$$\underset{\text{Area}}{(3 \times 6{\cdot}178)} \times \frac{\overset{\text{Leverage}}{(6{\cdot}178 \times 2 \times 2)}}{3} \times \overset{\text{Strain.}}{2000} = 305{,}336$$

Working this same girder by ordinary formula, taking the modulus of rupture as equal to the tensile strength, viz., 400lbs., the moment of rupture would be 160,000 lbs. If we consider another example of the same girder, but taking the tensile strength at 200, and the compressive strength at 2000 or as 1 is to 10, then we have—

$$10 \times (A \times X) \times \overset{\text{Leverage}}{(\tfrac{2}{3} X)} = \frac{\overset{\text{Area}}{(A \times (H - X)}}{2} \times \overset{\text{Leverage}}{\tfrac{2}{3} (H - x)}$$

$$10\ A\ X^2 = A\ (H - X)^2$$

$$X = \frac{-H \pm \sqrt{10\ H^2}}{9}$$

if $H = 1$, then $X = {\cdot}2402$, and the distance of neutral axis from the nearest edge $= 20 \times 2402$ or $4{\cdot}804$ inches; and the moment of rupture

$$= \underset{\text{Area}}{(3 \times 4{\cdot}804)} \times \frac{\overset{\text{Leverage}}{(4{\cdot}804 \times 2 \times 2)}}{3} \times 2000$$

$$= 3 \times 23{\cdot}078 \times 4 \times 2000 = 184{,}626$$

Then by ordinary formula the moment of rupture would be 80,000 lbs. If we take an example of a cast-iron beam, in which we may assume the tensile strength to be about 16,000 lbs., and the compressive about 96,000 lbs., that is to say as 1:6, we have $6\ aX \times \tfrac{2}{3} X = a \times (H - X) \times \tfrac{2}{3} (H - X)$

$$6\ aX^2 = a\ (H - X)^2 -$$

$$x = \frac{X \pm \sqrt{6\ H^2}}{5}$$

and if $H = 1$ then $= {\cdot}2899$, and taking the same girder that is $20'' \times 6''$ the distance of the neutral axis from the nearest edge $= 20 \times {\cdot}9$ or $5{\cdot}8''$ and the moment of rupture would equal

$$\frac{3 \times 5{\cdot}8 \times 5{\cdot}8 \times 2 \times 2 \times 96000}{3} = 33{\cdot}64 \times 4 \times$$

$$96000 = 12{,}917{,}760$$

Now, by ordinary formula, taking modulus of rupture as deducted from experiment by Rankine, we have

$$\frac{400 \times 6 \times 40{,}000}{6} = 16{,}000{,}000\text{lbs.}$$

which is a result slightly in excess of my formula, but this is easily explained by the great variation in strength of cast-iron. We shall now consider a Monier Plate girder tested by Mr. Roberts, of the Public Works Department, N.S.W., the dimensions of the girders being as follow :—Breadth, 18″; depth, 2″; distance from centre of bars to top 1·5625″ and area of bars ·392″. Let A = area of iron; b = breadth of girder; h = height of girder; X = distance of neutral axis from bottom of girder; C = tensile strain in iron (say five tons) = 11,200lbs; D = compressive strain in concrete (say 400lbs) ∴ C : D : : 28 : 1, and by formula

$$\text{C A X} = \frac{b}{\ } (h - \text{X}) \times (h - \text{X}) \times \text{D}$$

$$28 \text{ A X} = \frac{b}{3} (h - \text{X})2$$

$\text{A} = 0{\cdot}392$ $\qquad b = 18''$ $\qquad h = 1{\cdot}5625''$

$$\therefore 28 \times {\cdot}392 \times \text{X} = \tfrac{18}{3} (1{\cdot}5625 - \text{X})^2$$

$$10{\cdot}976 \text{ X} = 6 (1{\cdot}5625 - \text{X})$$

$$1{\cdot}8293 \text{ X} = (1{\cdot}5625 - \text{X})^2$$

$$\text{X}_2 - 4{\cdot}9543 \text{ X} = 2{\cdot}4414$$

$$\text{X} = {\cdot}555$$

Deducting ·555 from the real depth 1·5625, we have 1″ as distance from outer edge, and the moment of rupture will be

$$\frac{\overset{\text{Area}}{(9'' \times 1)} \times \overset{\text{Leverage}}{(1 \times 2 \times 2)} \times 2000}{3}$$

$$— = 24000\text{lbs.}$$

The girder was broken by 3900lbs. applied at centre, supports being 30″ apart; therefore bending moment

$$= \frac{3900 \times 30}{4} = 29250$$

which is slightly in excess of the result obtained by the formula. There is very little doubt, however, that the neutral axis does not deviate from the centre of gravity of the figure until the modulus of elasticity is exceeded. If this be the case, it would be advisable in a Monier beam to proportion the iron so that the neutral axis would be in the centre until rupture took place. The solution of the equation would then be :—Let A = area of iron ; d = depth of girder (measured from centre of iron to upper side of concrete) ; B = breadth ; C = compressive strength in concrete ; P = tensile strength in iron ;

$$P \times \frac{d}{2} \times A = \frac{(B \times d)}{4} \times \left(\frac{d}{2} \times \tfrac{2}{3}\right) \times C$$

$$P \times \frac{d}{2} \times A = \frac{B \times d \times d}{12} \times C$$

$$A = \frac{B \times d \times d \times C \times 21}{12 \times P \times d}$$

$$A = \frac{B \times d \times C}{6 \times P}$$

Take for example a concrete beam 2·5″ deep and 12″ broad. Then A would equal

$$A = \frac{12 \times 2{\cdot}5 \times 500}{11200 \times 6} = \frac{1500}{672}$$

$$= {\cdot}223 \text{ inches}$$

Say bars 5/16″ dia. = ·(767) ·2230 (3
2301

= 3 bars per foot 5/16″ dia.

I have included a table based on actual experiments on concrete beams, and it is interesting to note that, as the age of the concrete advances and the final strength is approached, the results obtained by experiment are almost identical with the results obtained by my formulæ, and are obtained with a substance having compressive strength nearly eleven times the tensile strength. Taking the last two examples, in one we have the moment of rupture by the above formula as 186,700 and by experiment 183,000lbs., and in the last experiment the moment of rupture by my formula is 202,184lbs., and as deducted from the experiment 218,484lbs. We are now in a position to examine closely the experimental arches before mentioned, also the arches and aqueducts being erected over Johnston's and White's Creeks atthe present time. The particulars of the arches experimented upon are given in the *Engineer* of February 21st, 1896, and are as follow:—Four arches each of 7.45ft. span and a rise of one-fifth the span, constructed of different materials, were tested to destruction in a quarry at Puckersdorf. Each arch was 6·65ft. wide. A platform supported on six sets of columns, the feet of which rested directly on the extrados of the arch, extending in each case from one abutment to the crown, and the testing was effected by piling rails on this platform. The first experiments were made upon an arch of cut stone and on one of brick. The stone used was a fairly hard limestone of excellent quality. The voussoirs were 1·97ft. thick at the crown, and 3·6ft. deep at the springings. The mortar used was mixed in the proportion of 5 cwt. of Portland cement to 35ft. of clean sand, or about 5 to 1. The brickwork arch had precisely similar dimensions to the foregoing; the same quality of mortar was employed. After the work was finished the centres were left in place for some weeks. The whole outer surface of the arches was then rendered with a thin coat of cement, so as to detect cracks more readily. The centres were then removed, and the work of loading the arches proceeded with. The stone arch gave way when the load piled on the platform reached an amount equivalent to 1·99 tons per foot run, and the brick arch when the load reached 1·81 tons per foot run. Up to the point of rupture the stone arch gave no signs of incipient failure, but in the case of the brick arch cracks declared themselves previously, which were apparently caused by the failure of the mortar, the bricks themselves being intact. After removing the ruins, a third arch of similar span and rise was constructed between the same abutments, the material being rammed concrete. The thickness of the arch ring was, however, uniform, being 2·3ft. The body of the arch consisted of 1 part Portland cement, 2 parts broken stone, 3 parts gravel, and 3 of sand, but for the intrados and extrados a higher quality of concrete was used, that for the former consisting of 1 part Portland cement, ½ part broken stone, ½ part gravel, and one part sand, whilst the latter consisted of 1 part Portland cement, 1½ parts broken stone, 1½ parts gravel, and 2 parts sand. The total quantity of concrete in the ring was about 50 cubic yards. Two months after completion, the centres were removed, during which time the arch was protected from the sun and frequently watered. The testing commenced three weeks after the centres were removed. Failure took place under a load equivalent to 2·24 tons per foot run on the loaded half of the arch. The next arch to be tested was constructed on the Monier system, the span and rise being as before, whilst the thickness of the ring was 1·97ft. at the springings and 1·15ft. at the crown. The concrete used consisted of 3 parts of river sand to one part of slow-setting Portland cement. The centres were removed at the end of two months, and arrangements made for testing. Failure took place under a load equivalent to 3·09 tons per foot run of the loaded half. Great difficulty was found in removing the ruins. The metal reinforcement was found intact, being bent, but not broken, at the points of failure. You will see by reference to the drawings that the Johnstone's and White's Creek aqueducts are wholly "Monier" structures. The arches have a clear span of 75ft. and a rise of 7·6ft., being one eighth of the span. The carrier is supported directly by the main arch at the crown and by jack arches and piers over the spandrils. The thickness at the crown of arch is 12in., and at the springings 14in. thick. In all structures of this description there is a great difficulty in treating the superstructure forming the carrier, more especially when composed of a material such as compo or concrete, which, we have seen, is subject to great expansion and contraction, the upper portion of the structure practically forming a rigid beam, which, unless precautions were taken, would effectually hamper the expansion and contraction of the arch. The crown of the arch requires to be free to rise and fall to some extent with the variations of temperature; and, small as these motions are, if the carrier was built continuous across the arches, the resistance produced would be very great, and increase the thrust on the arch; but, by putting cuts down the sides of the channel and filling in with some plastic substance, we destroy, to a great extent, this action. Experience has shown that concrete structures exposed to the sun will crack, unless due provision is made for the expansion and contraction in the same way as in an iron structure, although, through the greater thickness of the concrete structures, the changes due to temperature are not nearly so severe. In the outfall carrier of the western suburbs sewerage, the arches are 50ft. span and the whole structure is cut up into sections to allow for expansion.

In the portion of the triplicate sewer immediately adjoining, also for the most part a concrete structure, although of a different type, it is intended and was designed to be eventually covered in embankment. The embankment was temporarily omitted when the structure was built, as it could have been carried out much cheaper whilst the farm was being filled in. This has resulted in several cracks being caused, no doubt due to the expansion, and it is thought that it will be necessary to complete the embankment, and thus stop the movements due to the variation of temperature.

To find the lines of pressure in the various arches, I have used the method propounded by Dr. Sheffler, in conjunction with the formulæ in the previous part of this paper;

and, as in all the cases we have to consider, there are no horizontal forces due to the loads, all loads being directly transferred to the arch ring, this method should be correct. It is based upon the hypothesis of least crown thrust, and, according to this hypothesis, the true line of resistance is that for which the thrust at the crown is the least possible consistent with equilibrium. If space had permitted, I should have liked to go more fully into this, but a careful examination of the diagrams will make it clear, I think. I am sorry that the diagrams are not on a larger scale, but at the conclusion of the paper those members interested will be able to examine them more closely. The compression and tension at each joint has been worked out with the aid of formula 6 in the earlier portion of the paper. You will see, by reference to the diagrams, that in all the arches tested to destruction the pressure line fell considerably outside the arch ring before failure took place, whereas in the case of Johnstone's and White's Creek arches the pressure line (Plate X.) practically coincides with the centre line of the arch. The stone arch failed when the maximum compression strain was 490lbs. per square inch and the maximum tensile strain 341lbs ; the brick arch when the maximum of compression was 402lbs. and of tension 207lbs. The concrete arch withstood a tensile strain of 357lbs., and a compressive strain of 582lbs., and, lastly, the "Monier" arch withstood the tensile strain of 1260lbs. per square inch and a compressive strain of 1680lbs. per square inch before it collapsed—that is to say, the arch practically failed when the compressive strength of the compo was reached, which bears out the truth of the formulæ advanced. Comparing the four arches, the "Monier" withstood a strain about 4 times the stone, 5 times the brick, and 3 times the concrete, and it is probable, if the Monier arch had been made of the same dimensions as regards thickness of arch ring as the others, it would have carried proportionately a very great load.

Coming, lastly, to the White's Creek and Johnstone's Creek aqueducts, we will examine first the strain on the arch. The maximum pressure is reached at the springing, namely, 491lbs. per square inch, and this gives an average pressure of 304lbs. on the joint, the average pressure on the crown joint being 320lbs. per square inch. We have seen that an arch very similar to this, and loaded only on one half, withstood a pressure of 1680lbs., so that we may safely say that, without any tensile strain being allowed for, the factor of safety is 5. This is under normal conditions, but due to the very narrow width at the crown, which is only about ·5ft. During a storm there would be a bending moment tending to produce rupture at the crown and springing. To arrive at the amount of this moment we may consider the arch as a girder lying on its side, and having to sustain a load per unit equal to the wind pressure. On account of the form of the arch, you will see that this is not an equally distributed load, and a great portion of it is brought on the arch through the spandril piers. We must also consider the girders as a continuous girder. By reference to the diagram showing the bending strains, you will see the surface exposed has been cut up into numerous small portions, the bending moments of each being treated separately, and then the sum of the bending moments plotted producing the line of maximum bending moments. The diagram also shows the position of the point of contrary flexure. In order to be well on the safe side, I have taken 56lbs. per square foot as the wind pressure, although it will be generally conceded that it is too high, and that 35lbs. would have been sufficient. The maximum bending moment is 1,220,000lbs., the surface exposed to the wind pressure being 300 square feet. The moment of resistance at the centre of the span is equal to

$$\frac{C \times 12 \times 60 \times 60}{6}$$

Therefore R the modulus of rupture

$$= C.\ 72{,}000, \text{ therefore}$$

C, the cross breaking modulus

$$= \frac{1{,}220{,}000}{7200} = 169\tfrac{1}{2}\text{lbs. per sq. inch.}$$

and we have seen in a previous portion of this paper that the resistance of a rectangular beam is equal to the area of a triangle, the base of which is the width of the girder, and the height is equal to half the depth; therefore the depth of the arch ring being 12″ and half the width 30″ the total amount of compression would be

$$\frac{12 \times 30 \times 169.5}{2} = 30{,}510\text{lbs.}$$

The total tension would come to a like amount, namely, 30,510lbs, and the compression due to the thrust is equal to

$$\frac{329.20 \times 140}{144} \times 60 \times 12 = 230{,}400\text{lbs.}$$

Adding to this 30,510lbs, the compression due to the wind, we get the total compression on the arch ring, and the compression on the edge furthest from the wind would equal 320lbs.,the average pressure due to the thrust minus the 169.5lbs. pressure due to the wind, that is to say 150½lbs. per square inch. The pressure on the neutral axis line of the girder would not be affected, and would therefore be 320lbs.; and if we add to 320 the mean pressure, twice the difference between 320 and 150lbs. it willgive us the maximum pressure on edge nearest storm, or 660lbs; so that under the exceedingly severe conditions of a storm equal to a pressure of 56lbs per sq. foot, the maximum pressure would be 660lbs. By a reference to the diagrams of wind pressure, this will be made more clear. During a storm, therefore, the thrust on the side furthest from the storm will be decreased, and on the side nearest to the storm increased; at the point of contrary flexure the thrust will remain normal, and at the springing it will be 390lbs and 263lbs per sq. inch respectively. The calculations of the side walls and top plates I have made by the aid of the formula for "Monier" beams. The factor of safety, taking into account the tensile strength of the iron, would vary from eight to ten, and even without the iron mesh failure would not take place, although, no doubt, cracks would occur.

In conclusion, the author desires to express his thanks to Mr. R. R. P. Hickson, Under Secretary for Public Works and Commissioner for Roads, and Mr. J. Davis, Engineer for Sewerage Construction, for the plans lent from their Departments.

CEMENTS ONLY.

Age.	Tensile Strength in lbs. per sq. inch.	Compressive Strength in lbs per sq. inch.	Ratio C in Formula (1)	$\sqrt{C}$	X in Formula (1)	4 X2 in Formula (2)	Moment of Rupture by Formula (2)	Modulus of Rupture by Experiment.	Moment of Rupture by Formula (3)	REMARKS.	
Increasing from 3 to 12 months	134·7	1932.0	14·3	3·78	4 18	69·89	135,027·0	280·5	112,200	Beams 20″ x 6″	h = depth b = breadth
	143·5	1240·7	8·65	2·94	5·08	117·86	146,228·9	277·1	110,840		c = ratio of compression to tensile strain.
	201·14	1854·1	9.2	3·03	4·95	98·01	181,720·3	376·9	150,760	$X = \frac{h(\sqrt{C}-1)}{C-1}$ (1)	x = dist. of neutral axis from nearest edge.
	152·74	1100·1	7·2	2·68	5·42	117·51	129,272·7	288·6	115,440	Mt. Rupt. $= 4X^2 f$ (2)	
	172·	1617·0	9·4	3·066	4·92	96·83	156,574·1	373·3	149,320	Mt Rupt. $= 400f^1$ (3)	f = compressive strain.
	201·5	2068·7	10·26	3·20	4·75	90·25	186,700·2	457·5	183,000		f^1 = modulus rupture by experiment.
	224·4	2025·9	9·03	3·005	4 995	99·80	202,184·8	546·2	218,480		

TIMBER ONLY.

Grey Ironbark.	25080	10165	2·467	1·57	7·77	241·5	6,056,820	17,866	7,146,400	Formulæ as above, except c = ratio of tension to compression.
Tallow-wood.	16165	6753	2·394	1·547	7·85	246·5	3,984,673	15,257	6,102,800	
Blackbutt.	21708	7522	2·886	1·70	7·42	220·2	4,780,102	13,728	5,491,200	Results of tests are the averages of a large number of tests.