0·2 inch diameter = 4098 lbs. per square inch.

Increase due to spiral = 1312 lbs. per square inch.

Ratio of area of metal to concrete = ·022

$$\cdot 022 \times 39000 = 858 \text{ lbs. per square inch.}$$

$$\text{Ratio} \frac{1312}{858} = 1 \cdot 53$$

No. 4. Total strength of prism = 5270 lbs. per square inch.

Increase due to spiral and concrete from No. 2 test = 4098 lbs. per square inch.

Difference due to 6 rods of Bessemer steel ½ inch in diameter = 1172 lbs. per square inch.

Area of rods = 6 × 0·196
= 1·176 square inch.

$$\text{Resistance} = \frac{1 \cdot 176 \times 37770}{29 \cdot 5} = 1490 \text{ lbs. per sq. inch.}$$

Hence the rods contributed about 80 per cent. of their resistance.

No. 5. Total strength of prism = 4214 lbs. per square inch.

Resistance due to concrete and spiral from No. 2 test = 4098 lbs. per square inch.

Difference due to rods = 116 lbs. per square inch.

The 10 rods ⅜ inch diameter should have contributed 1400 lbs. per square inch, so that the longitudinal rods in this case did not contribute their proper proportion, due probably to initial stresses in consequence of shrinkage of concrete.

The experiments recorded in Tables IV. m. and IV. k. show that prisms of concrete reinforced with spirals of soft steel or iron possess considerable ductility and sustain large deformations before fracture. Moreover, cracks appear on the outer face long before actual fracture, thus giving warning of approaching danger.

The experiments also suggest that in order to obtain the maximum strength for the minimum cost in reinforced concrete columns, such as would be most suitable for fire proof buildings, the reinforcing should consist of soft iron or steel spirals having a longitudinal pitch of about 1/8 of the diameter, with longitudinal steel rods arranged on the inner side of the spirals, around the circumference of, say Bessemer or other steel having a high elastic limit. It has been shown by Considere that such a combination is approximately equal in resistance to riveted steel of the same weight. It has been suggested to submit such columns to a test load sufficient to crush the outer shell of concrete, and after the test to put on a concrete coating in which asbestos is substituted for the sand.

Coefficient of elasticity.—The curves Fig. IV. m. and Fig. IV. k. show that the coefficient decreases rapidly with the increase in the stress, when the loads are gradually increased to the breaking point, but if the loads are removed and reapplied the effect of the load first applied is to increase the coefficient for the second load. 4 Fig. IV. m., 5 Fig. IV. m., and 5 Fig. IV. k. illustrate this point. The application of the first pressure on a hooped concrete prism raises the elastic limit. The coefficient of elasticity obtained by repeated loading and unloading does not diminish to the same extent as when the loads are gradually increased to the breaking point.

## TRANSVERSE TESTS OF REINFORCED CONCRETE.

The first series of transverse tests considered in this chapter was undertaken by the author in order to compare the behaviour of reinforced mortar beams with similar beams not reinforced. Twelve beams were tested, six without reinforcement, and six reinforced, each with three rods on the tension side of Bessemer steel, ½ inch diameter.

The beams consisted of 1 of Portland cement to 3 of standard sand, and they were kept in water until tested. They were 45 inches long by 4 in. x 4 in. in cross-section, and were tested on a span of 40 inches with a load applied in the centre. Only the loads applied and the corresponding deflections were recorded, and the results are shown in the following tables which are arranged for purposes of comparison. The results for the four beams over 500 days old have also been added. Table XIV. shows the results of testing plain mortar beams, and Table XV. the results of testing similar mortar beams reinforced with round bars of Bessemer steel, ½ inch diameter. These results show that the strength of a beam reinforced in the manner described is about seven times that of a similar beam without reinforcement; also that the deflections before fracture in these reinforced beams were about ten times as great as in the beams without reinforcement.

The coefficient of elasticity and rupture in the Tables have been calculated in the ordinary way. In the case of the reinforced concrete beams the ordinary equations do not apply, and the moment of resistance must be specially considered. It is proposed to discuss this matter from the actual strains and stresses producing them, obtained experimentally, and afterwards to establish equations for the moment of resistance of a reinforced concrete beam.

## TABLE XIV.—TRANSVERSE TESTS OF MORTAR BEAMS.

### WITHOUT REINFORCEMENT, KEPT IN AIR.

Span 40 inches. Cross Section 4 inches x 4 inches.

| Age in Days | Load applied in lbs. | Total deflection in inches. | Coefficient of Elasticity in lbs. per sq. in. $\frac{WL^3}{dI} = \frac{10^3 W}{16\,d} \times 10^6$ | Breaking Load in lbs | Modulus of Rupture, lb. per square in. |
|---|---|---|---|---|---|
| 94 | 45 | 0·0000 | | 358 | 335 |
| | 89 | 0·0003 | | | |
| | 134 | 0·0025 | 3·35 | | |
| | 179 | 0·0056 | | | |
| | 224 | 0·0059 | | | |
| | 269 | 0·0071 | | | |
| | 314 | 0·0091 | 2·[illegible]5 | | |
| 154 | 0 | 0·0000 | | 336 | 315 |
| | 45 | 0·0002 | | | |
| | 89 | 0·0010 | 5·56 | | |
| | 134 | 0·0017 | | | |
| | 179 | 0·0022 | | | |
| | 224 | 0·0029 | | | |
| | 269 | 0·0039 | 4·31 | | |
| 180 | 0 | 0·0000 | | 324 | 303 |
| | 45 | 0·0004 | | | |
| | 89 | 0·0008 | | | |
| | 134 | 0·0017 | | | |
| | 179 | 0·0030 | 3·72 | | |
| 188 | 22 | 0·0000 | | 403 | 378 |
| | 45 | 0·0001 | | | |
| | 89 | 0·0001 | 5·56 | | |
| | 134 | 0·0007 | | | |
| | 179 | 0·0016 | | | |
| | 224 | 0·0023 | | | |
| | 269 | 0·0032 | | | |
| | 314 | 0·0042 | | | |
| | 359 | 0·0054 | | | |
| | 404 | 0·0057 | 4·43 | | |
| 518 | 22 | 0·0000 | | 448 | 420 |
| | 45 | 0·0004 | | | |
| | 89 | 0·0014 | 3·99 | | |
| | 134 | 0·0022 | | | |
| | 179 | 0·0032 | | | |
| | 224 | 0·0043 | 3·25 | | |
| | 269 | 0·0054 | | | |
| | 314 | 0·0068 | | | |
| | 358 | 0·0088 | | | |
| | 403 | 0·0103 | 2·44 | | |
| 519 | 45 | 0·0000 | | 488 | 457 |
| | 89 | 0·0012 | 4·63 | | |
| | 134 | 0·0022 | | | |
| | 179 | 0·0031 | | | |
| | 224 | 0·0043 | 3·25 | | |
| | 269 | 0·0054 | | | |
| | 314 | 0·0069 | | | |
| | 358 | 0·0080 | | | |
| | 403 | 0·0096 | | | |
| | 448 | 0.0109 | 2·57 | | |

## TABLE XV.

## TRANSVERSE TESTS OF MORTAR BEAMS REINFORCED WITH THREE BESSEMER STEEL RODS.

Kept in Air.

Span 40 inches. Cross-section 4 x 4 inches.

| Age in Days | Load applied in lbs. | Total deflection in inches | Breaking Load in lbs |
|---|---|---|---|
| 159 | 22 | 0·0000 | 2890 |
| | 134 | 0·0010 | |
| | 179 | 0·0016 | |
| | 224 | 0·0025 | |
| | 269 | 0·0039 | |
| | 314 | 0·0061 | |
| | 358 | 0·0065 | |
| | 403 | 0·0065 | |
| | 448 | 0·0070 | |
| | 806 | 0·0112 | |
| | 1075 | 0·0149 | |
| | 1344 | 0·0186 | |
| | 1613 | 0·0212 | |
| | 1882 | 0·0260 | |
| | 2150 | 0·0285 | |
| | 2330 | 0·0349 | |
| 161 | 0 | 0·0000 | 2240 |
| | 179 | 0·0019 | |
| | 358 | 0·0053 | |
| | 448 | 0·0064 | |
| | 806 | 0·0110 | |
| | 1075 | 0·0145 | |
| | 1344 | 0·0241 | |
| | 1613 | 0·0366 | |
| | 1882 | 0·0486 | |
| | 2150 | 0·0654 | |
| 187 | 22 | 0·0000 | 2598 |
| | 179 | 0·0011 | |
| | 358 | 0 0048 | |
| | 448 | 0·0059 | |
| | 806 | 0·0100 | |
| | 1075 | 0·0149 | |
| | 1344 | 0·0192 | |
| | 1613 | 0·0245 | |
| | 1882 | 0·0298 | |
| | 2150 | 0·0378 | |
| | 2240 | 0·0392 | |
| | 2509 | 0·0521 | |

## TABLE XV.—TRANSVERSE TESTS—Continued.

| Age in Days | Load applied in lbs. | Total deflection in inches | Breaking Load in lbs. |
|---|---|---|---|
| 192 | 22 | 0·0000 | 2777 |
| | 179 | 0·0010 | |
| | 358 | 0·0040 | |
| | 448 | 0·0071 | |
| | 806 | 0·0144 | |
| | 1075 | 0·0198 | |
| | 1344 | 0·0251 | |
| | 1613 | 0·0296 | |
| | 1882 | 0·0347 | |
| | 2150 | 0·0407 | |
| | 2240 | 0·0424 | |
| | 2688 | 0·0716 | |
| 529 | 45 | 0·0000 | 3405 |
| | 179 | 0·0023 | |
| | 358 | 0·0058 | |
| | 448 | 0·0076 | |
| | 806 | 0·0153 | |
| | 1075 | 0·0216 | |
| | 1344 | 0·0286 | |
| | 1613 | 0·0367 | |
| | 1882 | 0·0461 | |
| | 2150 | 0·0543 | |
| | 2240 | 0·0576 | |
| | 2688 | 0·0761 | |
| | 2958 | 0·0926 | |
| | 3317 | 0·1229 | |
| 529 | 45 | 0·0000 | 3394 |
| | 179 | 0·0026 | |
| | 358 | 0.0048 | |
| | 448 | 0·0084 | |
| | 806 | 0·0166 | |
| | 1075 | 0·0221 | |
| | 1344 | 0·0292 | |
| | 1613 | 0·0372 | |
| | 1882 | 0 0453 | |
| | 2150 | 0·0538 | |
| | 2240 | 0·0567 | |
| | 2688 | 0·0733 | |
| | 2958 | 0.0841 | |
| | 3317 | 0·0985 | |
| | 3405 | 0·1048 | |

b. The experimental determination of the neutral axis in a plain and in a reinforced concrete beam, also the curves of strain for loads increasing from zero to the load producing fracture.

In deriving the equations for determining the position of the neutral axis and the moment of resistance of a transverse section in a reinforced concrete beam, it has been assumed by all authorities up to the present, that a transverse plane section before flexure remains a plane section after flexure. On this assumption the curves of stress on each side of the neutral axis have been derived. The stress strain curve obtained from testing plain concrete prisms in compression under gradually applied loads, in which the abscissae represent the strains and the ordinates the loads producing them are of approximate parabolic form,[1] and this form is usually assumed for the curve representing the compressive stress from the neutral axis to the extreme fibre, where the maximum ordinate represents the intensity of compressive stress at the extreme fibre.

In order to test the accuracy of this assumption, ten beams were made 72 inches long, of square cross section 10 by 10 inches, one beam was of plain concrete, the others were reinforced each with three rods, varying in diameter from ⅝ of an inch to 1 inch. The beams were supported at points 40 inches apart and loaded at each extremity, so that the bending moments and corresponding stresses between these points of support were nearly constant. Four set of Martens' mirror extensometers[2] were arranged on each side of the beam to be tested, at equal distances from the centre of the beam, and Martens' sectors were arranged at the top and bottom of the beam in order to determine the strains produced by the loads applied, not only at the

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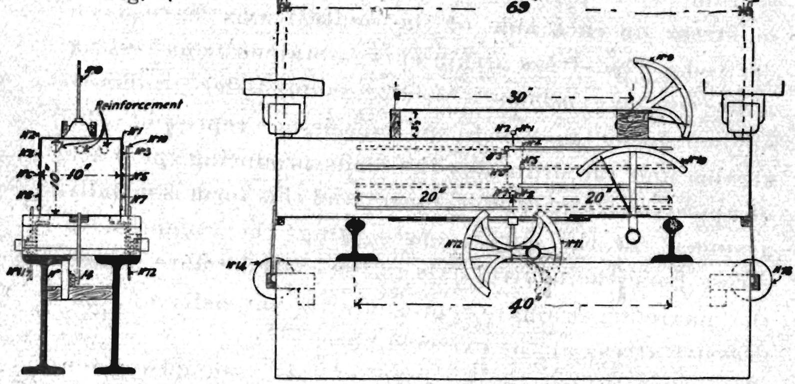
1 Further Experiments on the Strength and Elasticity of Reinforced Concrete.—Proc. Roy. Soc. N.S. Wales, Sept. 7, 1904.

2 Apparatus for ascertaining the minute strains which occur in materials when stressed within the elastic limit, by Prof. Warren. The theory of the Reflecting Extensometer of Prof. Martens, by G. H. Knibbs—Proc. Roy. Soc. N.S. Wales, July and August 1897.

extreme fibres, but at four other points in the depth of the beam on each side. Martens' sectors and dials were also attached to the beam in order to determine the end and centre deflections. The loads were applied at the ends of the beam by means of two hydraulic presses, and two rolled steel beams, resting upon a table of a special form of vertical Buckton testing machine, carry the knife edges upon which the concrete beam rests.

Fig. 3.




**Arrangement of Apparatus**

Nos. 1 — 8, Martens' Mirrors. Nos. 9 — 12, Sectors. Nos. 13, 14, Dials.

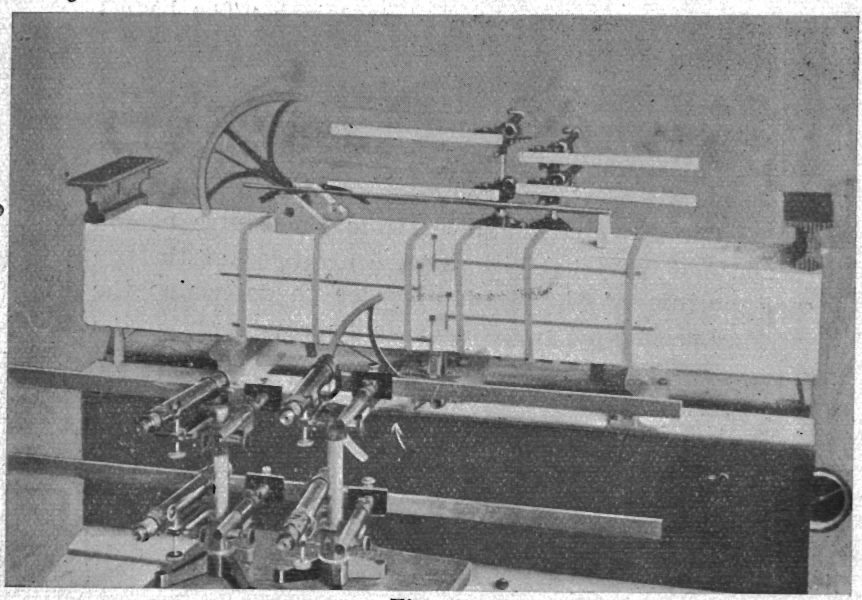
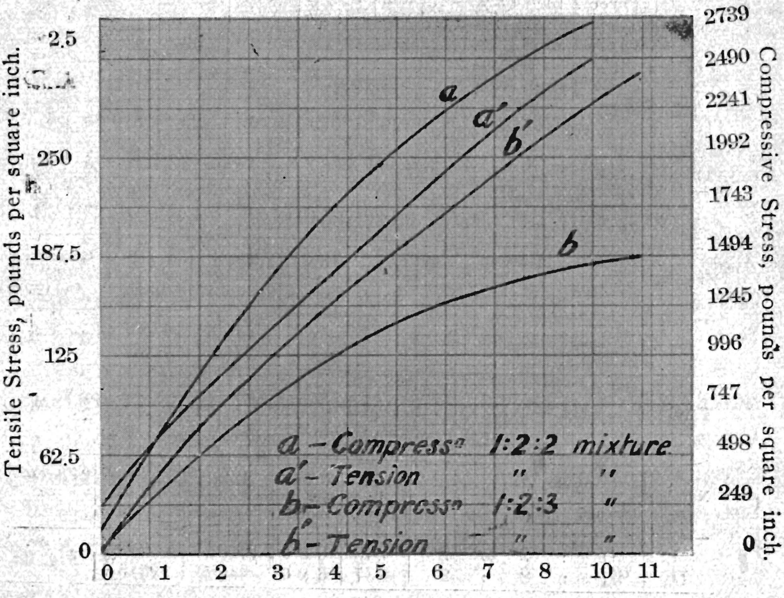


Fig. 4.

The arrangement of the fourteen instruments used in the determination of the various deformations is shown in Fig. 3, but the number denoting each instrument was used for convenience in tabulating the results from which the diagrams Figs. 5 to 8 have been plotted, and has no reference to the numbers on these diagrams. Fig. 4 is a photograph of the beam removed from the testing machine in order to show more clearly the instruments attached to it. In order to derive the stresses from the strains producing them, recorded in the manner above described, experiments were made, on plain concrete prisms, of the same age and composition as the beams under consideration, subjected to compression, and on briquettes of the same material subjected to tension; the results obtained have been plotted in Fig. 5, and from these curves the stress can be determined for each inch in the depth of the beam when the corresponding strain is known.

Fig. 5.—Stress-Strain Curves from Tensile Test of Concrete Briquettes and from Compressive Stress of Concrete Prisms (no reinforcement).




Compression per inch in units of .0001 inch.
Extension per inch in units of .00001 inch.

Some of the results obtained in testing the ten beams are recorded in Table XVI., and in the briquettes and prisms of the concrete used in the beams in Tables XVII. and XVIII.

### Table XVI.—TRANSVERSE TESTS OF REINFORCED CONCRETE BEAMS.

6 ft. by 10 in. by 10 in. Supports 40 in. centres. Loaded at ends. Reinforced with Bessemer steel bars, Elastic Limit = ——lbs. per sq. in.

| Number | Composition— Cement | : Builders' Sand | : ¾" Nepean Shivers | Age in Days | Maximum Deflection measured. Inches. | Load at which max deflec. was measured | Load producing fracture, tons. | B. Mt. at fracture = $14{\cdot}5 \times \frac{W}{2}$ inch tons. | Reference to Curve. |
|---|---|---|---|---|---|---|---|---|---|
| I. | 1 | : 2 | : 2 | | | Tons | | | |
| | No bars .. ... | | | 342 | 0·01 | 45 | 45 | 32 6 | Fig. 4 |
| II. | 1 | : 2 | : 2 | | | | | | |
| | Three ⅝ inch bars ... | | | 357 | 0·076 | 15 | 16 | 116 0 | |
| III. | 1 | . 2 | : 3 | | | | | | |
| | Three ¾ inch bars .. | | | 353 | 0·099 | 19 | 20 | 145 0 | |
| IV. | 1 | : 2 | : 3 | | | | | | |
| | Three ⅞ inch bars ... | | | 353 | 0·081 | 18 | 19 | 137·75 | |
| V. | 1 | : 2 | : 2 | | | | | | |
| | Three ⅞ inch bars ... | | | 357 | 0·146 | 21 | 22 | 159·5 | |
| VI. | 1 | : 2 | : 3 | | | | | | |
| | Three 1 inch bars ... | | | 365 | 0·157 | 22 | 22 | 159·5 | |
| VII. | 1 | : 2 | : 2 | | | | | | |
| | Three 1 inch bars ... | | | 369 | 0·109 | 26 | 28 | 203·0 | Fig. 5 |
| VIII. | 1 | : 2 | : 2 | | | | | | |
| | Three 1 inch bars ... | | | 314 | 0·074 | 24 | 26 | 188·5 | ,, |
| IX | 1 | : 2 | : 2 | | | | | | |
| | Three 1 inch bars ... | | | 320 | 0·079 | 26 | 29 | 210·25 | ,, |
| X. | 1 | : 2 | : 2 | | | | | | |
| | Three 1 inch bars ... | | | 3 19 | 0·069 | 24 | 27 | 195·75 | ,, |

### TABLE XVII.—TENSILE TEST OF CONCRETE BRIQUETTES.

| Number | Composition. Cement | : Builders Sand | : ¾in. Nepean Shivers. | Age in Days. | Cross Section inches. | Total Load pounds. | Load lbs. per square inch. | Reference to Curve. |
|---|---|---|---|---|---|---|---|---|
| .. | | 2 | : 2 | 341 | 4 x 4 | 6700 | 418·7 | Fig. 3a |
| I | 1 | : 2 | : 2 | 339 | 4 x 4 | 6450 | 403 7 | |

### TABLE XVIII.—COMPRESSION TEST OF CONCRETE PRISMS.

Length equal to 12 inches.

| Number | Composition. Cement | : Builders' Sand | : ¾in. Nepean Shivers. | Age in Days. | Cross section inches. | Total Load pounds. | Load lbs. per square inch. | Reference to Curve. |
|---|---|---|---|---|---|---|---|---|
| I. | 1 | : 2 | : 2 | 340 | 6 x 6 | 104832 | 2912 | Fig. 3a |
| II. | 1 | : 2 | : 2 | 337 | 6 x 6 | 94080 | 2613 | |

Fig. 6.—Distribution of Strain and equivalent Stress, over the Cross Section of a Plain Concrete Beam as experimentally determined.

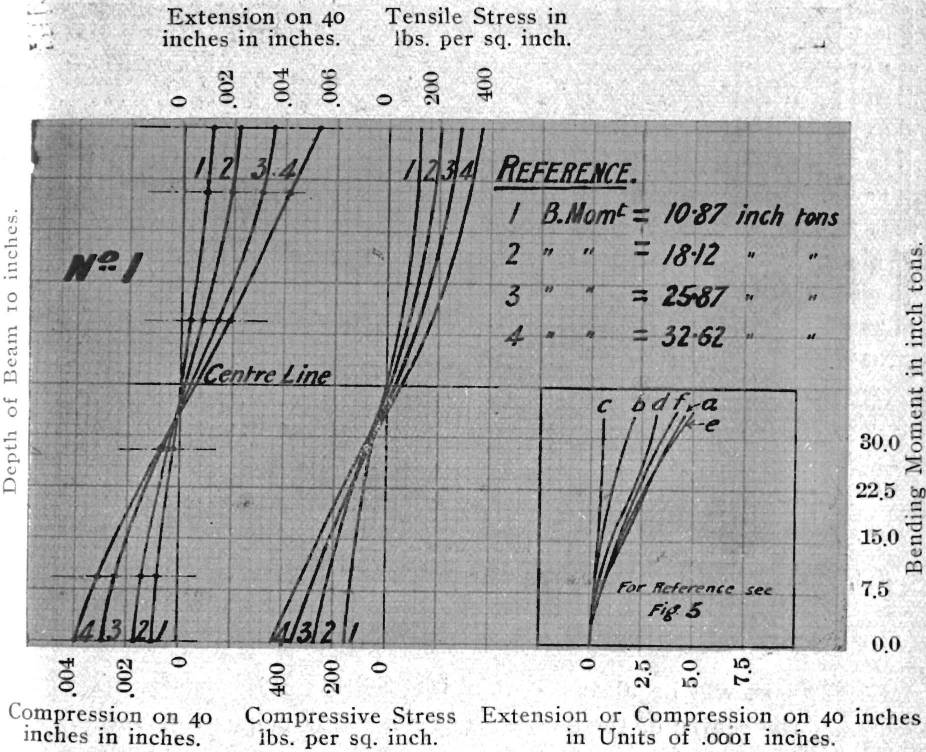
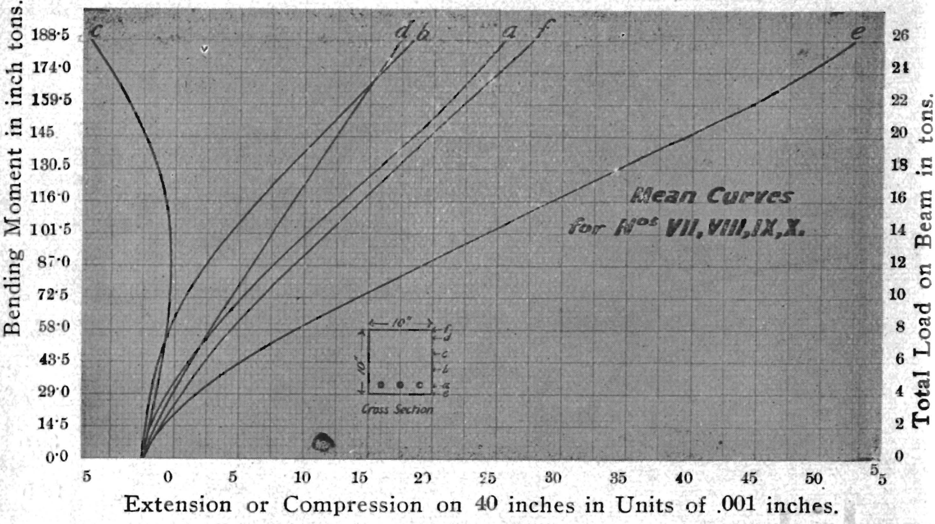



Fig. 6 shows the strain diagram and the stress diagram derived from it, by means of Fig. 5, for a plain concrete beam without reinforcement. The stress diagram showing the distribution of stress in the depth of the beam is shown on the right of the strain diagram.

The curves *c*, *b*, *d*, *f*, *a*, *e*, have been plotted from the deformations obtained by the various instruments attached to the beam at the position shown in Figs. 3 and 4, the exact distances from the top and bottom and centre of the beam are the same as described in the reference printed under Fig. 7. The results obtained have been used in drawing the strain curves 1, 2, 3, and 4.

Fig 7.—Curves showing the Extension (or Compression) of Fibres of a Reinforced Concrete Beam, measured at several points over the depth of the beam.




**Reference to Curves.**

a—Extension measured 1¼ inch from tension face of beam
b—Extension measured 1¼ inch from centre of beam on tension side
c—Compression measured 1¼ inch from centre of beam on compression side
d—Compression measured 1¼ inch from compression face of beam
e—Extension measured on tension face of beam
f—Compression measured on compression face of beam

It will be observed that the neutral axis remains in the centre of the depth up to a bending moment of one-third that necessary to produce fracture, and that it gradually moves towards the compression face of the beam as the bending moment increases, the maximum deviation being 0:8 of an inch. The strain curves 1, 2, 3, and 4, Fig. 6, show how nearly a plane section before flexure remains a plane section after flexure. The stress curves are more curved on the tension than on the compression sides, where they approximate very closely to a straight line. The strains obtained from testing a reinforced concrete beam are recorded in Fig. 7, which gives in each case the mean of tests of four beams of the same material reinforced in a similar man-