Maths for Einstein's Universe: Tools for Understanding Modern Reality. Part 1: Rationale

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Abstract

Aversion to mathematics is a recognised and widespread problem. Following a review of the literature on this subject, this paper presents an education program which has been developed to test the hypothesis that transferring attention from traditional school arithmetic to a broad range of mathematical skills relevant to modern science at an early age (ages 7-14) will improve students' attitudes to mathematics, reduce the incidence of maths anxiety and prepare students for mathematics topics normally introduced at more senior levels. The program entitled *Maths for Einstein's Universe* includes five modules covering extreme numbers, estimation, probability, vectors, and curved space geometry. All modules are taught through group activities, games, and plays. The modules complement appropriate early learning of modern physical concepts from the subatomic world to cosmology. While connected to science, the program aims to provide meaning and comprehension for socially relevant topics from national budgets to pandemics and opinion polls. The first part of the paper presents an overview of the program, encompassing the motivation for program development, the program's structure, and the teaching approach employed.

Introduction

Einstein-First project

In the early 20th century, discoveries in physics created a new understanding of space, time, matter, and radiation, collectively described as Einsteinian physics. These discoveries have given us knowledge of the smallest-scale phenomena in the universe through to the vast distances of the visible universe (Mourou, 2019; Weiss, 2018,). With this knowledge, we recognise the modern technologies on which our lives depend and with which we perceive the universe (Nita, Mazzoli Smith, Chancellor, & Cramman, 2023). The Einstein-First project is an educational response to this revolutionary change in human understanding which aims to develop a contemporary school science curriculum so that all students by Year 10, their final year of compulsory science education in Australia, achieve a basic understanding of our current best understanding of physical reality. An essential component of this understanding is mathematics (Popkova et al., 2023, Saeki, Ujiie, & Tsukihashi, 2001).

The distinguished Harvard and Oxford educator Jerome Bruner stated, "Any subject can be taught effectively in some intellectually honest form to any child at any stage of development" (Bruner, 1976). In the spirit of Bruner, this project is designed to determine an optimum approach and sequence of concepts to meet our goal. By necessity, the program must incorporate powerful, fun, and mind-expanding activities, all of which are based on peer-reviewed, published results.

Many studies have demonstrated that Einsteinian Physics can be successfully integrated into a curriculum at any level of schooling when taught at the appropriate level using approaches including activity-based learning with toys and models.

The **Einstein-First** project has pioneered a learning progression that starts in early primary school, aimed at ensuring every student develops a basic intuitive understanding of physical reality as explained by modern Einsteinian science. The program includes activity-based modules for every school level from Year 3 to Year 10 (ages 7-16). The seamless eight-year spiral curriculum is described in (Kaur et al., 2023), including evidence for learning outcomes when implemented by teachers trained through professional development programs. It starts (year 3) with the quantum description of physical reality - atoms, molecules, photons, phonons – introduced through toys, song and activities. Year 4 introduces both microscopic and macroscopic forces. In year 5, photons are introduced, including the statistical aspects of quantum interference, as well as Einsteinian gravity and its connection to spacetime curvature. By year 6, students are ready to be introduced to the breadth of the photon spectrum and the photon-molecule interactions that cause greenhouse warming. Years 7-10 build on the above knowledge base, with deeper exploration of Einsteinian gravity, Einsteinian energy ($E=hf$ and $E=mc^2$), fundamental particles, and finally a unified treatment of climate science and cosmology.

Evidence from the works of Choudhary, & Blair, 2021, Popkova et al., 2023, Lonshakova et al.,2025, suggests that the early introduction of modern physics concepts does not cause cognitive overload. On the contrary, since children have not yet been exposed to Newtonian physics, they do not have resistance to perceiving ideas such as photons, curved space-time, and the probabilistic nature of quantum reality.

One component that has not been investigated in the above research is the mathematical concepts that need to be developed in parallel with the concepts of Einsteinian physics. The need for modernised mathematics in schooling is partly motivated by trials of the above program where limitations occur because of the need for students to grasp extreme numbers, statistic concepts, vectors and curved space geometry. Examples include the extreme geometric distortions in images of distant galaxies caused by the curvature of space, and the extreme ranges of scales in the universe that exceed our ability to describe them using simple language. Extended mathematical concepts are also highly relevant to modern life like pandemic prognosis or financial risk.

The necessity to rework the STEM curriculum is highlighted in the literature (Xu, Fang, & Hobbs, 2023), with a focus on making mathematics more relevant to students' lives (Dogan, 2020; Koul, Fraser, & Nastiti, 2018; Sparrow, 2008) and connected to science (Darby, 2007). However, to the best of our knowledge (see the literature review), a structured, systematic, and prolonged learning sequence has not been proposed yet.

For this reason, in parallel with the fundamental modernisation of science education, Einstein-First introduced a goal of re-thinking mathematics education, with a program entitled *Maths for Einstein's Universe (MEU).*

Aiming to make the mathematics needed for describing physical reality more intuitive, more relevant to students' experience, and less dependent on rote learning, we ask whether Fermi estimation (Ärlebäck & Bergsten, 2010) might be more important than multiplication tables, and whether the conceptual understanding and representation of scale might be more important than arithmetic.

Preliminary trials of certain mathematical activities accompanied by physics concepts demonstrated very promising results (Popkova et. al., 2023).

The MEU program is also motivated by the steady decline in mathematical performance of Australian students in international assessments like PISA (Programme for International Student Assessment, 2024). This trend, also observed in other developed countries, is linked to decreasing enrolment in mathematics-intensive subjects (Blair et al., 2024) and is of concern because of the need for science and mathematics skills in modern advanced economies.

Maths aversion and Maths anxiety

Math anxiety (Commodari & La Rosa, 2021) is recognised as a widespread issue for all age groups. For example, approximately 93% of adult US Americans indicate that they experience some level of math anxiety (Luttenberger, Wimmer, & Paechter, 2018). Maths aversion and anxiety are widely investigated psychological problems (Stodolsky, 1985), which are not only affecting the psychological well-being of children (Winarso & Haqq, 2019) but also affect students' performance at school (Furukawa, 2018; Passolunghi, De Vita, & Pellizzoni, 2020). The Programme for International Student Assessment (PISA) studies in 2012 investigated maths anxiety amongst 15–16-year-old students in parallel with their maths performance. Researchers found a strong negative correlation between students' achievements and reported math anxiety (Luttebberger et al., 2018). Furthermore, this correlation remained stable for several assessment periods (OECD, 2013 Lee, 2009).

Research has shown that maths anxiety is linked to an immediate, instinctive aversion to seeing a math problem on a test, even before attempting to solve it (Pizzie, & Kraemer, 2017). A highly cited study by Bursal and Paznokas (2006) provided evidence that attitudes toward mathematics and maths anxiety can influence attitudes toward science, indicating that maths anxiety might have broader implications beyond mathematics education alone. Even teachers with maths anxiety experience a loss of confidence, making them less effective in teaching science compared to those who do not have such anxiety (Pizzie, & Kraemer, 2017). The long-term consequences of maths anxiety among students include avoiding mathematics and mathematics-related subjects in school, as well as an aversion to pursuing STEM professions in the future (Ashcraft & Krause, 2007). One of the effective strategies to confront maths anxiety used hands-on and whole-body learning in lessons (Thuneberg, 2016). Researchers argued that human cognitive processes depend on physical perception and consequently, body movement (Valentini, 2022). A well-known historical example is finger-counting. Using the body as an effective tool for understanding reality makes advanced mathematical concepts less abstract. Whole-body learning has been recognized as a powerful method for the comprehension of some mathematical concepts such as fractions (Isbister, 2018). Maths for Einstein's Universe uses hands-on and whole-body learning to help minimise maths anxiety.

Another successful strategy for alleviating maths anxiety is connecting maths to real-life situations (Maass, Geiger, Romero Ariza, & Goos, 2019). Making mathematics relevant to students' interests is seen as a useful tool to build students' confidence. Mathematics examples that are related to students' interests include estimation of pocket money, mapping, and maths for sporting competitions.

The approach presented here emphasises mathematics as a tool for the world. Abstraction is avoided by using toys and activities, connecting mathematics to physics concepts and capitalising on students' interests in questions such as the number of galaxies in the universe, the size of atoms or the total mass of all humans on Earth.

One of the negative factors which contribute to maths anxiety is when students are embarrassed because they cannot find the answer and feel unable to solve a problem. Children are familiar with toys and children naturally know how to play. Consequently, Maths for Einstein's Universe uses toys for teaching which may result in greater emotional positivity about learning. Research with Einstein-First, for instance, has shown that when learning with toys and activities, low-achieving students have similar improvements to higher-achieving students (Popkova et. al., 2023). Once students have improved their maths performance, the positive outcome encourages future learning.

According to Blazer (2011), female students experience maths anxiety more often than male students. The success of the Einstein-First program regarding girls' performance (Kaur et. all., 2020) indicates that the same approach used in science may be beneficial for alleviating female maths anxiety. The Einstein-First program starts its learning progression in Year 3 (7-8 years of age), a year before Blazer suggested that maths anxiety in female students begins to be manifested. We hypothesise that activity-based mathematics learning may help ensure that students develop a positive attitude to maths, preventing maths anxiety before it starts.

Additionally, the findings of Pizzie and Kraemer (2017) highlight the need for teacher education programs to address math anxiety as well. Educators can improve preservice teachers' attitudes and skills in both mathematics and science by integrating teaching methods, offering combined math-science courses, and including content area reading.

The structure of the *Maths for Einstein's Universe* program

To meet the goal of introducing all students to the core understandings of modern physics by Year 10, we identified five areas in which mathematical concepts need to be introduced early in schooling. They are summarised below —first in brief, and then each is expanded to explain the links and the learning sequence. Finally, we summarise the module content in Table 1. The five areas of understanding are the development of:

1) logarithmical thinking to facilitate understanding of scale (Mahajan, 2018);
2) estimation skills to allow students to understand magnitudes without detailed arithmetic;
3) vector thinking to allow students to develop skills in a symbolic representation for understanding the addition of forces, momentum, waves, and quantum probability;
4) probabilistic thinking (Johnson-Laird, 1994) to facilitate understanding of the intrinsic probabilistic nature of the quantum world;
5) the use of the geometry of curved space to allow students to understand Einsteinian gravity, gravitational lensing images and time dilation.

Furthermore, Module 1: *Extreme Numbers* is invaluable for understanding numbers beyond a million, which is introduced in the Australian mathematical curriculum starting in Year 5. This module is particularly important due to the lack of specific guidance on how to introduce the concept of extreme numbers. It also provides a deep understanding of decimals and powers of ten, which are typical learning outcomes for students aged 10–12.

Vectors (Module 3) are first introduced in the Australian curriculum in Year 2 for mapping locations. However, this learning can be extended to 9-year-old students to connect with the science curriculum on forces. The advanced part of Module 3 aligns well with the Year 9 Australian science curriculum, because it connects closely to quantum concepts.

The study of parallel lines and the angle sum of a triangle being 180° can be enhanced by exploring *Curved Space* geometry (Module 5) in Year 7 for students aged 12-13, aligning with science lessons about space.

The concept of Probability (Module 4) is a fundamental part of the Australian curriculum across all year levels. The *Probability* module should be introduced before ages 15-16 to prepare students for understanding quantum ideas.

Finally, *Estimation* (Module 2) aligns with the Australian trend of incorporating mathematical modelling into the curriculum and practicing estimation skills across all year levels (Government of Western Australia, 2024). It is well-suited for students aged 12-16 after they have adopted the powers of ten technique.

The overarching aim of ***Maths for Einstein's Universe*** (MEU) is to teach students that mathematics is more than just numbers and that early introduction of interesting mathematics can create positive anticipation of future learning. The modules of MEU are described using child-friendly names as shown in Table 1. Each module can be delivered at higher or lower levels of complexity according to the student age group. The duration of the standard component for each module is 6-8 hours over one or two months depending on the age and abilities of the students. Each module also includes a 2–6-hour extension for students who demonstrate additional interest.

Table 1. Five modules for *Maths for Einstein Universe* provide mathematical curricula for a better understanding of Einsteinian physics concepts from the theory of relativity and cosmology to the quantum world.

Module Name	Child-friendly Name	Module Description
Module 1. Extreme Numbers	Powers of the Universe	This module uses activities to introduce students to the world of tiny and huge numbers from the size of atoms to the age of the universe. *Powers of two* activities provide a step from linear thinking to logarithmic thinking. The transition to powers of ten reinforces and extends students' logarithmic mental number line, allowing the powers of ten notational tool to be used to conceptualise the extreme scale range of the universe. Students develop comparison and estimation skills for extreme numbers, as well as learning how to multiply and divide these numbers, making extreme numbers easy and manageable with simple arithmetic. They are empowered by a new-found ability to estimate the vast scale of the universe.
Module 2. Estimation	Roughly right is better than exactly wrong	Fermi's statement *Roughly right is better than exactly wrong* defines the theme of this module that builds on Module 1, emphasising powers of ten notation as tools for obtaining "roughly right" answers to estimation problems, combining it with statistical sampling concepts. Students learn how to estimate extreme numbers, such as the number of *stars in the universe* or *the number of solar photons hitting their palms.* Through an experiment, they discover the $\sqrt{N}$ rule for sampling

		which is relevant to both opinion polls and photon imaging in telescopes.
Module 3. Vectors	Maths of arrows	Students learn to connect physical concepts with symbolic representations in the form of arrows. They discover experimentally that the addition of arrows is commutative. Through whole-body force activities and the graphical addition of arrows, they recognise that both scalars and vectors are needed to describe the world. Vector understanding is applied to navigation, acceleration, forces, magnetism, wind and waves, and classical and quantum interference, all represented graphically. They are introduced to the phasor wheel, a rolling wheel toy used to connect wave motion to a rotating vector.
Module 4. Probability	Maths of chance	From Newtonian certainty to Einsteinian uncertainty: the intrinsic probabilistic nature of our world is revealed by considering photons reflecting off windows. We see quantum uncertainty in beam splitters, light interference, and radioactivity – they are all *chance machines*! This is contrasted with uncertainty for coin tossing and dice where randomness comes from variable initial conditions. Students play games that reveal the concept of statistical distributions and do experiments to reveal the $\sqrt{N}$ rule. Applying this to extreme numbers, students learn how chance connects to *statistical certainty*.
Module 5. Curved Space	Curved space	This module generalises geometry to the real non-Euclidean geometry of spacetime. Students study geometry on curved surfaces to learn that geometry reveals the shape of space. They study the properties of straight lines in curved space, discover that *straight is curved if space is curved,* and discover the violation of Euclid's third postulate and the general breakdown of Euclidean geometric formulae. A role-play asks *what is the space we live in?* and reveals how concepts of space and gravity changed over the years, culminating in the modern understanding that gravity is curved spacetime. The tiny magnitudes of curved space errors on Earth are contrasted with their dramatic effects in astronomical images.

To develop a learning sequence for the five core topics/modules, we took into account a) the current school mathematical curriculum, b) results from trials in which we assessed students' ability to conceptualize the relevant mathematical ideas, c) the mathematics requirements needed to match and enrich the already developed Einstein-First year 3 to 10 science learning progression, and d) the progression of mathematical learning from module to module.

The five modules form an interrelated set which naturally forms a sequence while allowing for repetition, extension, and reinforcement of concepts throughout the program. The connections between the five modules are illustrated schematically in Figure 1.

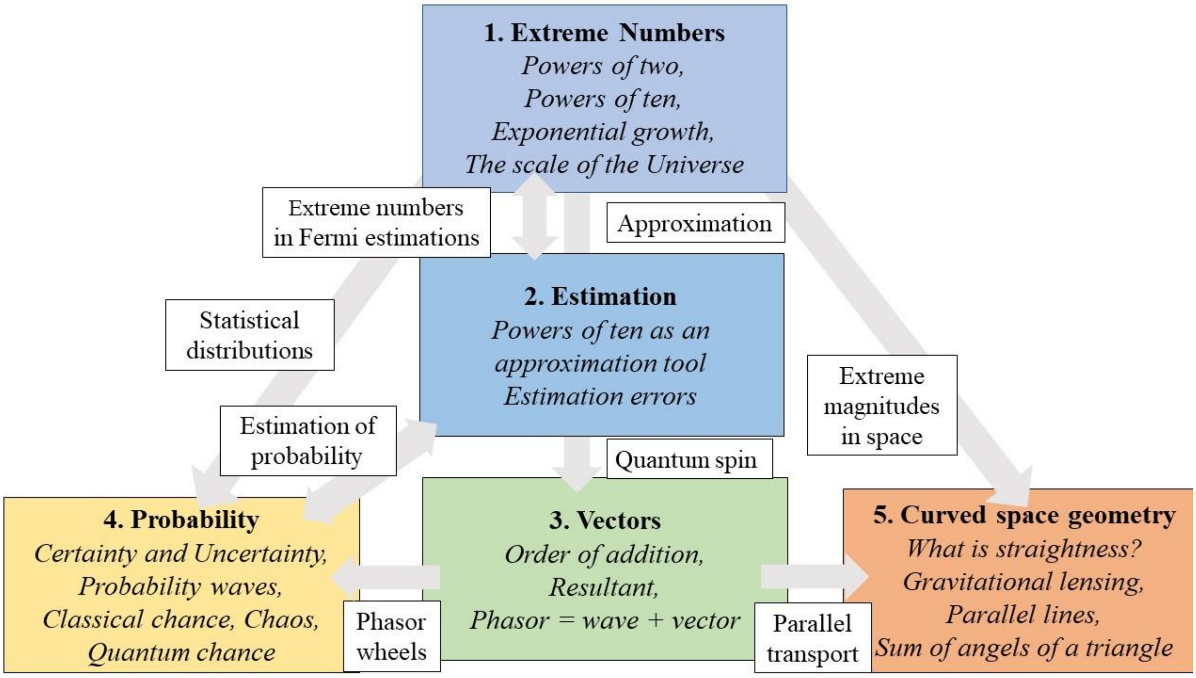

Figure 1. Maths beyond arithmetic: teaching sequence and interconnections between five modules of *Maths of Einstein's Universe*. The grey arrows in-between modules indicate topics that can be developed only with pre-knowledge from other modules.

We start with *Module 1: Extreme Numbers*. All modern areas of physics involve enormous numbers whether it be cosmological distances or the number of atoms in a cup of water, as well as the tiny numbers needed to describe the quantum world. All of these are beyond our direct perception but are accessible through mathematical reasoning if we first introduce powers of ten and the logarithmic number line, as described in Table 1.

The first module leads into *Module 2: Estimation,* because powers of ten notation always involves approximation in the form of rounding. Approximation is a challenging topic for students for whom mathematics has focused on finding single correct answers. However, powers of ten exercises in which numbers are approximated to the nearest power of ten and placed on a logarithmic scale emphasises the connection between powers of ten and estimation. Once students grasp powers of ten thinking for understanding the scale of the Universe (*Module 1*), this skill contributes to *Module 2* which is designed to deepen their understanding of estimations. Students practice the power of ten arithmetic and solve problems with numbers that are inaccessible for exact calculations or experiments. One example of this sort of problem is to estimate the number of galaxies in the visible Universe from the Hubble Deep Field image that shows thousands of galaxies in a tiny patch of sky (Williams et al., 2000).

Modules 1 and 2 focus on numbers, but many quantities are much more powerfully represented by arrows in the form of vectors, which is the subject of *Module 3*. Vectors are a higher-level representation where direction and magnitude are equally important. Their use is familiar in weather forecasting, but many aspects of physical reality, from forces to magnetism, are naturally vector quantities. Also, vectors can represent the spin of objects, whether they be planets or the extremely tiny spin of single electrons which add up to create macroscopic magnetic forces.

Module 4: Probability focuses on the uncertainty of events (processes) in our universe and connects with *Module 2.* We identify two types of probability: 1) the intrinsic statistical nature of all quantum processes (quantum probability) such as the chance that a photon will arrive at a certain location, and 2) probability associated with imprecise knowledge and complex dynamics (classical probability), such as knowledge of life elsewhere in the universe or the motion of a rolling dice. As an illustrative and interesting example of mostly classical probability, we use the Drake equation for estimating the number of civilisations in the Milky Way galaxy. This connects probability to the other module topics: estimation and extreme numbers. We also use videos of single photon interference (Rueckner & Titcomb, 1996) to allow students to visualise the transition from quantum probability to *statistical certainty* when the N-value becomes large enough.

Module 5: Curved Space extends geometry from the idealised geometry of Euclid to the actual geometry of curved space, as observed throughout the universe. The module links both to extreme numbers and to the understanding of vectors. Extreme numbers connect to the mass range of observed black holes where spacetime curvature has dramatic and surprising effects, to the enormous rigidity of space and to the tiny curvature of space and stretching of time in the weak gravity of the Earth. The displacement of vector directions when they are *parallel transported* on curved surfaces allows curvature to be identified.

Maths for Einstein's Universe: Activity-based learning

The goal of *Maths for Einstein's Universe* is to give students awareness of the breadth and scope of mathematics, to open their minds to the power of mathematics, to enhance their understanding of interesting things, and to discover maths that does not stress the exactness of traditional arithmetic. Activities with toys and models focus on tangible physics concepts. For example, as discussed further below, an activity about photons and beam splitters exposes the statistical nature of quantum reality while motivating the learning of probability. Activity-based group learning is free of the stress of obtaining exact answers and instead creates an enjoyable and memorable experiences.

Each lesson begins with an activity that, where possible, involves multi-sensory learning. By placing the activity first, strong student engagement is secured, creating the ideal platform for exploring the mathematical and physical concepts that the activity was designed to illustrate.

The activities used in MEU can be classified in terms of the dominant learning instrument involved. We divide them into four categories: 1) the use of toys and models, 2) mathematical games, 3) role-plays with songs and poems, and 4) the history of discovery. Each of these approaches provides complementary student experiences involving tactile, auditory, social, and story-based learning. Table 2 presents some examples in each area.

Table 2. Four main components are employed to construct activity-based lessons for Maths for Einstein's Universe in a non-abstract and age-appropriate way by using toys, games, drama, and history.

Method	Examples
1. Toys and Models	Lycra fabric models of orbits in curved spacetime, spinning tops and gyroscopes for understanding spin vectors, toy atoms and molecules as tangible representations of extremely small sizes, and toy cars to define straight line trajectories on curved two-dimensional spaces, and learning the right-hand rule to find spin vector.
2. Mathematical games	Exponential dice, a game in which dice sides represent powers of ten, and scores move the player up or down a logarithmic number line that extends from -10 (atomic dimensions in meters) to +26 (scale of the visible universe). Rules allow students to develop and practice their skills and match dimensions to known objects. A rice grain game on a chess board gives students a tangible understanding of doublings and powers of two. The tug of war game explicitly introduces the concept of vector addition and the idea of the resultant vector.
3. Role plays and songs	Discovery of Zero is a role-play about a birthday party set 4000 years ago when zero was invented as a placeholder to prevent errors in arithmetic (Blair, D, 2023). Ten Times Alice is a role-play based on Alice in Wonderland in which bites of magical food cause Alice to change size ten times. Songs and imagination reinforce the rules of division and multiplication and allow students to practice logarithmic thinking (Blair, D, 2023).
4. Historical content	The history of zero as an operator enables logarithmic thinking and allows us to imagine the entire universe. The millennia-long quest to determine the value of π to greater and greater precision includes feats of memory and supercomputing. The history of the understanding of space from Euclid to the precise observation of curved space at Wallal (Blair, Burman & Davies, 2022). including Gauss's Mountain top triangle experiment where he explored the understanding of geometry, the testing of Euclid's geometry and the concept that light trajectories define straight lines.

To further illustrate how activities designed with these learning instruments can elucidate mathematical concepts in an age-appropriate and engaging way, we give three examples. Firstly, an exponential growth activity relating to *Modules 1 and 2*; secondly, an activity about vectors in an abstract context (*Module 3*); and thirdly, an activity on probability for a quantum world (*Module 4*).

1.*The mathematician tricks the emperor*: The rice-on-the-chessboard activity is a realisation of the well-known story of the mathematician who tricks the emperor (Demi, 1997). Following the mathematician's demand, students place one grain on the first square and double the number for each square. Students typically manage to fill about 10 squares (2^{10} =1024 grains). Having physically realised the first ten squares, the activity progresses using estimation and imagination. Students have previously learnt through other activities and repeated here, that $2^{10} \sim 10^3$. They apply this approximation to the chess board, recognising that $2^{20} \sim 10^6$, $2^{30} \sim 10^9$ etc, ultimately determining the number of rice grains for square number 64 to be ~16 x 10^{18} grains.

This physical activity connects powers of two to powers of ten, and utilises both estimation and extreme numbers, all connected to the concept of exponential growth. It provides a framework for developing abstract representation and logarithmic understanding of other domains such as time and distance. If one rice grain represents one second, the age of the universe is represented by the square number 60 ~10^{18} seconds. If one grain represents 1 meter, then square number 40 takes you to the edge of the Solar system and square number 54 represents the distance to the nearest star. The combination of activity, imagination, estimation, and simple arithmetic has transported the students from pure numbers to astronomical scales of time and space.

2. *Phasor wheel vectors*: In *Module 3*, vectors are first introduced as tangible representations of forces as described in Table 2, item 2 (Tug of War game). We also introduce two abstract representations. First, we used phasor vectors as a tool for understanding the interference of light. Students observe the striking light interference patterns when a laser pointer is reflected off a soap film. The patterns of light and dark are taught as a vector addition process. A toy called a phasor wheel is introduced, which uses a rolling wheel with a rotating phasor vector and a cam bar attached (Choudhary et al., 2021). It shows the connection between a rotation, wave motion (the displacement of the cam) and the phasor (a radial arrow on the wheel). Bright spots arise when the phasor arrows from two paths are aligned.

Secondly, we used vectors to describe spin. We use the term spin, rather than angular momentum because we wish to introduce the concept of quantum spin using the analogy with classical spin. To introduce the spin vector, we simply introduce the right-hand rule and use simple toy tops to study the physical concept of spin, including the addition and cancellation of spin and the precession of spin vectors when forces are applied.

3. *Quantum probability and partial reflections*: Students investigate the reflection of a laser pointer from a window or else they photograph coloured objects seen directly in a mirror and in a partial reflection from a window. Pre-knowledge is introduced, that light comes as photons and their energy depends on their colour. The key observation made by students is that partial reflections do not change colour – they only change their intensity. This means that all the photons are reflected "whole". How does each photon "know" where to go? It's a matter of quantum chance. For normal window glass, the probability of a single photon being reflected very roughly equates to the probability of rolling a double six with a pair of dice. By rolling dice and tabulating double sixes versus all other scores, students obtain an experiential connection between photon reflection, the statistical nature of quantum physics, and the concept that image brightness represents the number of photons.

Further activities are presented in Lonshakova et al. (2025) and Lonshakova et al. (in press), the latter in the second paper titled *Maths for Einstein's Universe. Part 2: Development using the Model of Educational Reconstruction.*

Conclusion

We have presented a hypothesis that the modernisation of mathematics content to make it more relevant to student interests and to science, combined with less focus on arithmetic, is likely to help reduce maths anxiety. We have suggested that maths anxiety could be reduced by emulating the approach used in the Einstein-First program which has shown the significant benefit of an activity-based modern curriculum in improving teenage girls' attitudes to science.

We have presented five modules for the *Maths for Einstein's Universe* program. They focus on key mathematical areas that a) enhance understanding of the physical universe, and b) change the emphasis of mathematics from exactness to a broader conceptual foundation. It is suggested that by enhancing the understanding of modern physics concepts and associated mathematics from the early primary school level, we will be providing students with intellectual tools that are of value throughout our modern world while also preparing them for exciting future learning. The results of the first trials are presented in the second part of the paper.

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Ethics Approval and Consent to Participate

The participants involved in this study gave their informed consent for this publication. The research was carried out under the University Ethics Approval No. 2019/RA/4/20/5875.

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