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Maths for Einstein's Universe. Part 2: Development using the Model of Educational Reconstruction

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Abstract

This paper examines how the Model of Educational Reconstruction (MER) served as a guiding framework for developing and refining *Maths for Einstein's Universe* — a program integrating modern scientific concepts into primary and middle school mathematics. The structure and rationale are detailed in the accompanying paper, *Maths for Einstein's Universe – Tools for Understanding Modern Reality. Part 1: Rationale*. The central research question is whether the MER can effectively support the design of a program that fosters both conceptual understanding and positive attitudes toward mathematics. To explore this, the program was developed through iterative cycles: creating learning sequences, testing them in workshops and classrooms, collecting feedback, and revising accordingly. The MER structured both content development and evaluation. Evaluation tools included knowledge tests, attitude questionnaires, and feedback from students, teachers, and other stakeholders. The focus of this paper is on how the MER enabled continuous improvements across more than 300 hours of teaching. The findings demonstrate that the MER can support the integration of innovative, conceptually challenging content into school mathematics. Students showed measurable gains in understanding and engagement, and teachers reported increased enthusiasm and confidence. This work presents an application of the MER in mathematics education and highlights its potential for curriculum innovation grounded in modern physics.

Introduction

Although quantum physics and relativity underpin much of today's technology—from GPS and semiconductors to medical imaging and quantum computing—these foundational scientific ideas remain largely absent from school curricula. This disconnect stems from multiple obstacles: the perceived conceptual difficulty of Einsteinian and quantum ideas, a lack of suitable age-appropriate teaching materials, and minimal teacher training in modern physics (Bouchée et al., 2023; McInerney & Sutton, 2023). As a result, students often complete their schooling with no exposure to the scientific framework that underpins the technology they use every day. In a systematic effort to address this situation, the Einstein-First program has spent more than ten years trialling and implementing learning modules, culminating in the implementation of a spiral learning progression that builds a modern understanding of physical reality across the age range 7-15 years. (Kaur et al., 2024, Einstein-First-Annual-Report, 2023).

A crucial element needed for understanding Einsteinian physics is appropriate and relevant mathematics, which is also an essential tool for understanding many other aspects of the modern world. In the accompanying paper *Maths for Einstein's Universe - Tools for Understanding Modern Reality: Part 1 Rationale* (hereafter described as Part 1) we presented the motivations for this program, as well as its learning sequence and approach.

Because relevance has been identified as a key factor in improving student attitudes and performance in mathematics (Arthur, Owusu, Asiedu-Addo, & Arhin, 2018; Sasidharan, & Kareem, 2023), our goal was to use the relevance of modern science and social applications to motivate primary and middle school students, with the aim of bridging the gap between abstract mathematical principles and real-world applications. Also, we chose to use group learning through games and activities to provide tangible real-world learning experiences. The five modules of MEU are described in Part 1: 1) Extreme numbers, 2) Estimation, 3) Vectors, 4) Probability, and 5) Curved space. For example, Extreme numbers examines the scale of the universe, bacterial division, and population growth, while Probability explores chance from coin tossing to quantum probabilities of photon trajectories.

MEU takes an interdisciplinary approach by not only connecting mathematics to science, but also utilising drama and songs to reinforce the relevant language and concepts. These pedagogical approaches deepen students' understanding by extending the range of relevance (Quinnell, 2019), as well as enhancing their ability to apply mathematical knowledge across various contexts. This approach is particularly valuable in the context of teaching Einsteinian physics, where complex ideas are often best understood through a combination of mathematical reasoning and demonstrative physical activities, which have already been shown to have great explanatory and motivational power (Popkova et al., 2023)

Trials of this MEU program played a key role in its development, allowing the choice of mathematical concepts to be optimised along with development of the learning instruments. The testing of our innovative approaches through careful observations and testing of many classes allowed us to verify student engagement and learning of mathematical concepts normally considered too advanced for our primary-middle school target age group. Our review of the literature indicates that the concepts addressed in the *MEU* program have not previously been systematically introduced into the primary or middle school curriculum.

The MEU program was developed and optimized using the Model of Educational Reconstruction (hereafter denoted MER). This model requires multiple trials within various settings to enable optimisation. Approximately 400 students were involved in the trials. To describe the cyclical nature of the MER process, the results for key parameters across different modules, the Results section will present key parameters across different modules, including descriptions of the learning process, conceptual understanding, attitudes toward activities, attitudes toward learning through activities, and overall impressions of the program based on stakeholders' reflections. This overview of the program development will be supplemented with future papers giving quantitative results for each module, along with research on attitudes and the program's impact on alleviating maths anxiety.

The MER encompasses professional development that helps teachers integrate science concepts with practical teaching strategies (Duit, Gropengießer, Kattmann, Komorek, & Parchmann, 2012). In parallel with testing the program with school students, we delivered professional development sessions and micro-credential courses (UWA Plus Micro-credentials, 2024), which are themselves a significant part of the MER process. The aim of these sessions was not only to introduce teachers to the concepts, but also to demonstrate implementation of activities that are used to deliver the content and concepts. The teacher professional development also collected teacher feedback, and teachers' evaluation of the modules. The purpose of this paper is to show how the MER has been an effective framework for introducing *Maths for Einstein's Universe*. To the best of our knowledge, MER has not previously been applied to the development of a cross-curriculum program.

In the subsequent sections, we will present an overview of the MER process and provide a rationale for its suitability in the development of the MEU program. Following this, we will detail the trials conducted during the development process, describing the methodology employed for development. Finally, we will give specific examples that demonstrate how iterative cycles often contributed to the refinement and optimisation of the program.

Model of Educational Reconstruction: Literature review

The Model of Educational Reconstruction (Duit, 2007) is a widely recognized framework designed for adapting and introducing new concepts into classroom science curricula, particularly those not yet widely taught in schools. One of the key strengths of the MER approach is its close integration of scientific content and empirical research (Duit, 2012). By systematically combining these two elements, MER enables the optimized development of learning activities and teaching methods for a wide range of science concepts.

Many authors have used the MER approach specifically within science education, where it supports the introduction of new and conceptually challenging scientific ideas. Sam, Niebert, Hanson, and Twumasi (2015) used the MER to introduce new coordination chemistry concepts; Reinfried (2006) used the MER to improve the teaching of geoscience concepts such as plate tectonics, and Felzmann (2017) employed the model to enhance learning about glaciers and ice ages. González-García, Marbà-Tallada, and Espinet (2023) integrated green chemistry into school curricula using the MER, while Jáč (2024) used the MER for introducing biological concepts such as cell theory, evolution, genetics, and biodiversity. In physics, the MER has been successfully applied to novel learning domains like chaos theory, nanoscience, and climate change (Kersting, Henriksen, Bøe, & Angell, 2018, Niebert, & Gropengiesser, 2013). Kaur et al., 2024) reported on the steps used to introduce Einsteinian concepts across eight years of schooling. This paper is the first paper to our knowledge that reports the use of MER for mathematics education.

Figure 1 illustrates the MER approach. Its structured methodology allows complex scientific ideas to be effectively taught by breaking down the process into the five steps illustrated in the figure.

The first step involves clarifying the content, where scientists and educators collaboratively analyse and select important, relevant concepts not yet included in school curricula. This involves appropriate simplification of concepts for students. The second step, elementarization, involves developing models and analogies that make complex topics more accessible by linking them to everyday experiences. This process is central to the Einstein-First approach, which uses physical models and analogies to clarify key concepts and support conceptual understanding (Popkova et al., 2023). This often includes identification of weaknesses in the chosen models and how these can contribute to learning (Lonshakova, Adams & Blair 2025). The third step identifies students' misconceptions through pre-tests, allowing the teaching approach to be tailored to their specific needs. In the fourth step, a coherent learning environment is created, with lessons following a logical progression. Teaching methods are developed to ensure effective content delivery. These steps were used in parallel and are closely interconnected. We first developed the learning sequence, followed by the design of a pre-test based on our assumptions about students' prior knowledge and the intended learning outcomes of the program. The learning sequence was then refined in response to the insights gained from the pre-test. The final step is the evaluation of learning outcomes through tests and interviews, which measure the success of the program, assess students' and teachers' attitudes. This data feeds into the next cycle of improvements.

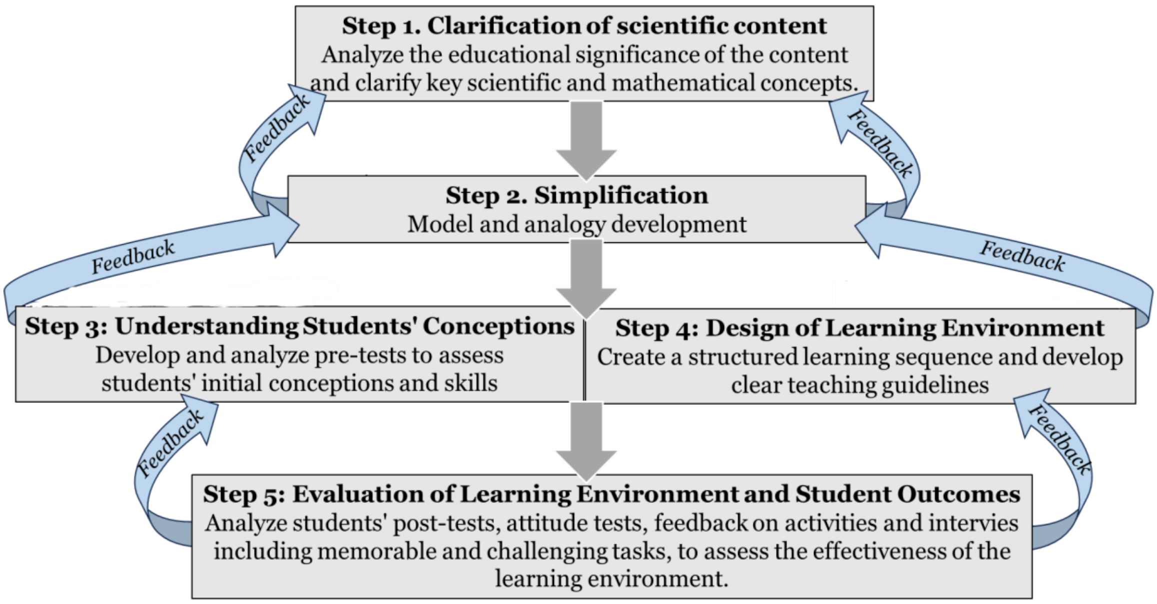

Figure 1. The iterative process for implementing the Model of Educational Reconstruction

The iterative approach adjusts the learning environment and teaching strategies. Because the teaching strategies are refined based on input from both teachers and students, as well as data from the learning environment, the approach should ensure that the program improves with each iteration, becoming more refined and better suited to meet students' needs, thereby enhancing both engagement and understanding. The MER approach enables the creation of a well-structured learning sequence, teacher guidelines, and assessment procedures specifically designed for the targeted age group.

Since feedback loops are a key component of the MER approach, it is essential to discuss the evaluation procedure. Duit et al. (2012) discuss the wide range of methods, from qualitative to quantitative, that can be used to study learning processes. There are two significant challenges in evaluating the program. First, since many of the scientific concepts are being demonstrated to students in this age group for the first time, there are no clear expectations for how children of a certain age will comprehend them. Second, the interdisciplinary approach makes it difficult to align the program with discipline-defined parts of the curriculum. Consequently, during the pilot testing phase, there were no prior benchmarks, nor expectations, to guide evaluation.

Our research addresses programs that teach fundamental concepts, that we determined to be very important for improving numeracy and scientific literacy, while minimising the risk of provoking maths anxiety, as discussed in Part 1. Generally, the material is new to the age group being tested (as measured by the mean post-test/pre-test ratio). Of particular importance is that the programs should provide engaging and valuable learning experiences for *all* students.

Based on this rationale, we designed tests where the results are not expected to follow a normal distribution, but rather to achieve high post-test scores by a large majority of students. Uniform post-test scores indicate that the new concepts introduced are accessible to all students. Detailed analysis of results by concept category then allows easy identification of areas where the teaching approach needs to be refined. Our approach in the MEU program contrasts with the more common approach in which educators design tests to provide strong discrimination of student learning. (Aitkin, and Longford, N. (1986)). Such tests by necessity must combine a mean score with an adequate standard deviation (Kurtz and Mayo, 1979).

Implementation of *Maths for Einstein's Universe*

The challenge in applying the Model of Educational Reconstruction as a framework lies in the need for multiple trials with various age groups and formats to successfully complete cycles of improvement. For initial trials we offered approximately 40 diverse 45-minute out-of-school workshops covering components from each of the five modules as described in Part 1. These workshops attracted over 700 participants. Following the initial trials (not reported here), both long-term and short-term programs were designed. Table A (see Appendix A) summarises the details of the programs relevant to the five modules: 1. Extreme Numbers; 2. Estimation; 3. Vectors; 4. Probability; 5. Curved Space.

To reduce selection bias and ensure a diverse participant base, the trials involved students with a wide range of academic performance levels, from underachieving to high-achieving groups (see Table A in Appendix A). The program was delivered both as an optional enrichment activity and as part of the standard curriculum in whole-class settings. Following the MER, the content and approach of *Maths for Einstein's Universe* were taught to teachers through workshops and accredited micro-credential courses (UWA Plus Micro-credentials, 2024), summarised in Table B (see Appendix B).

Methodology

Data collection
In line with the MER, data collection focused on capturing various dimensions of the program. Classroom observations provided insights into real-time learning dynamics. We assessed students' pre-existing conceptions using knowledge pre-tests and evaluated their learning outcomes through post-tests. To understand how students perceived the activities and the overall program, we administered attitude surveys. We also gathered teachers' perspectives through structured interviews and questionnaires. In total, we collected data from nearly 400 students, 50 teachers, and 10 parents.

Observations
Observations of the learning process in every program were made by author Anastasia Lonshakova. Data regarding students' misconceptions were obtained from the initial preliminary trials. Other observations were based on the trials listed in Table A (see Appendix A), that included students across an age range from 7 to 15 years. Because the program covered material normally outside the school curriculum, we had no benchmarks or prior data to calibrate our expectations. For this reason, observations and qualitative measures were essential for testing and refining lessons, evaluation of the optimum age for introducing learning content, and evaluation of student responses.

The qualitative observations are drawn from post-lesson notes for each workshop or lesson as well as some video and photographic data. We differentiated the MEU activities to suit various age groups and ability levels, while in some cases mixed ages allowed direct comparison of age-related engagement. The observations allowed us to obtain the following general description of student performance and age-related aptitudes, providing data that were used to define the next iteration of the MEU program.

We report observations under the six parameters listed below, categorised by age groups Years 3-4, Years 5-6, and Years 7-9:

a) students' pre-conceptions and misconceptions
b) engagement with toys, mathematical games and role-plays
c) duration of concept engagement
d) language development
e) age-dependent learning styles, and
f) learning retention.

Tests

Tests were created to assess students' understanding of the key concepts before and after the program. Generally, all questions on these tests were directly related to the activities covered in the module. According to the MER framework, the pre-test also aimed to identify students' initial misconceptions before the program began. Attitude questions were used to evaluate students' attitudes towards *Maths for Einstein Universe*, as well as preferences for specific activities. Tests were administered both before and after each module, with a time allocation of approximately 20 minutes for completion. A group of eight researchers in physics, mathematics, and education, as well as by teachers reviewed and validated all test items. For the knowledge and attitude questionnaires, student data were anonymized and processed by team members in accordance with the established marking guidelines.

Stakeholders' reflections

Part of the MEU program development was analyzing stakeholders' reflections on the program by collecting responses from parents, students, teachers and those who completed micro-credential courses. In the initial response cycle, we conducted free-form discussions about the program to identify key themes frequently highlighted by participants. Their verbal responses were transcribed, reviewed and validated for accuracy. In subsequent cycles, we used these parameters to develop an interview questionnaire, enabling a more structured and targeted collection of evaluative data.

Application of MER in Program Development.

In this section, we will use examples to describe the implementation of the MER process under four headings A) Preliminary observations for program development, B) Knowledge testing for program improvement based on Module 1, C) Attitude testing for program improvement based on Module 3, and D) Stakeholder responses based on all five modules. Each heading presents examples that demonstrate how aspects of the program were refined through the MER approach.

A) Preliminary observations for program development

Here we report observations under the six observation parameters described above. The first, pre-conceptions and misconceptions, was used for designing pre-tests and is based on observations of the preliminary trials. The five other parameters reported below were based on the trials listed in Table A (see Appendix A).

a) ***Students' misconceptions***

To design suitable pre-tests, it was first necessary to have initial data regarding student pre-conceptions and misconceptions. This was obtained through discussion and observation during the preliminary trials described above under 'Implementation'. Workshops played a significant role in identifying and exploring students' misconceptions and preconceptions. For example, during one of the workshop discussions, it became clear that students expected future technologies to enable interstellar travel. Even though they were aware of the speed limit imposed by the speed of light, they had little appreciation of the actual scale of cosmic distances. They did not realise that such journeys would span many generations, interstellar travel fundamentally impractical under current physical constraints. In response, a section titled *"Where is Everybody?"* within the *Estimation* module was developed (Blair, expected in 2025). It includes rapid estimations with powers of ten of ratios—such as the maximum possible speed compared to interstellar distances—to help students grasp the true scale of the universe and understand the physical limits it imposes. Discussions with students revealed numerous pre-conceptions and misconceptions for which we provide important examples in Table C (see Appendix C). Notably, since Einsteinian concepts are currently absent from the school curriculum, no age-dependent differences in misconceptions were observed during classroom observations, as students across all year levels appeared similarly unfamiliar with the content.

b) ***Engagement with toys, mathematical games***

From observations of students in Module 1, we concluded that abstract concepts like powers of two and powers of ten can be taught effectively using tactile models such as doubling rice grains on a chessboard, playdough bacteria models (Figure 2), and paper tape measures (see tape measure halving activity in Table C in Appendix C). They were particularly effective for students in Years 3-4, providing hands-on, engaging ways to visualize exponential growth. For Years 5-6, play-acted representations of family trees (see ancestor counts activity in Table C Appendix C) combined with paper representations proved highly engaging, capturing students' interest, and making the progression of powers easier to understand. For students in Years 7-9, interactive activities like scissors-paper-rock knockout tournaments (which halve competitor numbers) combined with analytical components helped deepen comprehension by combining conceptual learning with an element of competition and collaboration. A detailed description of the above activities and others is given in Lonshakova et al (2025).

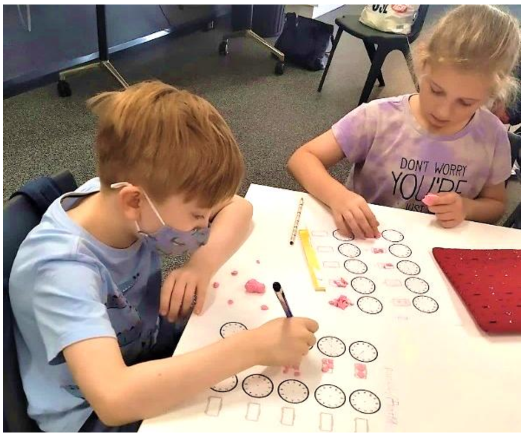

Figure 2. Students engaging in the investigation of exponential doubling processes of replicating bacteria using playdough.

c) Activity Engagement Duration

It was noted that activities could transition into prolonged, unstructured play which shifts the focus from learning to recreation. To address this, the program was adjusted to include structured, sequential tasks and time limits for each activity. This ensured that students remained focused on learning objectives, which improved the educational outcomes by helping students maintain attention on the concepts being taught.

d) Language development

We found that role-playing is highly effective for younger children, particularly in Years 3-5, as a method for learning new vocabulary in context. Integrating songs and storytelling into these role-plays has proven beneficial for language development, allowing students to grasp new terms such as "photon" and "atom" through repetition and immersive learning. This teaching approach reinforces vocabulary and also deepens students' comprehension of the underlying scientific concepts.

For Module 1, we developed and included two short role-plays: *Ten Times Alice,* which explores the concept of scale and reinforces skills in dividing and multiplying by ten and understanding scales in powers of ten; and *Discovering Zero* which introduces the discovery of the concept of zero as a symbol to represent an empty set (Blair, 2023).

e) Age-dependent learning styles

A striking observation was that older students tended to resist unfamiliar or unconventional concepts. For example, the rule for logarithmic rounding—where 4.5 is considered closer to 10 on logarithmic scale than it is to 1—can be challenging for them to accept. In contrast, younger students tend to embrace such ideas with greater ease.

The learning environment was adapted to suit different age groups, ensuring that older students were given analytical tasks, such as calculating the distance of a light-year or estimating the days needed for an entire city to be affected by a virus with specific contagion factors. The same tasks could also be used with younger students to challenge and engage particularly curious individuals. Providing real-world examples, such as population growth or scientific data applications, helped older students see the practical value of logarithmic estimation and clarified the logic behind this approach.

f) Learning retention

Observations revealed that students in Years 3-4 may struggle with retaining certain concepts, such as the rules for multiplying powers of ten. Guidelines to enhance learning retention were created for teachers on structuring activities to strengthen connections between toys, games, and mathematical concepts. Teachers supported these connections through guided discussions, developed worksheets and structured tasks, helping students progress from concrete to abstract thinking. Role-playing activities, complemented by teacher-led reflections, provided engaging, memorable experiences that deepened students' conceptual understanding and recall (Blair, 2023.)

B) Knowledge testing for program improvement

Based on the knowledge of students' misconceptions discussed above, a pre-test and learning sequence were created for Module 1, aiming to achieve three outcomes: a) understanding powers of 2 through doubling and halving processes, b) application of powers of 10 plus multiplication and division. and c) expanded logarithmic thinking through an activity book called *Powers of the Universe* described below. Four of 12 Module 1 activities (Lonshakova et al., 2025) linked to these outcomes and pre- /post-test questions are presented in Table D (see Appendix D). Q4 is connected to the *Powers of the Universe* activity book, for which a pre-test was not appropriate.

For this program, we undertook a sequence of four trials in which changes and improvements were made on the basis of post-test results. Trial results for four individual questions are shown in Figure 3, from programs d, g, e, i, respectively (see Table 1 in Appendix A). In all cases, class learning is obvious from the difference in scores between the pre-tests and the post-tests. Based on post-test results and class observations, changes were made to the learning sequence with the goal of achieving improved results in the next trial.

For the first question, about the doubling of ancestors in each generation (Fig. 3a), the post test score was very high, so that no improvement was required. In the second example which tested understanding of powers of two by halving paper tape measures (Fig. 3b) the first trial had a relatively low score of about 50%. It was clear that students needed to absorb the idea of estimation, being roughly right, to accommodate the concept that ten powers of two are roughly three powers of ten. This gave improved results in the second trial and, with further refinements, class marks increased to 86%.

The question for Figure 3c tested students' ability to multiply using powers-of-ten. The first test score of less than 50% indicated that the addition rule for multiplication had not been sufficiently emphasised. Thereafter more time was allocated to *Lazy Numbering* activities in which students practiced multiple examples of multiplication and division. After Trial 2, an additional 15 minutes were allocated, leading to further improved results, with final scores reaching 95%. Figure 3d presents results on student ability to think logarithmically by comparing numbers in the *Powers of the Universe* book. For the first three trials students used small notebooks in which they had labelled the pages in powers of ten. Significant improvement was achieved when a professionally designed child-friendly book was created for Trial 4. Furthermore, the implementation of Trial 4 in standard classroom settings demonstrates the program's accessibility for typical student populations.

Overall, there was no significant difference in the scores of students from high-achieving classes and those from mainstream classes. This aligns with the central goal of the *MEU* program, which is to make complex concepts accessible to all students through simple, engaging activities.

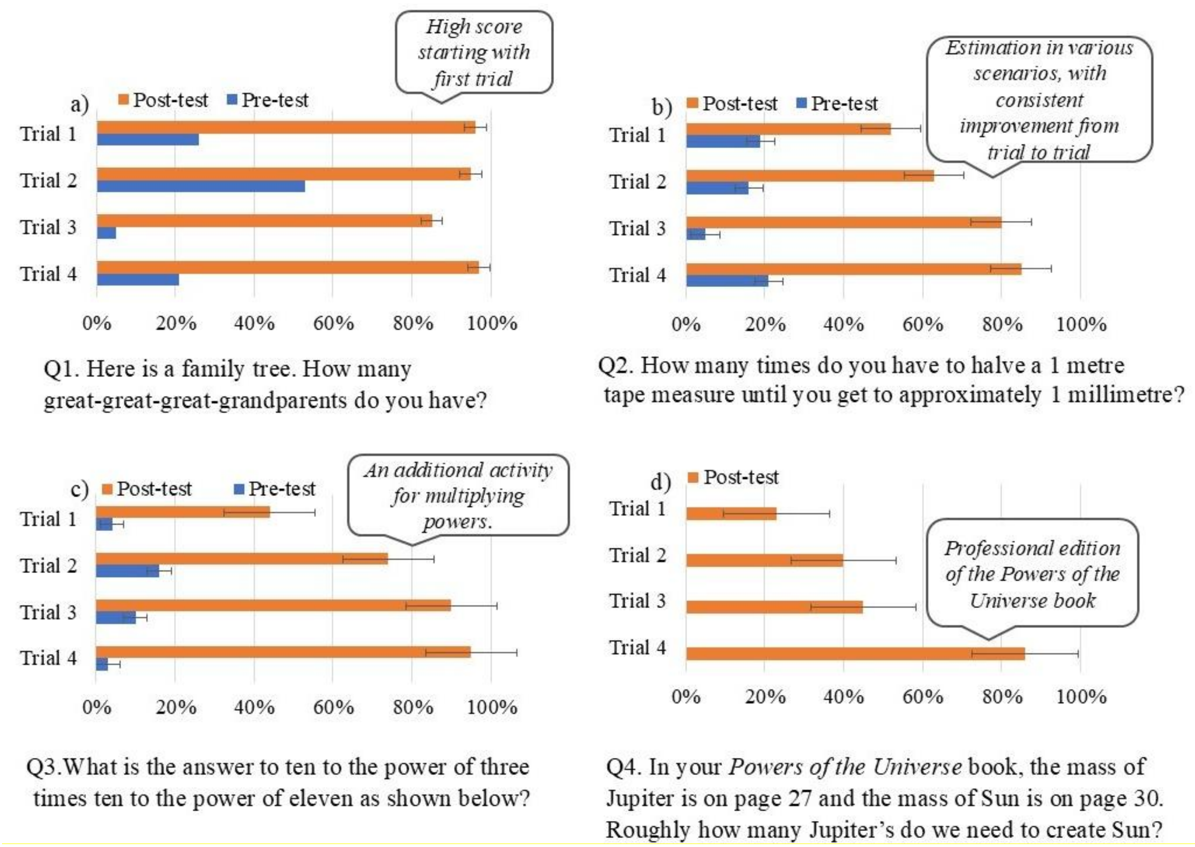

Figure 3. Trial results from Module 1: Extreme Numbers. Pre- and post-test results are shown for four questions (see Table D in Appendix D). The module was trialled in four programs: D, E, G, and I, corresponding to Trials 1 to 4 (see Table A in Appendix A). Error bars represent the standard error of the mean. Question 4 was assessed in the post-test only.

C) Attitude testing for program improvement based on Module 3 -Vectors

As noted in the Part 1, a primary goal of the program is to change students' perceptions of mathematics with a view to reducing the risk of maths anxiety. To evaluate student perceptions, we evaluated students' "yes" / "no" responses to the statement *I like learning maths through activities* for programs e), f), and i). The percentage of students responding positively to this question was relatively high, at 78%, 80%, and 100%, respectively. Although the programs vary significantly in duration and participant demographics, making direct comparisons challenging, the 100% positive response in the longest program, which lasted 100 weeks, suggests that the extended format may have contributed to the overwhelmingly positive feedback. It is also worth noting that during this extended program, four students (16% of the initial cohort) left the school and discontinued their participation, while four new students joined the program at later stages. For this reason, we report this observation qualitatively rather than treat it as a statistically verified finding.

A second example of attitude testing was asking students to recall *three activities they remember and rank them from most interesting to least interesting*, for two cycles of Module 3, *Vectors* that were parts of programs e) and h) in Table A (see Appendix A). The main activities are listed in Table E (Appendix E), with selected examples illustrated in Fig 4.

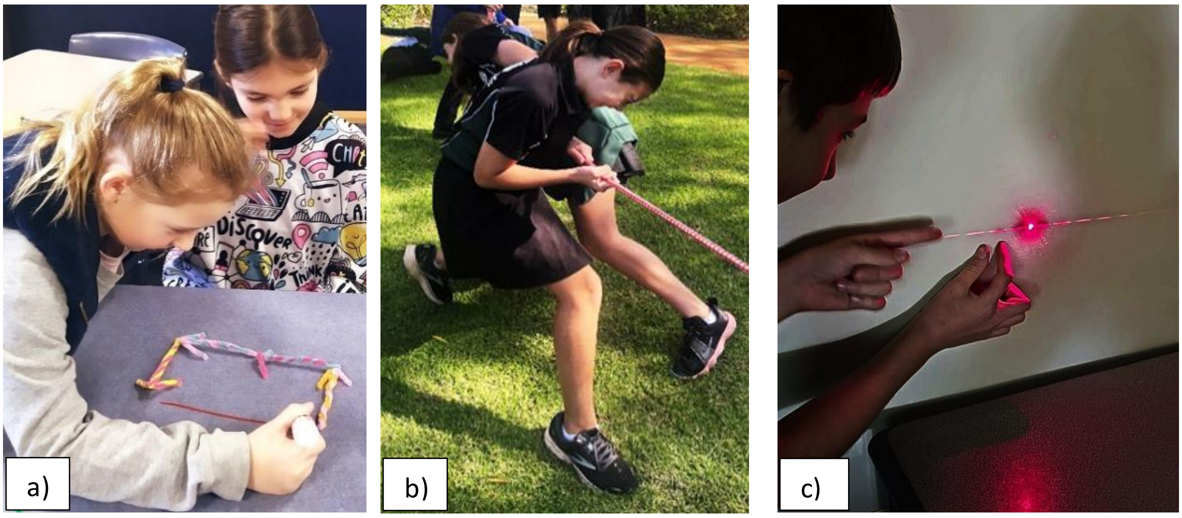

Figure 4: a) Students practice vector addition using toy arrows. b) Students explore the concept of resultant force through a tug-of-war game; c) a student estimates the gap between two interference minima;

Each of the key activities were frequently chosen as top preferences. Students particularly favoured activities such as *Vectors for Forces* in Trial 1, with 35% selecting it as their first choice. After a discussion on real-life applications of quantum spin, the *Spinning Tops and Gyroscopes* activity rose in popularity, becoming the favourite in Trial 2 with 35% of students choosing it as their top activity. A similar trend was observed with the *Vectors for Interference* activity; an additional discussion on light and photons increased first-choice selections from 15% in Trial 1 to 23% in Trial 2. The *Phasor Wheel* and *Arrow Addition* activities, which include analytical components, were initially challenging for many students. When asked *What was the most challenging activity?,* 58% identified *Arrow Addition* and 28% chose *Phasor Wheel* as the most difficult in both trials. However, after modifying the delivery in Trial 2 by adding an additional 20 minutes for paired work, 67% of students selected *Phasor Wheel* and *Arrow Addition* as one of their top three choices, compared to only 47% in Trial 1.

D) Response to Stakeholder Feedback for Modules 1-5

An important component of program development was analysis of stakeholders' reflections on the program. We collected written responses from 14 teachers who delivered the program or assisted in the teaching, including a STEM coordinator and a teaching principal. We also collected selected verbal responses (recorded and transcribed) from a random sample of 10 parents and 15 students. Due to time constraints within the short intervention period—particularly because of the administration of knowledge and attitude surveys—a limited number of interviews could be conducted. Parents and students were selected randomly based on availability, for example, during student pick-up times, rather than on the basis of student performance or attitude. Interviews with parents and students were conducted with participants from the cohort involved in program e), while interviews with teachers were conducted with those who participated in programs d), e), g), h), and i) (see Table A in Appendix A). In addition, many students and teachers provided informal verbal feedback during workshops or post-lesson discussions, although these comments were not systematically recorded. As such, we do not claim that the 39 written reflections are statistically representative of the full participant population. Rather, they are presented as illustrative qualitative insights that contributed to the iterative refinement of the program in accordance with the MER framework.

For the first cycle of responses, we used free-form discussions about the program to identify common parameters that participants frequently mentioned. In subsequent cycles, we used these parameters to develop an interview questionnaire.

According to the stakeholders, MEU was a successful and engaging program for students that provoked their interest in science and mathematics. A school principal who observed the program stated, "*I would have not believed that [this program] could be taught to the target age group (7-14-year-old students) if I had not witnessed the astonishing comprehension that even middle primary students have demonstrated*". To identify areas requiring improvement, we itemised stakeholders' opinions under the five parameters given in Table 1, in which examples of responses are given, labelled T, P and S according to category teacher, parent and student, and include the percentage of positive comments.

A notable exception to the high level of positive responses was teacher confidence. Despite interest in delivering MEU (parameter 4), the 50% score for item 5 indicated that teachers require additional support to develop the confidence to deliver the program. Subsequently the content was included in a teacher upskilling micro-credential courses (UWA Plus Micro-credentials, 2024) that included video instructions for teachers.

After completing a micro-credential course, 88% of the 26 participating teachers reported confidence in delivering the program in the classroom (Blair et al., 2024). This outcome suggests that a structured teachers' education approach is more effective than learning through participating and assisting an external presenter.

Conclusion

We have shown how the Model of Educational Reconstruction framework was used effectively to develop and optimise the *Maths for Einstein's Universe* program for primary and early high school students. Based on observations, tests analyses and stakeholders' reflections, the MER framework was applied to develop and refine modules over several cycles of testing and improvement.

Table 1. Stakeholders Reflections and Citations from Teachers (T), Students (S), and Parents (P).

Reflection parameters	Examples of reflection (exact quotations)	% of Positive comments
1) Program structure and way of learning	**T:** *"A good blend between concepts, activities, games and assessment."* **P:** *"I am truly impressed by the wide range of activities."*	**86%** **100%**
2) Enthusiasm and engagement	**T:** *"Students' attendance remained high even though it was an after-school program."* **P:** *"At home, he always talks about the activities."* **S:** *"It was so cool for all of us to push the car together!"*	**86%** **100%** **100%**
3) Effectiveness	**T:** *"The review questions asked at the beginning of the sessions were a further indicator that students were attending and understanding what had been previously covered."* **P:** *My son had no problems with advanced scientific problems in school extension classes…….*	**86%** **100%**
4) Future collaboration/program delivery	**T:** *"I look forward to teaching it again."* **P:** *"She wants to learn science and maths in this way."* **S:** *"I want to play with toys to learn maths and science."*	**100%** **100%** **100%**
5) Teachers' confidence	**T:** *"I did need to ask her* [Einstein-First team member] *a few times whether what I was teaching was correct."*	**50%**

Note: Responses were collected- from 14 teachers, 10 parents, and 15 students.

We demonstrated how an activity-based learning environment was initially created using preliminary observations, allowing a range of visual, play-based, and analytical activities to be tailored for students in the age range 7-14 years. This enabled identification of appropriate guidelines on activity duration, language development, and strategies for retention.

Our findings confirmed that successive trial iterations can improve learning outcomes, for example, in achieving over 85% proficiency in understanding powers of ten. Attitudes toward the program and its activities were explored, showing that science and real-world connections within activities enhance student interest, and that working in pairs can further boost engagement in challenging analytical tasks. Approximately 80% of students responded positively to the activity-based learning approach—the central methodology of the *Maths for Einstein's Universe* program—used in the short interventions. These results suggest that extending the program could lead to even higher levels of engagement, potentially approaching 100%.

Stakeholder reflections also identified key areas for improvement, such as teacher confidence, and we showed how structured learning through micro-credential courses can significantly enhance this confidence.

The present study is primarily aimed at examining whether the Model of Educational Reconstruction (MER) can serve as an effective framework for developing the *Maths for Einstein's Universe* program. It also investigates whether iterative refinement cycles can support students in mastering conceptual content that currently lies beyond the standard school curriculum. As a result, many of the analyses presented are illustrative in nature.
At this stage, the reported trials are predominantly focused on primary education, as the program's development begins with younger learners and gradually extends to older age groups. Detailed findings—including age-specific methodologies, attitudes, misconceptions, and learning outcomes—will be published separately for each module.

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Ethics Approval and Consent to Participate

The participants involved in this study gave their informed consent for this publication. The research was carried out under the University Ethics Approval No. 2019/RA/4/20/5875.

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References

Aitkin, M., & Longford, N. (1986). Statistical modelling issues in school effectiveness studies. Journal of the Royal Statistical Society: Series A (General), 149(1), 1-26

Arthur, Y. D., Owusu, E. K., Asiedu-Addo, S., & Arhin, A. K. (2018). Connecting mathematics to real life problems: A teaching quality that improves students' mathematics interest. *IOSR Journal of Research & Method in Education*, *8*(4), 65-71.

Bouchée, T., Thurlings, M., de Putter-Smits, L., & Pepin, B. (2023). Investigating teachers' and students' experiences of quantum physics lessons: opportunities and challenges. *Research in Science & Technological Education*, *41*(2), 777-799. https://doi.org/10.1080/02635143.2021.1948826

Blair, D. (2023). Playing Einstein. Perth: Einstein-First.

Blair, D., Lonshakova, A., Santoso, J., Kaur, T., Adams, K., Boublil, S., ... & Scott, S. (2024, September). Collapsing school physics enrollments: Micro-credential courses for upskilling teachers in Einsteinian physics (Empowering educators). In *Proceedings of The Australian Conference on Science and Mathematics Education* (pp. 99-108).

Blair, D. (expected in 2025). *Where is everybody?* Perth, Australia: UWA Publishing

Duit, R. (2007). Science education research internationally: Conceptions, research methods, domains of research. *Eurasia Journal of Mathematics, Science and Technology Education*, *3*(1), 3-15.

Duit, R., Gropengießer, H., Kattmann, U., Komorek, M., Parchmann, I. (2012). The Model of Educational Reconstruction – a framework for improving teaching and learning science. In: Jorde, D., Dillon, J. (Eds) Science education research and practice in Europe. Cultural perspectives in science education, vol 5. Rotterdam: Sense Publishers. https://doi.org/10.1007/978-94-6091-900-8_2

Einstein-First-Annual-Report-2023. Retrieved July 22, 2023, from https://www.einsteinianphysics.com/wp-content/uploads/2023/07/Einstein-First-Annual-Report-2023.pdf

Felzmann, D. (2017). Students' conceptions of glaciers and ice ages: Applying the model of educational reconstruction to improve learning. *Journal of Geoscience Education*, *65*(3), 322-335. https://doi.org/10.5408/16-158.1

González-García, P. J., Marbà-Tallada, A., & Espinet, M. (2023). A model of curricular content for the educational reconstruction of Green Chemistry: the voice of Chilean science teachers and science education researchers. *Chemistry Education Research and Practice*, *24*(4), 1190-1203. https://doi.org/10.1039/D3RP00063J

Jáč, M. (2024). What can educational case studies reveal about how teachers implement the Model of Educational Reconstruction in biology instruction?.*New perspectives in science education 13th Edition*, 141.

Kaur, T., Kersting, M., Adams, K., Blair, D., Treagust, D., Lonshakova, A., ... & Venville, G. (2024). Developing and implementing an Einsteinian science curriculum from years 3-10. https://doi.org/10.1088/1361-6552/ad6dcb

Kersting, M., Henriksen, E. K., Bøe, M. V., & Angell, C. (2018) "General relativity in upper secondary school: design and evaluation of an online learning environment using the model of educational reconstruction." *Physical Review Physics Education Research, 14*(1): 010130.

Kurtz, A. K., & Mayo, S. T. (1979). Statistical methods in education and psychology (2nd ed.). In *Statistical methods in education and psychology* (pp. 3-8). Springer.

Lonshakova, A., Adams, K., & Blair, D. (2025). Introductory learning of quantum probability and quantum spin with physical models and observations. *American Journal of Physics*, *93*(1), 58-68. https://doi.org/10.1119/5.0211456

Lonshakova, A., Blair, D., & Treagust, D. (in press). *Powers of the Universe: Empowering primary school students with powers-of-ten notation*. *International Journal of Science and Mathematics Education*.

McInerney, C., & Sutton, P. (2023). Einsteinian gravitational concepts throughout secondary school. *Physics Education*, *59*(1), 015001. https://doi.org/10.1088/1361-6552/ad001a

Niebert, K., & Gropengiesser, H. (2013). The Model of eEucational Reconstruction: A framework for the design of theory-based content specific interventions. The example of climate change. *Educational design research–Part B: Illustrative cases*, 511-531.

Popkova, A., Adams, K., Boublil, S., Choudhary, R. K., Horne, E., Ju, L., ... & Treagust, D. F. (2023). Einstein-First: Bringing children our best understanding of reality. In *The Sixteenth Marcel Grossmann Meeting on Recent Developments in Theoretical and Experimental General Relativity, Astrophysics and Relativistic Field Theories: Proceedings of the MG16 Meeting on General Relativity Online; 5–10 July 2021* (pp. 2438-2452). https://doi.org/10.1142/9789811269776_0194

Quinnell, R. (2019). Shifting from STEM to STEAM-reflecting on academic identity and scholarship. *International Journal of Innovation in Science and Mathematics Education*, *27*(7). https://doi.org/10.30722/IJISME.27.07.001

Reinfried, S. (2006). Conceptual change in physical geography and environmental sciences through mental model building: The example of groundwater. *International Research in Geographical & Environmental Education*, *15*(1), 41-61. https://doi.org/10.2167/irgee186.0

Sam, A., Niebert, K., Hanson, R., & Twumasi, A. K. (2015). The model of educational reconstruction: scientists' and students' conceptual balances to improve teaching of coordination chemistry in higher education. *International Journal of Academic Research and Reflection*, *3*(7), 67-77.

Sasidharan, S., & Kareem, J. (2023). Student perceptions and experiences in mathematics classrooms: A thematic analysis. *International Journal of Innovation in Science and Mathematics Education*, *31*(2). https://doi.org/10.30722/IJISME.31.02.004

UWA Plus Micro-credentials (2024). Retrieved July 22, 2024, from https://www.einsteinianphysics.com/micro-credentials/