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# INSTRUMENTAL CALCULATION AS APPLIED TO MODERN ENGINEERING.

(BY F. ERNEST STOWE.)

IN introducing to your notice Instrumental Calculators—a subject, though old, is to many quite a novelty—we do not for a moment anticipate or desire that anyone may be too ready to seek these aids in the most important branch of the Engineering Profession—Calculus. There are dangers, as well as disadvantages, from the use of these aids in unpractised hands, and the Slide Rule especially, becomes quite a failure if used as a mere mechanical instrument. But when the specialist tendency of modern times shall have thoroughly permeated this branch of science and produced special instruments for every requirement, these instruments may be used quite mechanically, in the sense that the operator may or may not be acquainted with the formula involved in each. Thus, there is a Slide Rule, designed by Mr. Ganga Ram, an Indian engineer, which embodies a formula expressed as an equation

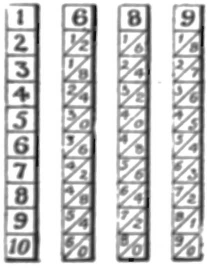
$\sqrt{\frac{2\,H}{3\,W\,(q+\frac{1}{8})}+\frac{t\,3}{2}}-\frac{t}{2}=x$, being that for the calculation of the mean statical thickness of retaining walls; yet, in using this rule, the operator need have no knowledge of the nature of the calculation, it merely requires that the data be read in the right place and the answer sought after certain self-evident manipulations of the instrument.

Perhaps the earliest form of calculator was the Abacus, said to have been known to the early Egyptians, consisting of rows of beads in a frame, and practised with, could, no doubt, be made to serve in the addition of many figures to great advantage.

Historically, the Slide Rule has been somewhat unfortunate from its association with, and semi-eclipsed by, the more widely-known, though less wonderful system, of Logarithms. It has been hinted by the German manufacturers of the Slide Rule, that it was probably known to the ancient Chinese; but Baron Napier, of Murchison, may well be given the meed of praise due to the inventor of this benefaction. He it was who, realising the magnitude of the work in astronomical calculations and the great loss of time, even in checking the finished work, with the instinct and impatience of true genius, mentally anticipated that which only of recent years has been accomplished in the Arithmometer—the rapid, mechanical, and unerring solution of vast arithmetical problems.

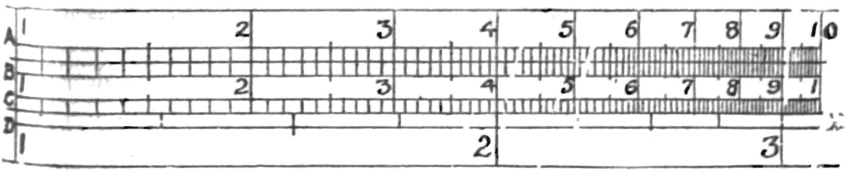
But Napier failed to realise his anticipations in this respect; left us only his "bones" to grace an old Scotch oath ("by Napier's bones") and to assist in the development of instrumental calculation. Napier's Bones are not, as Sir Walter Scott assumes, his physical bones, but certain sticks with figures arranged thereon in a peculiar manner, and so-called from the fact of their being constructed of bone or ivory. (Fig. 1.)

FIG 1



It will be seen that the first of these called the "index rod," is marked with a series of numbers advancing by units, while the others, headed 6, 8, 9, are marked with the multiplication tables of these numbers respectively, and their number may be increased to any extent. To perform the operation of multiplication the multiplier is taken on the index rod, say 8; then whatever multiplicand is required is obtained by ranging the other rods with the heading numbers constituting the multiplicand, then the product is read opposite the multiplier, thus in the example, Fig. 1, the product of 689 × 8 is found opposite 8 thus: $| 8 \,\|\, {}^{4}/. \,\|\, {}^{6}/_{4} \,\|\, {}^{7}/_{2} |$ the 2 being units, the 7 and 4 tens, the 8, 6, and 1 carried, hundreds, the 4 and 1 carried, thousands, giving 5512, the answer. The method seems cumbrous, but, as, with the Abacus, practice would enable the operator to work with speed and accuracy, the mind being less burdened and the possibility of error very small. Though these sticks were the key to the solution of Napier's puzzle, he fell short of accomplishment, not from incompetency, but because he saw the possibility of a Logarithmic System, the perfection of which henceforth became his chief aim, and with the assistance of Briggs, the Mathematician, the System was advanced to the stage that now, after more than two hundred and seventy years, has not been improved upon, and is only of recent years being superseded by mechanical calculators. The result of this combination of genius, and the once useful purpose of these well-nigh forgotten bones, is best known to those who know and appreciate the use of Logarithms.

It is worthy of remark that the Slide Rule in its inception and subsequent development has nearly been associated with some other practical result of true genius—thus Napier is credited with invention of the decimal point as well as Logarithms; and Edmund Gunter, who invented the Sector, the Surveyor's Chain, and other mathematical instruments, was the first to connect up the two links of Napier's genius, and combine the practical, mechanical bones with the theoretical Logarithms, the result being what is known as "Gunter's line of numbers." Again, Watt, in his avocation of rule maker, led the way in the production of fairly accurate rules, and in the application of these to engineering work, though, necessarily, it was not till the advent of Engine-divided rules that that degree of accuracy was reached as would justify their general use.




Gunter's line of numbers (Fig. 2) is of importance, inasmuch as it is the model after which Slide Rules generally are constructed, and because its characteristic feature is really the cause of much of its unpopularity where verbal explanation is lacking.

Now, let us consider two series of numbers, one in arithmetical progression and the other in geometrical progression, both sides representing equal parts of two lines, thus:—

ARITHMETICAL PROGRESSION.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | Logarithms. |
| 1 | 2 | 4 | 8 | 16 | 32 | 64 | 128 | 256 | Natural numbers. |

GEOMETRICAL PROGRESSION.

If any collection of units in the upper line be added to any other number of the same units, the increment gives the mechanical operation of addition:—thus, if the length of line from 0 to 3 be added to that from 0 to 4, the total length of line will be from 0 to 7 or 3 plus 4. But although a similar operation be performed with the lower line of numbers, as far as length of line is concerned, the resultant is the product of two of the numbers represented:—thus, by adding the same lengths of line as before, viz.—0 to 3 plus 0 to 4 on the upper line, or 1 to 8 plus 1 to 16 on the lower line, the increment of line results in multiplication, giving in this case the product of 8 by 16 equals 128. A result so astonishing that it is really difficult to comprehend its possibility; and conversely division may be performed by substraction. This practical method of asigning length of line to values gives the best possible idea of the system of Logarithms, though it will be readily understood that it is merely elemental.

If, instead of taking the natural numbers in the above order, we take them consecutively—1, 2, 3, etc., and assign to each their logarithmic value to the ordinary base, we must then take a logarithmic unit of very small dimensions, and mark the natural numbers off by it, thus from 1 to 2 (Fig. 2) there are, though unperceived, 301 of these units; from 1 to 3 477 units, from 1 to 4 602, and from 1 to 5 698 units, and so on from 1 to the several marks, including the consecutive number of units in the respective logs. Hence, Gunter's line of numbers is a line graduated with logarithmic units, but marked only where the natural numbers occur in the progression, and the marks being identified by the natural numbers only, the practical result being that, by the conjunction of two such lines and by sliding one against the other, results in multiplication, division, and proportion are instantly obtained. Again, if the logarithmic units be multiplied by the index 2 or doubled, the whole of the divisions must be extended to twice the length, as is the case with the D line (Fig. 2), the conjunction of the ordinary line C with the line D, giving proportion of squares and square roots, and all questions involving this proportion are instantly solved; thus, if with the ordinary slide rule the value .7584 is brought over 1, the whole of the areas of circles are found above their diameters, thus—

| | | | | |
|---|---|---|---|---|
| line, C. | ·7854. | 3·14 area. | 4·91 area. | 63·6 area, &c. |
| line, D. | 1· | 2″ dia. | 2·5 dia. | 9″ dia., &c. |

The results, as will be seen, are tabulated, and the operations, if such a simple process as shifting a line can be called an operation, are equivalent to the performance arithmetically of diameter 2 x .7854, the fact availed of being that areas of circles are proportionate to the squares of their diameters.

Nor is the construction of these lines limited to a comparison or ratio of squares, cubes, or simple proportion; and here is where the future of the Slide Rule is of vast importance to the engineer; these lines may be graduated by taking a fractional index to the log, thus giving results hyperbolically proportional as are the ordinates in the indicator diagram. Moreover the line used may be graduated, as is that used in the Excise Department, for getting the contents of casks, quite empirically and by practical experiment.

It is this, practical experiment has again and again demonstrated that simple arithmetical formulæ are but wide approximations, and with more accurate methods of obtaining data—with more extended and economic engineering methods there must be more accurate formulae. But this again involves operations so complex that accuracy becomes, if not impracticable, certainly so inconvenient that the engineer must often sacrifice accuracy for convenience. Hence, in all branches of engineering, the anomaly is common to a greater or less extent—the practical man with his empirical, handy rule; the theoretical man with his laborious, elaborate formula. But this anomaly must disappear before the advance of instrumental calculation, for there is not a formula so complicated that a slide rule may not be designed to admit of a solution by any practical man of ordinary intelligence, in a few seconds.

As showing the result of some of the earliest careful experimental enquiries, and the possibilities with the Slide Rule in this respect, there is the discovery of Mr. Eaton Hodgkinson, who found that solid columns of cast-iron, as regards stability, when both ends are rounded, are in the proportion of $14{\cdot}9 \frac{D^{3.76}}{L^{1.7}}$ where $D =$ diameter in inches, and $L =$ length in feet; a proportion which places the solution beyond the range of ordinary arithmetic, but a Slide Rule could be designed with a graduation in the ratio of $\frac{D^{3.76}}{L^{1.7}}$ so that any co-efficient could be used instead of the 14·9, and results obtained instantly, with diameters, lengths or loads tabulated as required.

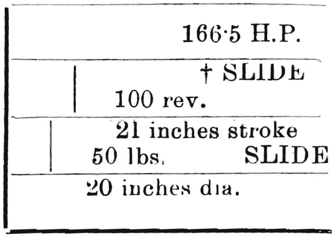
It will be readily understood that there are many difficulties in the way of the successful use of the Slide Rule, the first and greatest of these being the reading of values thereon, for since each of the divisions from 1 to 2, 2 to 3, etc. (Fig. 2), are equal to say, units, it follows from what has been said that these spaces also represent the differences of the Logarithms, which get less as the values increase and equal units are consequently not represented by equal spaces—thus, referring to Fig. 2 again, it will be seen that the space from 1 to 2 on the line A is equal to the space, or sum of spaces, from 5 to 10, the former including one unit, the latter five.

Again, the ordinary small Slide Rule cannot be used where the question requires a nearer approximation than 1/1000th, and since accuracy becomes a question of greater length in the instrument of the typical line, inventors have sought means whereby this may be obtained in compact space; with the result that we have Fuller's Spiral Slide Rule, equal to a Slide Rule 84 feet long, though contained in a box 4 inches square by 18 inches long, and it is claimed that an approximation of 1/10,000th may easily be obtained, or answer may be read to six figures. The result is obtained in this way. If, as is indicated in a series of tables given on the instrument, a piece of wrought iron plate 1ft. x 1ft. x 1in. weighs 40lbs., to obtain the weight of a plate 6.5ft. x 3.25ft. x $\frac{3}{4}$in. the operation may be obtained in a few seconds in this way —a pointer is set to 1 and another to 6.5, the barrel, on which is the line of numbers, is moved round so that 3.25 is under the first pointer, then under the second pointer is 21.125 square feet—the area; again, the first pointer is brought to 1, the second to 21.125, the barrel is turned to bring 40 x $\frac{3}{4}$ equals 30, under the first pointer, the answer being at once indicated under the second pointer, 683.75lbs., the same as would be obtained by the process of arithmetic in a very much longer time.

The Spiral Rules are now being generally used for ordinary engineering and business calculations, though the ordinary rule, less approximate, is still more convenient in smaller calculations, and should be thoroughly understood before it is attempted to use the Spiral, or, in fact, any description of calculator.

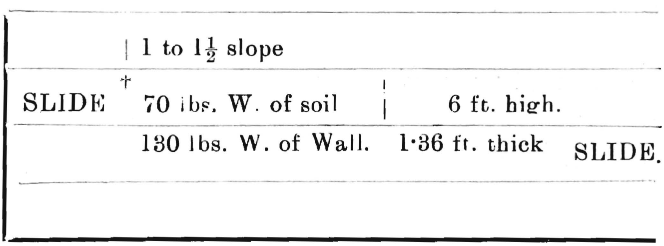
The contention that the Slide Rule is not accurate enough has thus been met by inventiveness, and to maintain compactness it is only necessary to have specially designed slide rules, which from their special construction give the desired compactness with the necessary degree of accuracy; such an one is Hudson's Horse-power Computer, involving the formula well-known to the engineer $\frac{\text{PLAN}}{33000} = \text{H.P.}$ The length of this instrument is $4\frac{1}{2}''$

and is made of cardboard or ivory, yet almost as quickly as the data can be stated, the operator may work out the solution—thus, if dia. equals 20in., mean pressure 50 rev. 100, stroke 21 inches, the result 166.5 H.P. may be obtained by bringing these values into the positions indicated by shifting the slides. Moreover, as showing the utility of these rules in machine design it is only neces-




sary, in this particular class of work, to set the arrow to the H.P., and any of the particulars indicated below will give an engine of the fixed H.P., thus any particular requirements of stroke, diam., revolution, etc., may be tested, and the most suitable taken, a very small space of time being taken in experiment. This rule may be used to also obtain the ratio of compound cylinders, or design them of a certain ratio. The Engineer (more especially the Marine Engineer) will readily perceive the wonderful utility of this instrument. But, more wonderful, is the Planimeter, an instrument, perhaps, unequalled in the whole range of ingenuity, giving, as it does the area of any regular or irregular figure by a measurement of its boundary, as it may easily be seen, a mathematical impossibility. The latest form of this is the Diagram Averager, which, by tracing round the boundary of the diagram with one of its points, gives at once the mean ordinate or mean pressure in the cylinder.

Other calculators of special construction include one for the strength or design of shafts and girders—for obtaining the flow of water in pipes, and one for the mean statical thickness of walls before-mentioned, and which well illustrates the extent to which the design of special slide rules may be carried, and with what advantage. On a reference to the formula it will be seen that the solution of x involves considerable arithmetical work; yet with an unerring something (one may be pardoned for calling it an instinct), this rule gives us the tabulated results of which one alone would be a tax on one's patience to produce arithmetically. The illustration is that of a retaining wall required to maintain a bank, the soil whereof forms a natural slope of $1\frac{1}{2}$ to 1, and weighs 70lbs. per cubic foot, the brickwork of the wall weighs 130lbs. per cubic foot, and from this data, the thickness (mean statical) is obtained for any height, thus—for 6ft. high, the thickness is 1.36ft.; for 20ft., 4.525ft., and so on; and these results may be manipulated so that the thickness is easily obtainable if the wall is to be battered, buttressed, diminished or arranged in any other way.




Another rule described before the English Institute of Civil Engineers gives the smelting charges, with the necessary proportions of fluxing materials, for any description of slag, totally superseding all other methods of the calculation of furnace charges.

Coming now to the general calculations of great magnitude, and where the system of Logarithms has been employed, we have the Arithometer, an instrument, or to speak more correctly, a machine capable of performing operations of multiplication and division involving many figures, mechanically, unerringly, and rapidly as the turning of a small handle can be accomplished; and although one meets few who not having seen will believe there are hundreds now in use making continually the heavy calculations of the Actuary, Engineer, and Astronomer, and that kindly lent by the Colonial Sugar Co. for illustrating this paper, bears very evident marks of the use that can be made of these instruments in the Chemists' Laboratory. Though one of two, and though only in use about three years, some of the metal parts are worn out, and the handle has cut into the metal case considerably. And while speaking of the Sugar Co., it is worthy of remark that the Laboratory, one of the chief sources of the Company's success, is replete with calculations, both Mr. Walton and Mr. Steel being, as most men who have realized the value of the Slide Rule, are, enthusiastic; hence it has become there a commercial principle, and it would be safe to say that arithmetic, though certainly not a lost art, is an unpractised art.

The latest Calculator in the order of review is the Comptometer, of American origin, and though he could not speak from personal experience, many wonderful things are claimed for it, as without the handle turning of the Arithometer, rapid results are obtained, the method being to press keys of the typewriter form. Both these instruments are operated mechanically, the construction being somewhat similar to that of the printer's numbering machine—the action that of a series of wheels carrying the symbols, each single motion of each being an increment of 1 which may be one ten, one hundred, etc., according to which wheel is moved. The practical result being that values can be manipulated, giving results in addition, subtraction, multiplication, and division, involving 20 figures; in fact, there is no limit of extent, and no result need require more than a few seconds to obtain.

It is contended that the use of these Calculators takes away from the arithmetical capabilities of the user, but with the Arithometer this cannot be true, since the operator is simply doing the work as if it were arranged to be worked out by the old method, and with reference to the Slide Rule, it can only be true where the underlying principle of construction is lacking; in reality, it is one of the best assistants the young engineer can have, and, if properly understood, is a wonderful exponent of our decimal system of numeration. Thus, its use in devising formulæ may be shown in the case of levers; instead of burdening the memory, the first principle is investigated by considering a single case of the first order of levers, and it is found that the least of the parts is F to W, least of forces P, least of motions that of W, and then with the knowledge of construction, and that the least values are kept on the same line, one gets instantly and without any equating the whole of the formulæ, thus:—

Line A. F to W. P. Motion of W.
Line B. F. to P. W. Motion of P.

And even if one has not the Slide Rule to do the work with, the arithmetical methods are all indicated at a glance, showing that—

$$\text{F to W} = \frac{\text{P} \times \text{F to P.}}{\text{W}} = \frac{\text{Motion of W} \times \text{F to P}}{\text{Motion of P.}}$$

and $\text{P} = \dfrac{\text{F to W} \times \text{W}}{\text{F to B.}}$ &c., all by inspection; and to show the application of the rule, assume F to W=28″, F to P 3″ P=640 lbs., the lever being of the third order—a safety valve lever, the result is 68·5 lbs. W, obtained thus:—

Line A. 28=F to W 640=P Motion of W 1·5″.
Line B. 3=F to P find 68·5=W Motion of P ·14″.

Again, to take the case of Boiler Construction, the formula for Stayed Plates of less than $\frac{7}{16}''$ thick is expressed $\frac{C \times T^2}{P^2}=$ working pressure in lbs. per square inch. Where C=90, T= thickness of plate in 16ths., and P=pitch of stays. Instantly the formula can be arranged to give any of the several results with sufficient data given, thus:—

| | | |
|---|---|---|
| g | Constant. | Working pressure. |
| D | T. | P. |

Thus, if the plate be $\frac{3}{8}''$; pitch 8″, the working pressure is obtained by arranging the values thus:—

| | | | | |
|---|---|---|---|---|
| g | 90 | 50·5 lbs. answer. | | 70lbs. |
| D | 6 | 8″ | | 6·8″ pitch. |

and at a glance it can be seen that if the working pressure be increased to 70 lbs., the pitch must be 6·8″.

Moreover, its value in experimental work is great, and amply repays the trouble of learning its use, but it would be quite impossible to illustrate one-half of the uses of the ordinary Slide Rule where reason rather than memory must play the chief part.

From all of which it will be seen that the Science of Instrumental Calculation is advancing rapidly, and bids fair to play a most important part in the education of the engineer ,and to assist in the more scientific practice of the profession, and he thought the time had come for the question to be answered by Engineering Associations and kindred interests. "Should the use of these aids be given general encouragement?" and it is not asking, under the circumstances, too much, that older members of the associated professions throw aside all bias, and but lend encouragement to younger members in the study of this worthy subject. He knew that too much is made of the statement, that anyone can use it, for, like all good things, the use of the Slide Rule cannot be accomplished without much patient study and continual practice for a time.

The first of these interests to answer the question in English-speaking countries is the City and Guilds Institute of London, giving in the Engineering Section of their Examinations, extra marks for every question worked out by the Slide Rule, and in Sibley College, Cornell University, every student is obliged to provide himself with one for laboratory use.

In closing, thanks are again due to the Colonial Sugar Co., for the Arithometer; to the Australian Mutual Provident Society, through D. Carmont, Esq., Actuary for the Comptometer; and to J. Fitzmaurice, Esq., your Secretary, for Electrical Calculators used to practically illustrate this paper, and in apology for the lack of practical demonstration, he could only say that it is well-nigh impossible to maintain interest in working solutions, the question invariably following each is "How did you do that?" a question not easily answered, unless to those having a thorough grasp of the instrument used, so he had endeavoured to describe the nature and construction of these instruments, an.. to explain their utility to the modern engineer.