No relation	Relation
No relation	118	37
Relation	20	13
Chi-cuadrado(a)	4,491	
Significância asintótica	,034	

a Corrected by continuity

Analysis of the connection "circumference-ball"

In the beginning of the research the existence of this connection was identified in the cognitive structures of some students, although it is considered erroneous. After the analysis of the Pathfinder Associative networks, there was a decrease in the number of connections between “circumference” and “ball” (Table 6). At the percentage level an incidence of 30.32% was found in the pretest and of 21.28% in the postest.

Table 6: Average number of connections between “circumference” and “ball”

Connection “circumference-ball”	Mean	N	Standard deviation	Standard error of the mean
Pretest	,3032	188	,46086	,03361
Postest	,2128	188	,41036	,02993

By performing the McNemar Test (Table 7) we found a decrease in the number of connections established between “circumference” and “ball” from the pretest to the postest. It has been observed that in the pretest 44 individuals presented the connection and in the postest only 27 started to present it. A decrease of the existence of the connection occurred in 17 individuals, which is not statistically significant (p = 0.058).

Table 7: McNemar Test of the connection between “circumference” and “ball”

	Postest	
Pretest	No relation	Relation
No relation	104	27
Relation	44	13
Chi-cuadrado(a)	3,606	
Significância asintótica	,058	

a Corrected by continuity

Analysis of the Connection between “radius” and “central angle”

In the investigation an increase in the number of connections between “radius” and “central angle” has been observed (Table 8). The pretest showed an incidence of 14.36% and the postest one of 20.74%.

Table 8: Average number of connections between “radius” and “central angle”

Connection “central angle-radius”	Mean	N	Standard deviation	Standard error of the mean
Pretest	,1436	188	,35164	,02565
Postest	,2074	188	,40656	,02965

In the McNemar Test (Table 9) we observe an increase in the number of connections from the pretest to the postest between “Radius” and “Central angle”, with 21 subjects presenting the connection in the pretest and 33 who began to present it in the postest. With an increase in 12 individuals, this is however not statistically significant (p = 0.134).

Table 9: McNemar Test of the connection between "Radius" and "Central Angle"

	Postest	
Pretest	No relation	Relation
No relation	128	33
Relation	21	6
Chi-cuadrado(a)	2,241	
Significância asintótica	,134	

a Corrected by continuity

Discussion

According to the objectives presented at the beginning of this study and according to the results obtained and the analysis of relations established between the concepts described in the previous section, we have confirmed that there were significant differences between the students' initial and final cognitive structures.

Since geometry is one of the subjects in which students obtain worse results than for other subjects, it is fundamental to identify the relationships that students establish between concepts. Through knowledge of these relationships it is possible to eliminate those that are wrong and to reinforce or create those that are essential and thus to carry out a more efficient educational intervention.

Our research confirmed that the learning process caused a decrease in the erroneous connections and a strengthening of the connections considered correct in the students' cognitive structure. Similarly, it was possible to demonstrate how the methodology used for data collection makes it possible to gather information referring to a large number of quantitative variables from a large sample in a practical and quick way. On the other hand, it allows us to study in detail the evolution of the cognitive structure of the individuals regarding the number and quality of connections shown. This allows us to evaluate very specific details of the students' learning process, beyond the commonly used methods.

This methodology also allows us to analyze the data per subject, comparing one subject with the others, or form groups of subjects and compare them. There are several studies which used the same methodology, (namely Casas, 2002; Casas & Luengo, 2004; Antunes, 2011; Veríssimo, 2013; Carvalho, Ramos, Casas, & Luengo, 2010; Veríssimo, Casas, Luengo, & Godinho, 2011; Veríssimo, Godinho, Casas, & Luengo, 2012; Godinho, Veríssimo, Casas, & Luengo, 2012; Casas, Godinho, Luengo, Veríssimo, & Carvalho, 2013; Carvalho, Luengo, Casas, & Mendoza, 2012; Almeida, 2014), in which we can observe the use of the Pathfinder Associative Networks to analyze various aspects of students' cognitive structures and their changes with the learning process. In most of these studies comparisons of Pathfinder Associative Networks in a pretest with a postest are performed.

These studies, when compared to others (Liu, 1994; Enger, 1996; Ruiz-Primo & Shavelson, 1997; Ruiz-Primo et al., 2001; Lavigne, 2005; Shavelson et al., 2005) which use more qualitative methods or focus on case studies and are more “invasive” to the subjects, have the advantage of allowing us to obtain numerical data and charts from a large group of subjects.

Therefore, this study makes an important contribution to educational research, allowing the use of large samples, unlike what is usual in qualitative studies, which have been characterized by providing very detailed information on participants, but using small samples.

Finally, with regard to the applications in the teaching-learning processes, by using a methodology for teaching a didactic sequence based on the assumptions of the TNC, the teacher is no longer focused only on the transmission of knowledge, but combines the concern to improve the students' own cognitive structure. This research fosters innovation in the teaching-learning process, since it changes the usual view of its dynamics, changes the way the teaching units are designed, the students’ role and changes the educational intervention itself, enabling targeted activities designed to promote the creation of meaningful and correct connections in certain mathematical concepts of interest.

Acknowledgments

This work was supported by the European Union and Junta de Extremadura under Grant FEDER (European Regional Development Fund) "Una manera de hacer Europa" GR10061.

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