

Exploring Electrical Resistance Shape Dependence through Inquiry-Based Learning: A Carbon Nanotube Approach

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Abstract

Objective: We design and practice lessons to imagine and understand the shape dependence of electrical resistance in two dimensions.

Methodology: A carbon nanotube paper resistor is the most suitable teaching material for these lessons because it can handle measured shapes and resistance values quantitatively, and experiments with reproducibility are possible. We designed a one-hundred-minute lesson based on the 5E instructional model, which strongly correlates with inquiry-based learning. These designed lessons were practiced in two junior high schools. The analysis of post-test average scores for experimental and control groups used the Mann-Whitney U test.

Results: The students were engaged in hands-on experiments and elucidated the two-dimensional shape dependence of electrical resistance by experiments using resistors adjusted by themselves with creative thinking.

Conclusion: The effects were confirmed in the degree of consent for combined resistance and the evolution problem of parallel circuits. We clarified that "Authentic learning", which has a deep understanding using knowledge, was realized. "Authentic learning" is one of the policies of the current Course of Study 2017 by the Ministry of Education Japan (Ministry of Education, Culture, Sports, Science and Technology Japan, 2019). It is an innovation for physics education to understand electrical resistance with an image of dependence on its two-dimensional shape.

Introduction

In response to the COVID-19 epidemic, the use of online educational content is booming (e.g.,

Jose, Kochandra, & Daniel, 2021). Although student performance may show similar learning effects in online learning experiences compared with previous face-to-face lab activities, students strongly agree that the online learning experience had significant limitations when compared to the live activities (Klein, et al., 2021). According to Hofstein & Lunetta (2004), the laboratory has been given a central and distinctive role in science education, and science educators have suggested that rich benefits in learning accrue from using laboratory activities. Hands-on lab activities are essential for science education. In particular, it plays an important role in understanding scientific theories with geometric images. The property that electrical resistance value changes depending on a shape of a resistance element (linear with respect to its length and inversely proportional to its cross-sectional area) is a good example.

Previous research has focused on resistors that are closely related to daily life and whose shape can be easily changed. For example, some science education studies use pencil-drawn resistors (Woolf & Streckert 1996; Derman & Goykadosh 1999) and mechanical pencil leads (Chiaverina, 2014; Küçüközer, 2015) as teaching materials. Not only do they facilitate experimentation, but they also help students to understand resistor shapes and the type of connections (Kamata & Abe, 2012). This is because the resistor drawn in pencil can be considered a pseudo-2D image, whereas the actual shape of the resistor is 3D. In particular, a carbon nanotube paper resistor is argued to have a quantifiable measure of its length and is highly reproducible in shape (Shintsuruta, Okubo, & Iwayama, 2021).

In Japan, the shape dependence of electrical resistance is learned in senior high schools. There is no set level for each grade, but it is included in the content of "fundamental physics" taken in the first or second year at many schools. On the other hand, in junior high schools, there is no formal explanation, and combined resistance is explained only with 3D images. It is difficult for many students to understand (Shintsuruta, Okubo, & Iwayama, 2022). Presenting a 2D image with carbon nanotube paper resistors might help students to understand shape dependence and also to understand combined resistance.

Our research question is "What is the learning effect obtained by treating the shape dependence of electrical resistance as a pseudo-two-dimensional model?" The aim of this study is to design new lessons and practice them to clarify their learning effect in junior high school science education in Japan. The learning effects to be investigated include knowledge and understanding of electrical resistance, calculation skills, degree of consent, and ability to apply knowledge and think.

Teaching material for understanding 2D shape dependence

Carbon nanotube paper resistors

Carbon nanotube paper is impregnated carbon nanotube. Figure 1 shows a piece of carbon nanotube paper and the resistors that use it with different widths. Carbon nanotube paper has excellent electrical conductivity (T T Trading Co., Ltd, 2009). The surface resistivity of a typical carbon nanotube paper is about $100 \Omega/\text{sq}$. Surface resistivity is a measure of the resistance of thin films that are uniform in thickness. It is defined as the electrical resistance of a sheet-like resistance element per unit area. Surface resistivity is a physical quantity with the same dimension as resistance, so in this paper, the unit is written as Ω/sq according to the convention.



Figure 1. Carbon nanotube paper and the resistors that use it with different widths

We can explain some advantages of using this experiment instead of others in the textbooks. The first is that it is possible to cut a piece of paper for students. Hands-on activities that make the most of the students' creativity are integrated into the existing science education content. The second is that the shape of the resistance element can be captured as a two-dimensional image. By quantitatively specifying the length and width of the resistive element, it is possible to perform experiments that ensure reproducibility. Moreover, it becomes possible for students to experiment in a quantitative and reproducible way in the classroom. The third is that it also becomes possible for students to learn about electrical resistance visually and actively through conducting experiments by themselves and undertaking discussions with other students, rather than through the traditional teaching methods: which are often characterized as teachers delivering information to students in lectures and readings practiced in many schools worldwide (Banilower et al., 2010). For example, there is a possible activity of handmaking resistors with arbitrary resistance values as part of inquiry-based learning. Resistors of the same

values will come in a variety of shapes, depending on the creativity of the students. This helps to foster students' creative thinking. It is expected to deepen students' creative thinking and extend learning using carbon-based paper (Shintsuruta et al., 2021), but it is inadequate verification based on practices.

Electrical properties of carbon nanotube paper

The electrical resistance value R of a uniform solid is generally given by

$$R = \rho \frac{L}{A} . \quad (1)$$

Here, ρ is the electrical resistivity, L is the length, and S is the cross-sectional area of the resistance element. In the case of carbon nanotube paper having a uniform thickness, the resistance value R is expressed as

$$R = \rho_s \frac{L}{W} , \quad (2)$$

$$\rho_s = \frac{\rho}{D} . \quad (3)$$

Here, ρ_s is the surface resistivity, W is the width, and D is the thickness of the resistance element. The surface resistivity is the resistance of a square piece of thin material of uniform thickness D in contact with two opposite sides of the square.

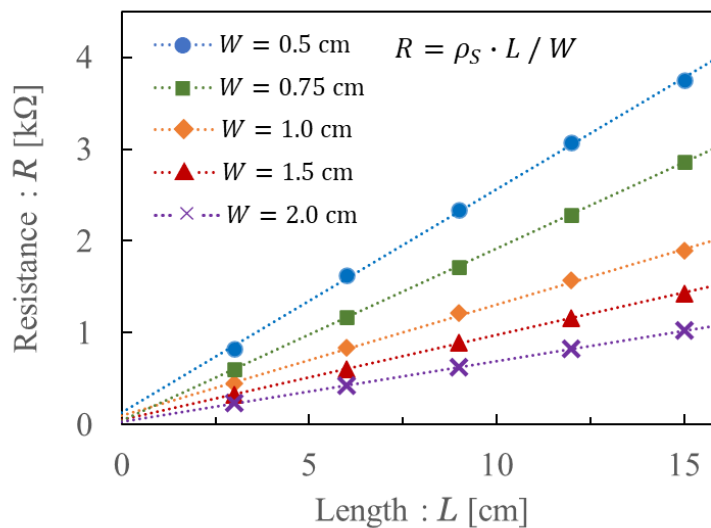


Figure 2. Results of measuring resistance values as a function of length for carbon nanotube papers of uniform thickness. The widths of the papers are 0.5 cm, 0.75 cm, 1.0 cm, and 2.0 cm.

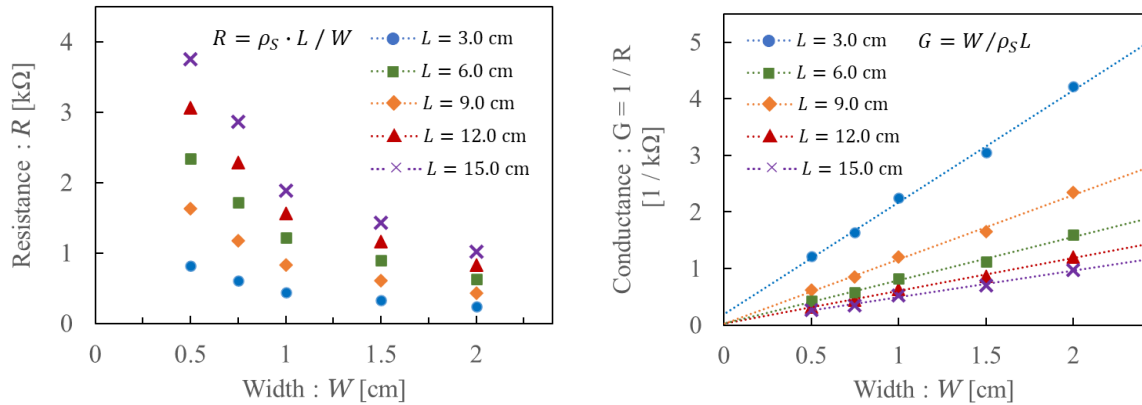


Figure 3. The replotted data of Figure 2 with the horizontal axis as the width of each length. The vertical axis is electrical resistance R and conductance. The conductance is expressed as the reciprocal of resistance.

Figure 2 shows the resistance values as a function of length for carbon nanotube paper resistors. A typical digital multimeter (Keysight, U1242B) was used to obtain this data. It is a good representation of the relationship that resistance value is proportional to its length each width. We replot the resistance values as a function of its width, as you can see in Figure 3. The vertical axis is the conductance in the graph on the right. Conductance is defined as the reciprocal of electrical resistance. So, these Figures clearly describe the relationship between the resistance and their width: inverse proportion. The measured values well represent the relationship shown by the theoretical Equation (2).

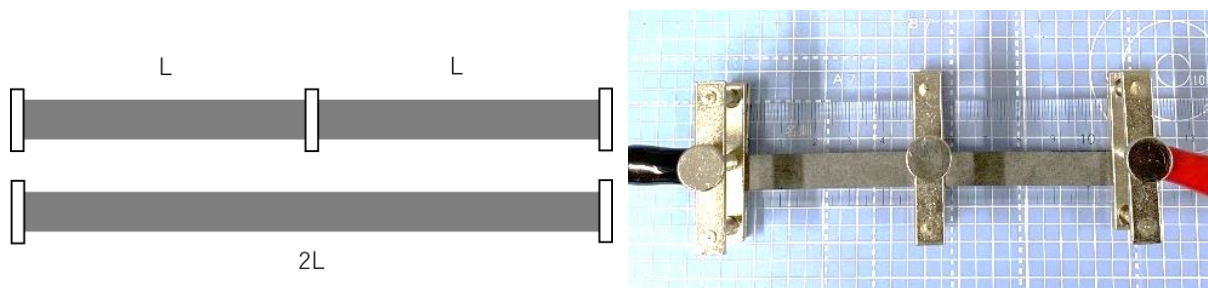


Figure 4. The combined resistance of two carbon nanotube paper resistors each with a length L in series. Both of them have the same width. The combined resistance value is equal to the value of one paper which has the length from the sum of the two papers.

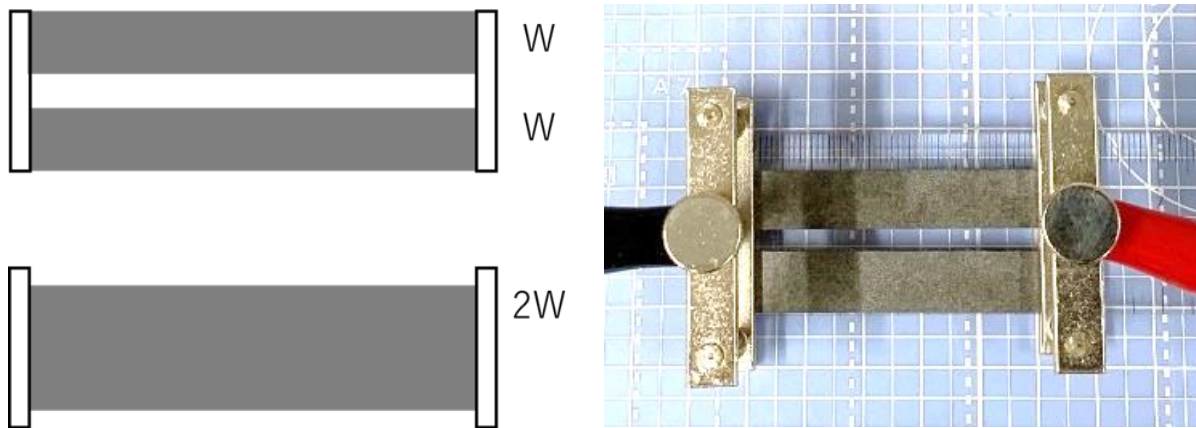


Figure 5. The combined resistance of two carbon nanotube paper resistors for parallel connection. Both of them have the same length. The combined resistance value is equal to the value of one paper which has the width from the sum of the two papers.

Application of 2D shape-dependent properties to understanding combined resistance

The combined resistance of two carbon nanotube paper resistors for a series connection is shown in Figure 4 and for a parallel connection is shown in Figure 5. Figure 4 represents that the combined resistance value and the resistance value of one sheet of paper which has the length of two sheets of paper are equal. Here, it should be noted that in the series connection of Figure 4, the range of the electrodes is not included in the length of the resistor. In a parallel connection, the combined resistance value is equivalent to the resistance value of one sheet of paper with the width of two sheets of paper, as you can see in Figure 5. The expression of these combined resistances is clear visually and quantitatively.

The use of two-dimensional shape dependence of electrical resistance can be applied to foster an understanding of combined resistance. It is well known that the formula for the combined resistance value R of two resistors which have resistance values R_1 and R_2 respectively is expressed as

$$R = R_1 + R_2 \quad (4)$$

for a series connection. Equation (4) is rewritten as

$$R = \rho_S \frac{L_1 + L_2}{W} \quad (5)$$

using Equation (2). Here, L_1 and L_2 are the lengths of resistance elements R_1 and R_2 respectively. Similarly for parallel connection,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \quad (6)$$

and therefore

$$\frac{1}{R} = \frac{W_1 + W_2}{\rho_s L} . \quad (7)$$

Here, W_1 and W_2 are the widths of resistance elements R_1 and R_2 respectively. These teaching methods on combined resistance are visually apparent and include activities that are validated by quantitative experiments.

Teaching method: designing new science lessons

We will attempt to integrate lessons for understanding to imagine the shape dependence of electrical resistance into science education in Japan. The target students are in the 8th grade (13-14 years old) of junior high school when the concept of electrical resistance is introduced.

Selection of teaching materials

To achieve this educational goal, the carbon nanotube paper resistor is the most suitable teaching material. It allows learners to visually understand the shape dependence of electrical resistance and combined resistance with 2D images. For students learning the concept of electrical resistance for the first time, it is convenient to imagine a two-dimensional shape. The specific benefits of a carbon nanotube paper resistor are summarized below.

- The shape of the resistance element can be visually grasped with a two-dimensional image since it is a thin sheet with a uniform thickness.
- Excellent electrical conductivity: the surface resistivity is about 100 Ω /sq. This makes it convenient and safe to use with low voltage power supplies and multimeters when conducting to the LED.
- Easy to shape: all you need is scissors or a craft knife.
- Experiments that ensure quantitative performance are possible.
- It is highly safe, and there is almost no risk of electric shock or burns.

Structure of lesson content

The educational goal of this lesson is to consider the shape dependence of electrical resistance using a two-dimensional image and apply it to understanding combined resistance. We structure an inquiry-based learning processes to engage students through hands-on experiments and activities. Students elucidate the two-dimensional shape dependence of electrical resistance by experiments using resistors adjusted by themselves with creative thinking.

At the beginning of the lesson, learners process carbon nanotube paper into arbitrary shapes and experiment with lighting LEDs to predict the determinant factors of resistance value. Students make measurements of resistance values to verify their hypotheses using digital multimeters. A discussion based on experimental results helps to understand that the resistance

value of carbon nanotube paper is proportional to its length and inversely proportional to its width. The above is new lesson content for learning the two-dimensional shape dependence of electrical resistance.

Table 1. Correspondence between the 5E instructional model and designed lessons

Phases	Learning Activities
Engagement	Accesses the learners' prior knowledge of electrical resistance. The learners observe that the brightness of the LED lights changes, by processing the carbon nanotube paper into an arbitrary shape.
Exploration	Make hypotheses using prior knowledge and test them through laboratory activities.
Explanation	Share and discuss the experimental results. The concept of 2D shape dependence on electrical resistance is introduced.
Elaboration	Extend students' conceptual understanding and applied to combined resistance. Measure the resistance values of circuits with two resistors connected (for series and parallel).
Evaluation	Students assess their understanding and abilities.

The lesson contents are structured as an inquiry-based learning process. We design two lessons of fifty minutes each based on the 5E instructional model which has a strong relationship with inquiry-based learning (Bybee 2002; Wilson et al., 2010). Many studies indicate that the 5E learning model cycle is an effective teaching strategy for enhancing students' understanding and achievements (Sari et al., 2017). Table 1 shows the correspondence between the 5E instructional model (Bybee et al., 2006) and the designed lessons.

The sequence of the curriculum

We describe the relationship between the teaching method and the sequences in the Japanese science curriculum. The science lessons established as a subject at school begin in the 3rd grade. According to the Course of Study in Japan, elementary school students are taught about electric circuits every year from 3rd to 6th grade (Ministry of Education, Culture, Sports, Science and Technology Japan, 2019). Utilizing the concept of electric current learned there, 8th graders study voltage and resistance in junior high school. We attempt to integrate practices including hands-on experiments using carbon nanotube paper resistors focusing on this period. There are two lessons: the shape dependence of electrical resistance and the combined resistance of series and parallel connections. Students who have taken these lessons will understand electrical

resistance with a two-dimensional visual image, enhance their creative thinking, and experience authentic (deep) learning. The "Authentic learning" approach includes deep understanding and consent to concepts. These acquired skills and learning experiences will be used to study the three-dimensional shape dependence of electrical resistance (Equation 1) and other electrical circuits in senior high school physics, as you can see in Figure 6.

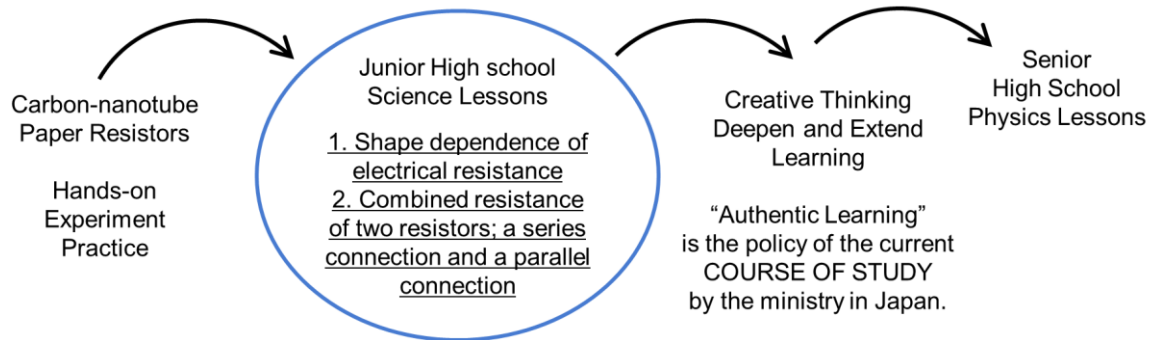


Figure 6. Curriculum content and sequence in designed lessons

“Authentic learning” is one of the policies of the current Course of Study 2017 by the ministry of education Japan. Three perspectives of class improvement have been proposed:

- Proactive Learning,
- Interactive Learning, and
- Authentic (Deep) Learning.

Learning that includes these three elements is called “Active Learning” in Japan (Ministry of Education, Culture, Sports, Science and Technology Japan, 2019).

Results and discussion of practices

Based on the teaching method above, we practiced these designed lessons at two junior high schools: School 1 and School 2. Each has 75 and 183 8th grade students, respectively. The classes are conducted by the persons in charge of the usual science classes. Two groups were set up in each school: an experimental group and a control group. Students in the experimental group undertook the designed experiment, while those in the control group undertook conventional lessons. All analyzed students completed questionnaires before and after classes. The Mann-Whitney U test (Irma et al., 2020) was used to analyze the results of the questionnaire survey, and no significant difference was found between the control and experimental group in academic ability related to electrical resistance before classes. Therefore, we assumed that the two groups were homogeneous before the classes.

The post-test includes content that questions knowledge and understanding of electrical

resistance, calculation skills, and degree of consent. The "degree of consent" is not merely a matter of memory but an indicator of the degree of consent with a scientific concept, the method for measuring it will be explained later. We decided that there was no need to compare differences in learning effects on shape dependence of electrical resistance in the post-test since the students in the control group did not learn it. The post-test has three tasks about the combined resistance of two resistors which are description formulas, calculation problems, and degree of consent. The calculation problems are combined with two resistors which have $10\ \Omega$ and $15\ \Omega$ for series and parallel connections. We measured the degree of consent about the formulas of combined resistance for each connection. The degree of consent was grasped by selecting one from the following four, strongly agree, agree, disagree, and strongly disagree. The items of this scale:

1. How convinced are you with the formula for combined resistance of a series circuit?
2. How convinced are you with the formula for combined resistance of a parallel circuit?

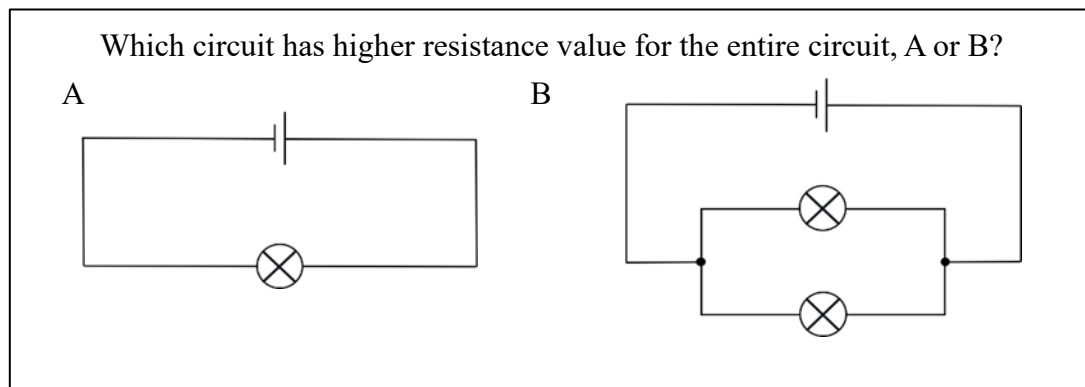
The results of the analysis of post-test average scores for experimental and control groups using the Mann-Whitney U test are shown with p-values and effect sizes r in Table 2. If the mean scores for the experimental and the control vary, with a probability of more than 99%, the p-value is less than 0.01. For this test, a p-value of less than 0.01 indicates that there is, in fact, enough variance in the sample to account for possible mean differences. 'Effect size' quantifies the difference between compared groups, in other words, the actual effect. To make a statement about the effect size in the Mann-Whitney U Test, the Standardized test statistic z and the number of pairs n are needed. The effect size r is given by, $r = |z|/\sqrt{n}$. The effects in School 1 were manifested in the degree of consent for series connections and the memory of formulas for parallel connections.

We added an application and evolution problem to the post-test content because students scored higher on the pre-test in School 2. Because the post-test questions (previously described) are basic and are expected to have a similar percentage of correct answers in high-performing schools. The evolution problem is posed as a question, as shown in Figure 7.

Table 2. The results of the analysis of post-test mean scores for experimental and control groups using the Mann-Whitney U test.

p r	Series Connection			Parallel Connection			
	Formula	Calculation	Consent	Formula	Calculation	Consent	Evolution
School 1	0.307	0.531	*0.000	*0.000	0.322	0.426	—
	0.146	0.089	0.730	0.839	0.142	0.148	—
School 2	0.574	0.149	0.463	0.150	0.355	0.358	*0.008
	0.066	0.169	0.088	0.168	0.108	0.123	0.313

*p < 0.01

**Figure 7. The evolution problem solved by the students in school 2**

To solve this evolution problem, it is necessary to apply the concept of the combined resistance of parallel connections. Details are as follows. The two loads of circuit B are at the same voltage as the load of circuit A. Therefore, the magnitude of the current flowing through each load is the same, and the current flowing through the entire circuit B is twice that of circuit A. In other words, circuit A has a higher overall resistance value. The reason why the total resistance value of a circuit with a large number of connected resistors is smaller cannot be answered without an authentic understanding of the combined resistance of parallel connections. As expected, the learning effect obtained in School 2 was the item solving the evolution problem. Note that the degree of consent was not investigated for the evolution problems.

The effects at School 1 are the degree of consent for series and memory of the formula for parallel connections. The effect at School 2 is to solve the evolution problem. The degree of consent is not increasing with only memory skills. It is impossible to solve the evolutionary problem with only a shallow understanding. The fact that we were able to confirm the effects in terms of the degree of consent and the evolution problem is one of the factors supporting the

implementation of authentic learning.

Conclusion

We designed new lessons to clarify the learning effect obtained by practicing classes that treat the shape dependence of electrical resistance with two-dimensional shapes in junior high school science education in Japan. As teaching material for the new lessons, the carbon nanotube paper resistor is the most suitable because it allows learners to visually understand the shape dependence of electrical resistance and combined resistance in two-dimensional images. A carbon nanotube paper resistor has excellent electrical conductivity and is made from familiar and inexpensive materials. It was shown that the measured resistance quantitatively and accurately expressed the shape dependence of the electrical resistance and had high reproducibility. This makes it possible to consider the combined resistance of series and parallel connections as a shape-dependent two-dimensional image. In particular, learners understand visually and quantitatively that a series connection increases the length of the resistance element, and a parallel connection increases the width.

We attempted to integrate lessons for understanding to imagine the shape dependence of electrical resistance into the 8th grade students of junior high school when the concept of electrical resistance is introduced. The educational goal of this course is to consider the shape dependence of electrical resistance using two-dimensional images and apply it to understanding combined resistance. We have built an inquiry-based learning process based on the 5E instructional model to engage students through hands-on experiments and activities. The content of the class is to clarify the two-dimensional shape dependence of electrical resistance through experiments using resistors adjusted by students themselves.

Based on the developed teaching method, these designed lessons were practiced in junior high schools. We set up an experimental group and a control group. All analyzed students completed questionnaires before and after classes. The analysis of post-test average scores for experimental and control groups used the Mann-Whitney U test. The effects in School 1 were manifested in the degree of consent for series connections and the memory of formulas for parallel connections. Solving the evolution problem was the item which obtained the learning effect in School 2.

The degree of consent is not increasing with only memory skills. It is impossible to solve the evolutionary problem with only a shallow understanding. The fact that we were able to confirm the effects in terms of the degree of consent and the evolution problem is one of the factors supporting the implementation of authentic learning. We clarified from the results of the practices that it is effective for authentic or deep learning. “Authentic learning” is one of the

policies the current Course of Study 2017 by the ministry of education Japan (Ministry of Education, Culture, Sports, Science and Technology Japan, 2019). The "Authentic learning" we support has a deep understanding and consent of concepts. Our approach involves inquiry-based learning including a proactive activity and interactive discussion, aligning with the educational policy of the Course of Study. The attempt to improve authentic learning by implementing inquiry-based learning utilizing two-dimensional resistors is an innovative approach in physics education.

Our findings show that the experiment is an effective way to introduce inquiry-based learning with creativity, including for those students who think that learning electrical circuits is hard. This new approach contributes to understanding and consenting concepts of electrical resistance.

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