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Training Teachers for New Ways of Understanding Physics Teaching from Its Mathematization

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Abstract

Using mathematics in physics teaching often becomes an obstacle to learning. Regarding this problem, we study the possibilities of training future teachers for new understandings about the relationship between physics and mathematics. Our main frame of reference is a research sequence developed in the Teaching and Learning of Physics research group. Data arose from participant observation in the “Didactics of Physics” course in an initial teacher training program. It was qualitative research of the case study type, with 18 students finishing their training process. We focused on addressing the "mathematization of physics for teaching" in three phases by developing ways to enrich classroom interaction using experimental resources, technologies, and literature. These three phases were 1) Phenomenological approach; 2) Physical systems observation and; 3) Conceptual modelling. Results show how students substantially changed the way of creating explanations in physics to the point of being able to work on topics that they did not understand at the beginning, such as Minkowski diagrams, quantum entanglement, and the concept of entropy. We found evidence of real possibilities to get out of the traditional way of presenting mathematics in physics, understanding the mathematization as a natural and mental process to see nature.

Introduction

Authors such as Redish, E., & Gupta, A. (2010); Uhden, O., Karam, R., Pietrocola, M., & Pospiech, G. (2012) have explored the relationship between physics and mathematics in education, highlighting that it extends far beyond mere equations. This research aims to investigate how to implement a process of mathematizing physics in education that surpasses the conventional technical connection between physics and mathematics focused solely on equations.

To address this issue, this work proposes an alternative approach to understanding the process of mathematizing physics in educational contexts. This approach is epistemologically distinct from historical perspectives on mathematization. Is physics just math? Is math just equations? Is physics teaching better without mathematics? The answer to all of these questions from common sense or even from the traditional way of teaching physics used to be ‘yes’. But from the research in physics education, the answer is ‘no’. The proposal consists in planning the teaching and learning process in at least three phases: 1) the phenomenological approach (awareness of the existence of the phenomenon); 2) the physical systems observation (identification of variables, parameters, constants, and initial conditions), and; 3) the conceptual modelling (synthesizing explanations and arguments in different types of language representations), as explained below.

This research took place at the Universidad Distrital Francisco José de Caldas, Bogotá, in a “Didactics of Physics” course into a program to educate physics teachers. Data arose from participant observation through class notes and audio-visual records. Content analysis techniques were applied to study the information collected. We can conclude that students really learn physics in a different way. Also, they believe that learning physics can be deeper and more impressive than simply memorizing and applying formulas, as well as having an awareness of the dynamic role of the teacher beyond a content transmitter.

The problem

It is common to think that "physics is mathematical" and "the conceptual domain of physics means good algorithms use". But these visions contain an unconscious imagination about the characterization, argumentation, explanation, phenomenological physical construction, or modelling of natural phenomena.

We present a case study in which an alternative perspective proposal tries to change the meaning of the mathematization of physics in the teaching and learning processes. We start from the assumption that the mathematization of physics should be conceived not as a tool or language but as a way of thinking about nature. Likewise, we believe that the mathematization of physics throughout history has been epistemologically and methodologically different from class processes, since the first one responded to a series of somewhat chaotic conditions of very different kinds at diverse moments in human history, while the physics construction in class responds to the students and their context characteristics. This context absorbs direct influence from various action fronts, such as educational policy, culture, current societal demands to shape a specific type of citizen, the particularities of the students' and teachers' lifestyles, and the prevailing notions of what is considered science in the classroom, among others.

Synthetizing, the main objective of this work is to show concrete results on new ways of understanding the relationship between physics and mathematics in educational settings.

Theoretical Framework

We study two thematic lines: first, the sense and meaning of mathematization of physics in the classroom, drawing on analyses by various authors, and second, previous work by our research group. Additionally, we adopt the dimensional perspective of the didactics of physics developed in research within our group, which is based on various theoretical references from Latin America.

Some reflections on the problematic relationship between physics and mathematics

The importance of mathematics in the historical development of physics is undeniable, as extensively discussed by Bochner (1991) and Holton & Brush (2001). They highlight the critical role mathematics has played in advancing science while cautioning against viewing physics merely as a collection of equations. Such a perspective can narrow the scope of physics education, reducing it to algorithmic procedures and overshadowing the importance of conceptual understanding and scientific thinking skills, as emphasized by Redish & Gupta (2010). Recent studies, such as Arenas (2019), suggest that the approach to mathematization in physics teaching should differ from that in the construction of physics theories. Teaching physics requires specific resources and must address contextualized learning objectives, integrating mathematical skills with a robust conceptual framework. This approach ensures that students develop a comprehensive understanding of physical principles and the ability to apply mathematical tools effectively.

On the topic of physics teaching, according to Karam and Pietrocola (2009) and Karam (2014), we must stimulate students to acquire structuring abilities. At the same time, they gain technical skills and construct scientific knowledge. On the other hand, the development of physics historically was a set of individual and collective actions from different people and diverse knowledge areas leading to the consolidation of one phenomenon representation that was studied by specific communities. Uhden, Karam, Pietrocola and Pospiech (2012) highlight the significance of modelling mathematical reasoning within physics education, advocating for the integration of mathematical modelling as a core pedagogical strategy to enhance students' conceptual understanding. Similarly, Greca and Moreira (2002) discuss the cognitive challenges students face when reconciling mental, physical, and mathematical models in physics, underlining the complexity of navigating these intertwined domains. These studies lay the groundwork for understanding the intricate relationship between mathematics and physics, setting the stage for exploring innovative teaching methodologies that can bridge these gaps.

Another consideration in teaching is that how pre-service teachers learn physics becomes the same methodology they adopt, as shown by Adelantado, Aleixandre and Gil-Pérez (1992) and Castiblanco and Vizcaíno (2018). In other words, it does not matter how many courses the student has taken regarding the didactics of physics or about teaching methodologies; they will only teach using the same methodologies in which they learned physics in their physics classes. This is a barrier to break with the tradition of assuming physics teaching from alternative perspectives.

Our sequence of research on the mathematization of physics for teaching.

Given that this is a relatively underexplored line of research within the Iberoamerican context, we have been actively developing various studies over the past 20 years to explore the subject more thoroughly. Below, we present the main results of each of our research efforts in their respective sequence.

In her master's thesis, Castiblanco (2003) explores the concept of 'mathematical beauty' as articulated by Paul Dirac. Dirac viewed mathematical beauty as an organizational criterion for modelling the laws of physics, particularly in the emergence of Quantum Physics. One of Dirac's key contributions was the Dirac equation, which introduced four new components to the electron's wave function based on the concept of positive and negative states of energy. This innovation allowed scientists to envision new properties of spin and magnetic moment for subatomic particles. Dirac emphasized the deep relationship between physics and mathematics, asserting that equations not only describe nature but also possess physical meaning and beauty.

In his doctoral thesis, Vizcaíno (2013) examined the role of mathematization in physics education. During his literature review of the past decade's studies on this topic, he identified three prevailing trends: the first trend views mathematical modelling as synonymous with the entire process of mathematizing physics; the second trend conceptualizes mathematization as a process involving physical systems and their representations, primarily mediated through experimentation; the third trend posits that mathematization entails synthesizing physical laws into equations, but only after a process of phenomenological construction.

Based on previous research, Ruiz (2019) and Perez (2019) found that even among students training to become teachers, the emphasis on mastering equations remains persistent. However, when these students engaged in metacognitive or critical exercises, they realized that mastering equations alone cannot adequately explain physical phenomena. This realization led them to question why they had never learned to think critically about the physical world, create their own languages to describe phenomena, or develop autonomy in constructing scientific knowledge.

Sierra & Castiblanco (2022) complemented these studies, showing that teachers must learn to develop their discourse on physics, which implies that they can envision the phenomena under study. Additionally, they must construct a certain logic to decide the best use of their language to describe phenomena and communicate ideas. Furthermore, they must learn to build explanatory models that make sense to them, beyond simply believing that "laws are true because a scientific authority says so." In all of these aspects, the students recognized their shortcomings and their desire to overcome them. Recently, two papers by Castiblanco and Vizcaíno (2024a, 2024b). were published characterizing in a more specific way the concept of mathematization in physics education.

From all these results, we believe it is necessary to propose alternative relationships between mathematics and physics in teacher training programs. These programs should offer opportunities for future teachers to develop the ability to structure their physical thinking and introduce methodologies that go beyond merely presenting an equation as a mathematical model. This approach does not imply eliminating equations; rather, it aims to give them meaning, allowing students to reach their own "mathematizations" of the physical world.

Research methodology

The study is qualitative research focused on a physics didactics course at the District University Francisco José de Caldas. It involved 18 students, with the professor serving as a co-researcher alongside a student researcher enrolled in the course. The student researcher conducted a participant observation from an ethnographic perspective over a period of 16 weeks of classes, six hours a week. The ethnographic research method, specifically participant observation, was employed based on perspectives outlined by Martínez-Otero (2004) and Kawulich (2005). These perspectives allowed for a more nuanced approach to capturing and understanding participant discourses within the study.

Data collection

Data collection was conducted through audio and video recordings, along with reports generated by observation grids. These instruments allowed us to study the debates and discussions present within each class. For data analysis, content analysis techniques were used to seek evidence of the role the concept of the mathematization of physics played in the students' discourse. This content analysis consisted of three stages: coding, where units of analysis were identified from all the constructed text; grouping, where units of analysis belonging to the same kind of evidence were clustered, allowing for the emergent identification of analysis categories; and finally, the interpretation of these categories. The observation grid for the classes was collaboratively designed by the researchers, using the characteristics of the three phases of the mathematization of physics process as a priori categories based on our theoretical framework. Below is the table containing the indicators. The observer was instructed to note each time any of these aspects were observed and to describe how they occurred.

Table 1. Categories and Indicators of a grid to observe the classes and analyse audio video recordings.

1. Phenomenological Approach			
Indicators	Yes	No	¿Why?
Provides feedback on their knowledge through class-led debates.			
Raises questions in the classroom, thereby enriching their training process and contributing to class dynamics.			
Recognizes the phenomena under study and links them to the description of physical models in class, verbalizing their ideas.			
Applies principles of epistemology, history, and philosophy in the treatment of physical concepts.			
Identifies the fundamental concepts associated with physical phenomena.			

2. Observation of the physical system			
Indicators	Yes	No	¿Why?
Identifies the magnitudes of physical variables and relates them to the concepts covered in class, discussing different interpretations with classmates.			
Understands the significance of "observability" in physical systems.			
Organizes and articulates ideas concerning the discussed physical phenomena.			
Adopts new methods of structuring their ideas in both physics concepts and physics teaching concepts.			
Recognizes the importance of systematic observation, hypothesis formulation, analysis, and interpretation of results			

3. Modelling the physical phenomenon			
Indicators	Yes	No	¿Why?
Understands the concept of mathematical modelling and effectively represents phenomena using equations.			
Develops processes for understanding phenomena, characterizing them, expressing doubts, constructing explanations, and presenting arguments.			
Uses the language of mathematics to organize thoughts for studying physical phenomena.			
Identifies the conditions and limitations of equations in describing phenomena.			
Explores various forms of representation to effectively communicate ideas and conclusions.			

The head professor of the course initiated the development of a concept called "the interactional dimension of the didactics of physics," aiming to train teachers to establish decision-making criteria for new physics teaching methodologies. To achieve this, she developed five modules. The first module focused on the instruction of physics mathematization in teaching, based on three classroom approaches derived from her previous research, with characteristics as presented in Table 2.

Subsequently, the professor presented three modules titled "Typologies of Literature," "Typologies of Information and Communication Technologies," and "Typologies of Experimentation." These modules, derived from his research, showcased a wide range of support resources for the classroom. These resources can be selected to enhance the implementation of the three phases of the mathematization of physics in the classroom. Table 3 provides a concise overview of these typologies.

Table 2. Main characteristics of phases for mathematization physics process in class.

1st Phase: Phenomenological Approach	**2nd Phase: Physics Systems Observation**	**3rd Phase: Modelling phenomena**
Consists of looking for relationships between the conceptual schemes of physics and sensory experience. Students have to understand their ways of understanding the phenomenon. It privileges reflection and debate for the resolution of problems formulated from reality, taking as a starting point the sensory-motor experience and its description. It focuses on the ways of thought works when describing a physics phenomenon.	Involves studying physics systems through processes that systematically organize ideas. Initially, it reflects on the roles of the “observer,” “the observable,” and “the observed.” Students learn strategies for reasoning to construct knowledge. The schematization of situations in the physics world is emphasized, starting with identifying a problem and formulating hypotheses.	The goal of the process is to understand the suitable mathematical methods for solving open physics problems, emphasizing deductive logical analysis. It aims to stimulate students' deductive reasoning, helping them choose the appropriate expressions to describe the behaviour of physical phenomena. This involves using their own and scientific language, and engaging in communicative interactions with their peers.

Source: author’s production

Table 3. Synthesis of the characterization of typologies of bibliographic resources based on their potential to enrich the mathematization processes of physics in the classroom.

Typology of literacy resources	**Application criteria as a support resource in the classroom**
University basic and secondary education **textbooks**. In print and digital format	It is permanent reference material for the teacher. Allows to identify points of view in agreement and disagreement, as well as successes and failures in the treatment of the contents, which can be used to illustrate or criticize along with their students.

Scientific Dissemination, in articles, magazines or books format.	It stimulates questioning in students when ideas that seem clear require dialogue with the teacher for deeper understanding.
Research Results in Physics. In articles or books.	This material is primarily designed for self-guided teacher training across a wide variety of topics. It serves as a resource for teachers who need to update their knowledge or deepen their understanding of specific topics in order to effectively explain them to students.
Research Results in Physics Teaching. In articles or books.	They offer a reference or inspiration on how to deal with certain topics in the class, since these results show successes and failures, as well as innovative teaching proposals.
Science Fiction, in short stories or books formats. Digital or printed.	Stimulates scientific reading comprehension, facilitates the development of activities based on creativity. It allows connecting with the interests and language of the students.
Encyclopaedic information on the web or in printed format.	It can be used in class to sharpen critical sense and analytical skills, as well as teach students to develop information selection criteria.

Source: (Castiblanco & Nardi, 2023)

Table 4. Synthesis of the characterization of Experimentation typologies based on their potential to enrich interaction processes in the classroom. Source: (Castiblanco, 2021)

Type of Experiment	Application criteria as a support resource in the classroom
Illustrative. Phenomenological analysis, stimulus to doubt.	The Professor describes and explains a physical phenomenon, teaches the student to doubt, offers resources to test a scientific discourse, stimulates the imagination, and strengthens scientific language. The student questions ideas or explanations, modify conditions of occurrence, thinks interdisciplinary, works as a team.
Mental. Representations of language, debate, tolerance.	The Professor contextualizes the mental experiment, teaches different forms of representation (pictorial, graphic, algorithmic, literary, experimental, conceptual, guides the coherence of language. The student uses various types of representation, socializes, creates his discourse.
Research. Formulating questions, constructing answers and socializing.	The teacher guides the formulation of a question, helps define its relevance, offers inputs for the solution. The student solves a research question, defines procedures, searches for information and resources, debates, analyses, and concludes.
Home. Teamwork, creativity, analysis.	The teacher designs, shows, and explains the assembly, attends to the concerns of students, proposes challenges. The student must overcome the assembly, identify variables, parameters, constants, elaborate criteria to make decisions and communicate.

Discrepant. Conceptual imbalance, questioning of common sense.	The teacher organizes experiments that unbalance ideas, asks for hypotheses about what will happen, stimulate metacognition, guide the debate. The student formulates hypotheses, elaborates explanatory models, puts the experimental assembly to the test, debates, argues, and agrees.
Virtual. Modelling of phenomena. Identification of epistemological obstacles.	The teacher analyses the explanatory model, questions its relevance, offers ideas for enrichment in use and design. The student models produce analogies, create, overcome epistemological obstacles.
Crucial. The non-linearity and the non-bias of science.	The teacher provides full descriptions of various paradigmatic experiments, poses questions about why they occurred, challenges students to replicate them phsyically or virtually. The student analyses paradigm shifts in Physics, develops their discourse, socializes, and evaluates.
Recreational. It amuses, stimulates questions, and enriches language.	The teacher guides the construction of experiments of a single staging, guides the definition of questions and answers about the assembly itself, leads recreational fairs with various dynamics. The student poses questions, answers question, interacts with peers, searches for information and resources.

Table 5. Synthesis of the characterization of ICT typologies based on their potential to enrich the mathematization of physics in the classroom.

Typologies of ICTs	Application criteria as a support resource in the classroom
Audio-visual, audio and visual material	They encourage debate and reflection. You can work on episodes from real life or completely fictitious scenarios that stimulate the critical position of the participants, identifying aspects such as academic, educational, political, economic, cultural, etc.
Photographic records	They approach the analysis of the reason for the occurrence of an event, recognizing that it is possible to go beyond what is seen with the naked eye. It allows discussing the limits of the "observer", "the observed" and "the observable"
E-learning platforms	It is a permanent source of self-taught training. It grants teachers intellectual autonomy to make up for their shortcomings or enrich their speech prior to their performance in class.
Interactive Software	It supports the development of experimental practice. Saves time on technical exercises. Promotes creativity. It does not replace real experimentation but enriches it with the proper guidance of the teacher.
Social networks and groups	Stimulates autonomous work. It is the new language to find out what others think of any idea you have, through unconventional processes of dialogue. It offers a new power to society to make its decisions.
Interfaces	Easily engage the interests of students. Facilitates the inclusion of students with visual and auditory functional diversity or any other diversity. It requires the direct application of learned concepts. It has a wide variety of forms (digital sensors, apps, video games, etc.)

Virtual and Augmented Reality	It broadens the teacher's vision of various ways of mathematizing physical phenomena through play, especially through digitized interaction. The logic by which augmented reality works is studied, with the potential that it offers when conceptualizing physics.

Source: Author's elaboration, based on (Castiblanco & Nardi, 2023)

In the fourth module, the teacher asks her students to design a class that applies the structure of the three phases of physics mathematization using various resource types. The teacher advises students during several classes dedicated solely to this preparation. Then, in the fifth module, each group acts as teachers to their peers, teaching them some physics concepts.

Results

This part consists of two sections. The first section provides a synthesis of activities proposed and developed by student groups during three class interventions. These exercises aimed to teach physics from a new perspective of mathematization. Notably, the students selected topics they perceived to have conceptual flaws for these exercises. The second section involves research interpretation, attempting to verify the impact of these activities on the learning process. The goal is to assess how the students' discussions and interventions influenced their understanding and engagement with the material.

Activities planned and executed by the students

Quantum Entanglement

1. Phenomenological approach
Historical analysis of the conditions leading to the emergence of the concept of quantum entanglement. Discussion on why two photons can be "connected" even when far apart. There was no definitive answer, so students collaboratively explored and built their understanding.

2. Physics system "Observation"
A thought experiment is presented to illustrate the phenomenon, prompting many questions about the logic behind it; if one particle is altered, the other reacts instantly. The debate focuses on how these rules differ from those in classical physics.

3. Modelling

The discussion centered on the new paradigms of "non-local correlation" and "quantum causality" as phenomena that are created to "exist" rather than being effects of known causes. Reflections on Bell's Theorem addressed the non-existence of local hidden variables in the quantum world.

Space-Time Concept

1. Phenomenological approach
Text analysis of original papers discussing the debate between Newton and Leibniz, focusing on their arguments for concepts such as absolute time and isotropic, homogeneous space.

2. Physics system "Observation"
Creating resources using elastic material (latex), where different figures are drawn and then deformed by stretching the latex in various ways. This serves as an analogy for the behaviour of deformed space and the factors causing such deformation, illustrating how relative distances between two points are created and, consequently, different travel times between them.

Additionally, they constructed physical models to represent Minkowski diagrams, which explain the geometry of space-time.

3. Modelling
A simulator is programmed to demonstrate the "twin paradox" using Minkowski geometry, based on the book "The Philosophy of Physics" by Maudlin (2019).

Entropy Concept

1. Phenomenological approach
A debate is sparked by a video demonstrating the reaction between citric acid and sodium bicarbonate, analysing the necessary conditions for effervescence to occur. Students then conduct several illustrative experiments, varying the materials to generate different chemical reactions, characterizing a kind of "irreversible disorder."

2. Physics system "Observation"
The concepts of macrostates and microstates of a system are introduced, and their relationship is explored, particularly in the context of the Carnot cycle.

3. Modelling
A probabilistic experiment is designed to discuss probability distributions and the concepts of permutation and counting.

Results Analysis

Pre-service teachers recognize that modelling holds profound significance, involving the structuring of thought for both the teacher and the student. In the process of mathematical modelling, teachers should avoid the misconception that it merely involves expressing phenomenological observations through equations. Instead, it is about analysing the relevance of these equations in explanations. This does not reduce the rigor of analyses but examines whether they enhance the understanding of the phenomenon in specific contexts, such as those presented below.

When discussing representation in physics, it is recognized that relying solely on mathematical expressions is insufficient. Likewise, interpreting a single model as absolute truth is discouraged. This perspective challenges traditional teaching methods where instructors often emphasize rigid reproduction of knowledge, sometimes merely transcribing textbook content without considering potential interpretative difficulties. Such approaches can lead to confusion among students and even teachers themselves.

Below, we present the testimony of a student reflecting on this topic.

> *Student (S) 8:* It is not sufficient to rely solely on a single representation when discussing quantum models, such as the wave model. For example, when explaining light teleportation, one might begin with wave models but ultimately switch to particle models to describe the final outcome, akin to constructing matter. This highlights the need to be flexible in imagining and understanding these complex concepts (...) Initially, atomic and subatomic particles were represented as discrete particles. However, Schrödinger introduced the concept of a probability wave, which challenges this particle-like representation. The idea evolved further into the concept of a rotating probability wave. These changing representations demonstrate the complexity and evolving nature of quantum mechanics (...)

> Sometimes, the conventional representations in quantum mechanics may appear inadequate. Therefore, it is essential to reconsider them from a philosophical perspective to gain a deeper understanding (...)
> Our current explanatory and representational models often fall short in fostering true understanding because they rely solely on mathematics. While mathematics serves us well and appears sufficient on the surface, there is more to the relationship between mathematics and abstraction that needs exploration.
> We must delve deeper into understanding how mathematics and abstraction interact to reveal coherent interpretations of phenomena. This exploration is crucial for bridging the gap between mathematical formulations and meaningful interpretations of the underlying physical reality.

We can note how this student raises an important question about the process of understanding and assimilating quantum phenomena. The first step in finding alternative representations in physics is recognizing that the current or generally accepted models may not ensure comprehensive understanding for everyone, particularly for pre-service teachers. Without fully grasping these accepted concepts, it becomes challenging for them to explain the phenomena in a way that differs from textbook descriptions.

When analysing the role of developing their scientific language and finding physical meaning in the representations they use for explanations, students agree that traditional classes do not support this development. Consequently, they propose participating in a collective construction of their ways of thinking about phenomena. Below is a dialogue where they engage as academic peers to clarify their ideas about the relativity of space and time, attempting to move away from traditional methods.

> *S11:* I'd like to share my perspective on the debate in our group, particularly in defence of Newton's ideas. Although there was no consensus reached, I believe Newton's approach is worth considering, especially when compared to Leibniz's perspective. From what I gathered, particularly from Student 12's explanation earlier, there's a notion that movement through space allows for traversing from one point to another. However, when it comes to time, there's a fundamental difference. Time has a distinct and privileged direction—unlike space, we cannot travel backward in time as freely as we might move left or right in space. Time defines the moments we measure in space, and these moments always progress in a specific direction. This understanding supports Newton's view. By separating these concepts and considering them in a four-dimensional framework, we can gain deeper insights into these fundamental aspects of physics.
>
> *S7:* When you mention moving backward or forward in space from one place to another, what frame of reference are you considering?
>
> *S17:* This is what we were discussing: when we talk about motion being absolute, the key question is: who is observing it?
>
> *S7:* That was the question, right?
>
> *S19:* If an absolute observer were out of space, maybe something like God.
>
> *S5:* We were discussing that Newton was very advanced for his time because he defined absolute space up to Saturn, and the theories he applied could be applied up to there, that is, it works.
>
> *S7:* We forgot to discuss the idea that the ether was considered to be superterrestrial. When Newton described the circular movement of planets, he attributed the same laws to celestial bodies even though they were no longer believed to be made of the characteristic matter he

> described. To explain further, imagine this triangle here (The student points to a point drawn within a triangle that was in turn drawn on an elastic rubber material). If I were to deform space-time—let's pretend we don't know about relativity—by stretching this triangle, no matter how much I stretch the material, the point in the middle will always remain inside the figure because the continuity of these lines is maintained.

In most of the evidence collected, the active intervention of the students was notable, reflecting their desire to deepen their understanding of the phenomena under study. At times, they acknowledged their lack of mastery, prompting the head teacher to intervene with specific information, which the students welcomed. Other times, they continued arguing not due to a lack of understanding, but because they felt compelled, in a non-traditional manner, to go deeper and ensure their ideas were understood. Ultimately, they recognized this as a win for their intellectual autonomy.

Conclusions

The results confirm the need for fundamental transformation in how physics education is approached by teachers. One crucial aspect involves challenging the prevailing discourses found in educational policies, textbooks, and curricula regarding the relationship between physics and mathematics. Typically, this relationship is narrowly viewed through the lens of using equations, which limits the broader understanding and interdisciplinary nature that could enrich physics education. This calls for a shift towards a more comprehensive and integrated approach that goes beyond equation-based teaching methods.

At the same time, they confirm that this perspective of going from starting from the phenomenological approach, going to the "observation" of the system, and ending with the modelling, all supported by typologies of technologies, literature, and experiments, is a more human and natural perspective of the way in which students construct scientific knowledge.

Students have undergone significant transformations in their approach to learning physics. They have shifted from being passive recipients of information to actively engaging in meaningful practices such as debating with self-generated arguments and expressing doubts without fear of judgment. This shift has fostered critical thinking and a deeper understanding of physics concepts. Furthermore, students have gained insights into the interconnectedness of "the observer" and "the observed" in modern physics, recognizing the complex relationships within scientific observation. They have also incorporated perspectives from history, epistemology, and philosophy to enrich their comprehension of physical phenomena. Additionally, students have developed specific criteria for identifying fundamental concepts within studied phenomena, emphasizing the importance of discerning variables, parameters, and constants in physical systems. Overall, these experiences have led to the creation of innovative teaching strategies that integrate mathematical reasoning with the study of natural phenomena, promoting a holistic approach to physics education and fostering deeper learning outcomes.

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