"MONIER STRUCTURES."

BY A. E. CUTLER.

Before discussing "Monier" structures it will be necessary to review as briefly as possible the different principles involved in designing masonry structures, although in doing so I have to go over ground which will be familiar to most of us, in order to make the latter portions of the paper as intelligible as possible. In all masonry structures there are at least two strains to be considered, the dead weight of the structure itself, and a certain strain for the withstanding of which the structure has been erected; if an arch, the load rolling or otherwise that the arch was built to support; if a pier, the load brought on to it by the arch, or bridge, together with the pressure on the side of the pier due to the wind, water, ice, etc., coming into contact with it; if a dam or retaining wall, the weight or pressure of the earth or water to be supported or kept back; but in all these structures, whether arch, dam, retaining wall, pier, etc., the same principles are involved. The first step, after having ascertained the bearing power of the soil on which the structure is proposed to be built, is in designing the footings. It is enough in the case of buildings to proportion the footings to the area of the load to be supported, but in building on treacherous soil it is most important that this should be done with great care, as, if one portion of the foundations is weighted more than another, unequal settlement will take place. For instance, if in a building one wall contains numerous windows or other openings, and another wall consists entirely of masonry, the footings should be so proportioned as not only will a saving be effected, but—what is of far greater importance—if settlement

does take place, it will then be equal, and no cracks will result.
Again, in transferring the load from the flooring joists to the
walls, it must be borne in mind that the footings under the walls
carrying the joists will have a greater pressure to bear, and
must therefore be increased proportionately. Moreover, care
must be exercised that the axis line of pressure should coincide
with the axis line of resistance; and it can be seen from this
that in all cases where one foundation supports two loads, one
much heavier or lighter than another, they should not be
connected together; and, moreover, to be on the safe side, it
will always be preferable to design the base so that the axis
line of the load will strike slightly inside the centre of the area
of the base rather than on the outside, as any inward inclination
is rendered impossible by the interior walls, etc., whereas any
outward tendency can only be counteracted by anchors or bonds
in the masonry; or, in other words, all foundations should be
so constructed as to compress the ground to a slight concavity
rather than to a slight convexity. Fig. 1 shows a case where the
axis line of pressure and resistance coincide; Fig. 2, where they
do not coincide, and the probable result; Fig. 3 shows where a
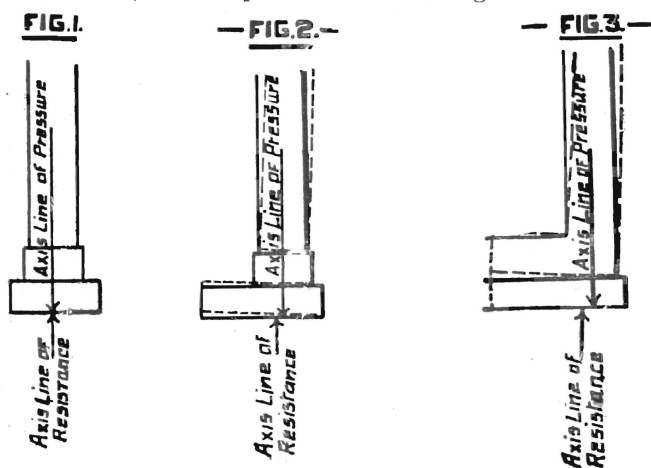
wall with a light load is joined to a pier taking a heavy load;

Fig. 4 shows a case where the axis line of pressure is outside the centre of area of resistance, and the cracks liable to result ; Fig. 5 shows a case where the axis line of pressure is slightly inside the line of resistance, and the tendency of the crown of the arch to close instead of to open. In all these cases the author has taken the simplest possible forms, but just the same law would govern the most complicated. The author may state that in anything approaching bad ground a very slight difference in the pressure will be sufficient to cause the bed to become convex upwards. At Chicago an omission of only 1 or 2 per cent. of the weight, such as openings for windows and doors, usually causes sufficient convexity to produce unsightly cracks, and it is here that the art of constructing foundations on compressible soil has been brought to a state of much perfection ; every tier of columns, each pier, each wall, etc., has its independent foundation, the
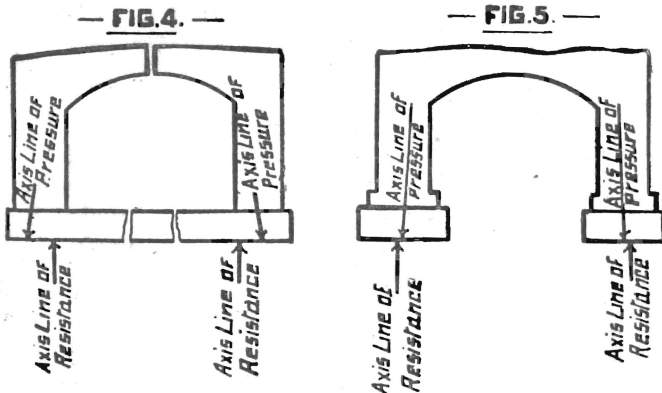

area of which is proportional to the load, and in which the centre of pressure coincides with the centre of resistance ; and with these precautions it has been found that, although considerable settlement takes place, no cracks result. Before leaving this portion of the subject, it is as well to consider the offsets in the footings. The portion of the footing course that projects may be considered as a cantilever loaded uniformly, and therefore the pressure on the ground multiplied by one-half of the length

of the offset is equal $\frac{1}{6}$ of the modulus of rupture multiplied by the breadth of the footing multiplied by the square of the thickness, or—calling (P) the pressure in tons per square inch ; (R), the modulus of rupture ; (b), the greatest possible projection of the footing in inches ; (t), the thickness of the footing course in inches, and expressing this relation as above and reducing, we get—

$$(1) \qquad b = t \; \sqrt{\frac{R}{41 \cdot 6 \; P}}$$

and therefore the projection required can be found. This will give a load on offset that would just produce rupture, so that the factor of safety must be allowed for. In concrete, taking the modulus of rupture as equal to 150lbs. per square inch and a factor of safety at 10, with a pressure on the offset of $\frac{1}{2}$ a ton per square foot, the offset will be 0·8 of the thickness, with 1 ton 0·6, and with 2 tons 0·4. It will be seen from the foregoing that the breadth of the offset is proportional to the square foot of the modulus of rupture, and, therefore, if we can increase the modulus of rupture we can also increase the set-off. It will be shown later on that by the "Monier" method it can be increased to equal the compressive strength of the concrete, say from 150lbs. to 2000lbs., or, in other words, the offsets can be increased from 0·8, 0·6, and 0·4 to 2·88, 2.16, and 1·44, which in a structure built on bad ground might be of great importance. Coming next to a dam, pier, retaining wall, or chimney, or any other structure subject to an over-turning moment ; in the case of the dam subject to water pressure, or a pier or a chimney to wind pressure, exactly the same methods may be employed. Of course, in both cases the centre of pressure due to the wind or water must be found. In the case of water, this will be at a height equal to $\frac{1}{3}$ of the total height of the water, and for the wind the centre of gravity of the section exposed. Let us suppose that we have a structure, a, b, c, d. Let E be the centre of pressure on the base due to the structure. Let H be the force acting on the face d, b, either horizontally or at an

angle thereto. Let $W =$ the total weight of the structure above the line a, b. $A^1, =$ the area of the horizontal cross section. $I =$ the moment of inertia of this section. $F =$ the distance a, b; $b, =$ distance a, e; N, the point on the base a, b, where the resultant of H and W cuts a; $a =$ distance N.E.; $M =$ moment due to the force H. Now, when the force H is not acting, the pressure on a, b, may be considered as uniform. This is not strictly true, as there is no doubt that in a structure of considerable weight and width of base the centre portion of the base is subject to a higher pressure; but when there is a force acting at H, it is seen that

FIG.7.—

FIG.6.—
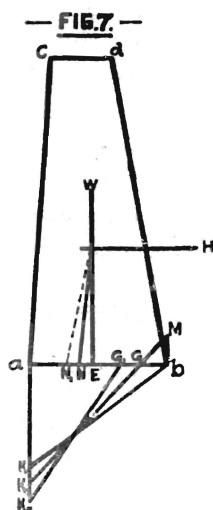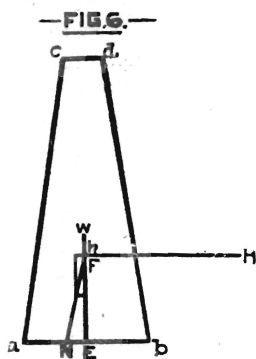
the pressure at a will be increased, and therefore the pressure at b decreased. To find the law of this variation of pressure, we will consider a, b, c, d as a cantilever; the maximum pressure at a will be the pressure due to the structure plus the compression due to flexure, and the pressure at b will be the compression due to the weight minus the tension due to flexure. W will be the uniform pressure due to the weight, and the strain at a is equal to

$$\frac{M.L}{2\ I}.$$ Therefore the maximum pressure per unit of area at a is equal to

(2)
$$\frac{W}{A_1} + \frac{M.L.}{2\ I} = P$$

and the minimum pressure at b will therefore be $\dfrac{W}{A_1} - \dfrac{ML}{2\ I}$

This formula is perfectly general, and is applicable to any cross section and any system of horizontal and vertical forces. Now, if we substitute in the above formula the value for I which in a rectangular section is $\frac{1}{12}\ b_1\ L^3$, where L is the length of the section and b_1 the breadth, we have

(3)
$$P = \frac{W}{A_1} + \frac{M.L.}{\frac{1}{12}\ b_1\ L^3} \qquad = \frac{W}{b_1\ L} + \frac{6M}{b_1\ L^2}$$

and substituting the value of M which is $H + h_1 - h_1$ being the length of the arm of the lever, we have

(4)
$$P = \frac{W}{b_1\ L} + \frac{6H \times h_1}{b_1\ L^2}$$

and by reference to Fig. 6 we see that
$$H:W \ ::\ NE : SE$$
Therefore $\qquad H:W \ ::\ d \quad :\ h.$
Or $\qquad\qquad Hh_1 = Wd$

(5)
$$P = \frac{W}{1b} + \frac{6\ Wd}{bL_2}$$

and if we call S pressure per square inch, we have

(6)
$$P = \frac{S + 6\ S\ d}{1}$$

And this equation is correct for the pressure between any two plane surfaces pressed together. And if we take an example where the pressure line cuts at $\frac{1}{3}$ from the outer end of the

base, that is to say when d equals $\frac{1}{6}$ L, the equation would become

$$\frac{S + 6S + \frac{1}{6}L}{L} = \frac{S + 6S \times L}{6L}$$
$$= S + S$$

That is to say, the pressure at a would be exactly double the average pressure, and the tensile strain at b would equal S—S, that is zero. And this is in accordance with what is known, in the theory of arches, dams, etc., as the principle of the middle third, for you will observe that if the resultant line passes beyond the centre third there will be a tensile strain at b. But it must not be forgotten that when the line of pressure falls on the centre third the pressure at that point has increased to double the average pressure, and that therefore, unless this has been taken into account, your factor of safety will be reduced by half. Now, it might be supposed from this that, if your structure can stand no tensile strain at b, as soon as the line of pressure passes the outer edge of the centre third the structure will be in unstable equilibrium, and on these lines most structures are designed by English engineers, although, if we consider the base of the wall as compressible, the factor of safety against rotating about the point a is equal to $\frac{1}{2}$ the length of the base divided by the distance between the centre of the base and the point where the resultant cuts the base, that is to say the factor of safety against rotating. $(7) = \dfrac{\frac{1}{2}l}{d}$

and when the line of pressure is on the centre third we have

$$\frac{\frac{1}{2}l}{\frac{1}{6}L} = 3$$

That is to say, the factor of safety would be 3, and before rotation would take place we could increase H until the line of pressure passed through a, when we would have

$$\frac{\frac{1}{2}l}{\frac{1}{2}l} = 0$$

But it can be shown that before rotation takes place the
structure must fail by crushing at A, which will shorten the
base of support, and then rotation will ensue. Coming back to
formula 6, which was

$$P = S + \frac{6\,S\,d}{1}$$

we saw that when $a = \frac{1}{6}1$ that at a, $P = 2$ S and b, $P = O$.
Showing this diagramatically, the strain would be represented
by the triangle $a\,b$ K, and it was also pointed out that if the
point N shifted further out there would be a tensile strain at
b, and the triangle a K G would represent the compression and
triangle G. M. b, the tensile strain, and this is strictly true,
provided that the line a K. represents the maximum pressure
that it is deemed advisable to put on the structure. If, how-
ever, this pressure can be increased, we can lay off from N^1;
the point where the pressure line cuts the base, a distance N^1
G^1 equal to twice $a\,N^1$. We have now a new base, and the
resultant line is on the centre third, and using the distance a
G^1 as L in formula 6, and provided the average pressure on $a\,G^1$
does not exceed half the maximum strain allowed, the structure
will still be in stable equilibrium, and no tensile strain need
be borne at b. You will also notice that when the
structure would just fail by rotation the line a K
would be infinite, provided no tensile strain was allowed
at b, and therefore the structure must fail before rotation about
point a could take place. From the foregoing it follows that,
although the line of pressure deviates beyond the centre third
and also the masonry can stand no tension, yet, provided the
crushing strength on the outer edge of the structure is not
reached, failure will not take place. This is equally true for
any kind of structure, and there are many arches in existence
that could stand practically no tensile strain and in which the
line of pressure deviates considerably from the centre third.
With the aid of the above formulæ the exact amount of devia-
tion that may be allowed with or without tension can be

calculated. It is my intention further on, with the aid of these formulæ, to work out in detail the different strains of some experimental arches 74'5 feet span constructed respectively of "Monier," concrete, stone, and brickwork ; but before doing so it will be advisable to consider a beam or girder built on the Monier principle, which is briefly a concrete or compo. structure fortified with an iron mesh comprised of bars laid longitudinally and transversely and bound at crossings with wire, the iron mesh being inserted on the side subjected to tension. It
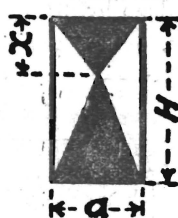
can be seen at once that if the tensile strength of concrete was as great as the compressive, instead of the modulus of rupture being about 150lbs. it would be nearer 2000lbs., and there seems no reason why iron should not be inserted for this object, more especially as the expansion of the iron and concrete is, under ordinary circumstances, almost identical, namely about ·000012 for every degree Cent.   There is also in favour of this class of structure the great amount of cohesion between the iron and the concrete varying from 350lbs. up to 500lbs., so that in rods of small diameter, if 20 times the diameter of the rod be imbedded in the concrete it will have the same holding power as the strength of the rod ; and, moreover, the cement protects the iron in such a way as to permanently arrest oxidation.   To arrive at the formulæ necessary to calculate the strength of a Monier plate or beam, we must first analyse the existing formulæ for girders. Taking the ordinary formula for a

rectangular beam, we have the moment of rupture

$$= \frac{C \times B \times D^2}{6}$$

where C = strength of material, B = breadth, D = depth. We may arrive at the same result by the following method. In any rectangular girder in which the material has the same tensile and compressive strength, the neutral axis must be the centre of the girder, and the outer edge has the maximum strain, which will diminish to nothing at the centre, and therefore the triangle ABC in Fig. 8 shows the section of material strained, and the line joining the centre of gravity of the two triangles represents the leverage—thus, taking an example, say a girder $20''$ deep by $6''$ broad constructed of iron, capable of standing, say, 5 tons per square inch, then by the ordinary formula we have—

$$\frac{400 \times 6 \times 5}{6} = \text{modulus of rupture}$$

By method No. 2. $= 2000$ tons.

Area of either triangle would equal 10 x 3, and the leverage would be equal to $\frac{2}{3}$ of 20 $\therefore$ modulus of rupture

$$= \frac{5 \times 10 \times 3 \times 20 \times 2}{3} = 2000$$

which is the same result. But in the case of a girder constructed of concrete, the tensile and compressive strength of which, we will assume, is as 400 is to 2000, it is natural to suppose that the neutral axis will, before rupture takes place, deviate from the centre of the figure as the girder will be in equilibrium round the neutral axis; therefore, in Fig. 9, if we call x the distance of the neutral axis from the nearest edge—A, the breadth of the beam and H the total height, we have $5 \times A \times X \times \frac{2}{3} X = A \times (H-X) \times \frac{2}{3}(H-X)$ $\therefore X$

$$= \frac{H \sqrt{5} H^2}{4}$$

and solving this we have

$X = \cdot 3089$ when $H = 1$. If we take a girder therefore $6''$ wide and $20''$ deep, the neutral axis would be 20 x $\cdot 3089$ or $6 \cdot 178''$ from the nearest edge, and the moment of rupture would be.