Using the same data as before and applying the equations to the beam tested we obtain:—

$x=0{\cdot}444$

$c=2750$ lbs. per square inch.

$\frac{M}{bh^2}=568{\cdot}512.$

Professor Hatt obtains:—

$$\frac{M}{bh^2}=\frac{5\,cx^2}{12}+pf(u-x)\ \dots\ (1)$$

$$\frac{2}{3}cx=pf\ \dots\ (2)$$

$$p\frac{E}{E_c}(u-x)=\frac{2}{3}x^2\ \dots\ (3)$$

(3) can be simplified by solving for x thus :—

$$x=\sqrt{\frac{3\,p\,E_s\,u}{2\,E_c}-\frac{3\,p\,E_s}{4\,E_c}}\ \dots\ (4)$$

(1) may be simplified thus:—

$$\frac{M}{bh^2}=pf\left(u-\frac{3x}{8}\right)\ .\ (5)$$

Using the same data as before and applying the results in a similar manner we obtain:—

$x=0{\cdot}4$

$c=2835$ lbs. per square inch.

$\frac{M}{bh^2}=567$

Professor Hatt, however, uses data which appear to differ from the author's, and if these had been inserted the results would not be the same. For instance he gives:—

$\frac{E_s}{E_c}=7{\cdot}5$ Instead of 12 or 15.

This appears to be due to a want of agreement in the definition of the coefficient of elasticity.

If we require the coefficient of elasticity in a concrete prism for a certain intensity of stress, according to Professor Hatt the prism should be loaded to this stress and the strain noted, then the load is to be removed to zero and the permanent strain noted. The ratio of the stress to the difference of the two strains is termed the modulus of elasticity.

The following Table XX. gives the modulus of elasticity obtained in this way by Professor Hatt:—

TABLE XX.

PROFESSOR HATT'S TESTS OF CONCRETE IN COMPRESSION.

Composition.	Condition	Age in Days	No. of specimens.	No. of determinations	Coefficient of Elasticity of 10,000. Av'g.	Max.	Min.	Ultimate strength in lbs. per sq. in. Av'g.	Max.	Min.
STONE.										
1 — 2 — 5	Wet	90	3	14	531	765	378	2483	2710	2268
1 — 1, 6 — 5	,,	90	3	10	431	504	326	2482	2985	2030
1 — 1, 6 — 5	Plastic	90	3	15	421	504	380	2576	2610	2530
Average of all, 90 days.				39	461			2513		
1 — 2 — 5	Wet	28	3	11	288	314	217	1836	1990	1730
1 — 1, 6 — 5	,,	28	3	24	374	515	270	2296	2598	2030
1 — 1, 6 — 5	Plastic	28	3	13	345	488	200	2070	2450	1730
Average of all 28 days.				48	335			2067		
Average of all stone					398			2296		
GRAVEL.										
1 — 6	Wet	90	4	23	474	1030	266	2855	3990	2290
1 — 5	,,	90	3	15	555	760	420	2933	3520	2289
1 — 5	Plastic	90	3	17	412	767	306	2625	2985	2240
Average of all, 90 days.				55	480			2804		
1 — 6	Wet	28	3	17	371	552	222	2293	2590	1890
1 — 5	,,	28	3	23	356	455	267	2290	3280	1490
1 — 5	Plastic	28	3	20	402	520	271	2136	2333	1890
Average of all 28 days.				60	379			2239		
Average of all gravel.					**429**			**2521**		

Average ratio of Modulus in compression to Modulus in Tension = 0.9

Average ratio of strength in compression to strength in tension = 8.5.

(For 1-1, 6-5), and 1-5).

Average ratio of Modulus of wet mixture to Modulus of Plastic mixture = 1.08.

Average ratio of strength of wet mixture to strength of Plastic mixture = 1.00.

Mr. Wentworth (*) has calculated the fallowing table for the depth of the neutral axis, the area of steel required for various intensities of stress in the steel reinforcement.

It will be noted that decreasing the intensity of stress in the steel, the stress in the concrete remaining contant, lowers the neutral axis, and consequently increases the resistance of the section.

TABLE XXI.

Material.	$x =$ Depth of the neutral axis from the compression side of the beam.	Area of Steel reinforcement required in sq. inches.	Moment of Resistance of the section in inch lbs.
Stone concrete	0·273d	0·051d	$744d^2$
Cinder ,,	0·333d	0·050d	$711d^2$
Stone ,, granolithic top	0·292d	0·060d	$870d^2$
Cinder ,, ,,	0·355d	0·059d	$826d^2$
		f=14,000 lbs. per sq. inch.	
Stone concrete	0·300d	0·0643d	$810d^2$
Cinder	0·364d	0·0623d	$767d^2$
Stone ,, granolithic top	0·320d	0·0755d	$944d^2$
Cinder ,, ,,	0·386d	0·0728d	$888d^2$
		f=12,000 lbs. per sq. inch.	
Stone concrete	0·333d	0·0833d	$889d^2$
Cinder ,,	0·400d	0·0800d	$832d^2$
Stone ,, granolithic top	0·355d	0·0976d	$1033d^2$
Cinder ,, ,, ,,	0·423d	0·0931d	$959d^2$
		f=10,000 lbs. per sq. inch.	
Stone concrete	0·375d	0·1125d	$984d^2$
Cinder ,,	0·444d	0·1066d	$908d^2$
Stone ,, granolithic top	0·397d	0·1312d	$1138d^2$
Cinder ,, - ,,	0·468d	0·1236d	$1043d^2$

Considère and Christophe use a ratio of 15 to 20 in order to allow for the reduction in the value of Ec before rupture, but again the rupture would be about one half as much if Professor Hatt's method were adopted.

(*) Discussion on Concrete Steel, Trans. Am. Soc. C.E., vol. LIV., Part I., 1905.

Professor Hatt has given equations for the determination of x, M, and c for loads less than those which produce a crack on the tension side, and points out clearly the necessity of using the correct values of the coefficient of elasticity for the particular stresses developed in tension and compression in a reinforced concrete beam. The real difficulty in obtaining correct results for steel concrete work consists in knowing accurately the strength and coefficients of elasticity of the various materials employed under the conditions existing.

According to Professor Hatt, recent experiments in the United States show that very fine hair cracks develop in concrete beams at a period of deflection which corresponds with a stress of not more than 2000 to 3000 lbs. per square inch in the steel. Professor Hatt, however, in his own laboratory tests of beam did not obtain visible cracks until a stress of about 20,000 to 27,000 lbs. per square inch was developed in the steel.

The author did not observe any cracks in the beams tested by him at a lower estimated stress than 20,000 lbs. per square inch.

Considère (*) first pointed out that armoured concrete can not only support without fracture much greater extension or stretching than that which breaks unarmoured concrete, but also possesses after these considerable deformations a resistance to tension comparable with, and perhaps equal to, that of concrete which has not undergone any previous deformation. These results have been criticised by several steel concrete authorities in America, and Considère (†) has since made further experiments which confirm his previous conclusions.

(*) Comptes Rendus de l'Academie des Sciences, Paris.
(†) Also 1905, Vol. CXL., pp. 291-5.

It is more or less true, however, that structures of armoured concrete generally show cracks, and the principal cause of this is that concrete, exposed to dry air after manufacture, shrinks considerably for some days and has but small resistance. If this contraction is hindered by the metallic armouring cracks are generally produced, at first invisible, but afterwards opening out and extending when the structure is subjected to tension. On the other hand, if the concrete is kept moist for a sufficient time after manufacture there is no shrinkage and no tendency to fracture while the material is acquiring resistance and ductility. The concrete steel tends to shrink when it is no longer kept moist, but it then possesses high resistance, and does not crack in spite of the opposition to shrinkage presented by the enclosed metal.

SHEARING STRESSES IN STEEL-CONCRETE BEAMS AND THE METHODS EMPLOYED TO RESIST THEM.

When a steel-concrete beam reinforced in a horizontal plane only is subjected to a uniformly distributed load, it tends to fail near the ends by cracking on the tension side in a direction inclined towards the centre of the beam, following the full lines in Fig. 17. The inclination of the cracks is 45 deg. To prevent this cracking, the beam should be reinforced in a vertical plane by means of bars arranged vertically or inclined at 45 deg. sloping away from the centre of the beam.

The cracking is more likely to occur with distributed loads than with a load concentrated at the centre as in testing, and it is more likely to occur in deep beams than in shallow beams, as in both cases the shearing forces are greater. In a steel-concrete beam properly designed, a crack should appear on the tension face before the elastic limit of the steel reinforcement is reached. Before a crack has developed, the internal stresses will follow the curved lines shown in the figure in which the full lines denote compression and the thin lines tension; these lines intersect the neutral axis at an angle of 45 degrees and equilibrium is established among the internal stresses, and no reinforcement is needed. When a crack has developed, the thick curved lines should be eliminated below their intersection with the neutral axis, and tangents to the curve at the points where they intersect the neutral axis continued to their intersection with the horizontal reinforcement at the under side of the beam, show the altered directions of the lines denoting the internal stresses. These inclined lines may be resolved horizontally and vertically at the points where they intersect the horizontal reinforcement, into horizontal and vertical components of equal intensity; the former are resisted by the horizontal reinforcement, the latter must be resisted by vertical stirrups or preferably by inclined bars, the sectional area and spacing of which should be made proportional to the shearing stresses developed.

If the beam is subjected to a uniformly distributed load, the distribution of bending moments along the beam is represented by a parabola, Fig. 18, the equation being:—

$$y = \frac{w}{2}(lx - x^2)$$

Where y = the bending moment at any distance x measured from the origin.

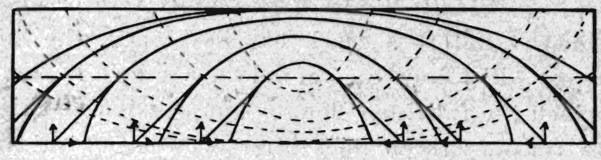


Fig. 17

Fig. 18

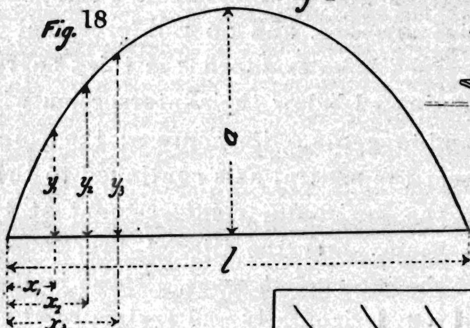
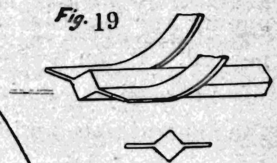


Fig. 19



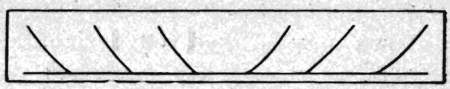


Fig 21

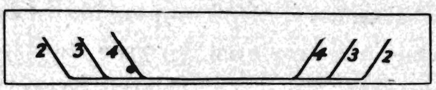


Fig. 20

w = the uniform load per unit of length.

l = the span.

If a = the area of the horizontal reinforcement in square inches required at the centre of the beam:—

Then a is proportional to the central bending moment when $= \frac{l}{2}$ *i.e.*, $\frac{wl^2}{8}$ and we may write the equation in terms of a thus:—

$$y = \frac{4a}{l}\,(lx - x^2)$$

For any length of beam denoted by $x_2 - x_1$, the area required to resist the shearing stresses arranged in a vertical plane is:—

$$y_2 - y_1 = \frac{4a}{l}\left\{(x_2 - x_1) - \left(\frac{x_2^2 - x_1^2}{l}\right)\right\}$$

and the area of rods arranged to slope at an angle of 45 deg. away from the beam, is clearly the value of $(y_2 - y_2)$ Sec. 45 deg. for the length of beam included between $(x_2 - x_1)$.

If we make the distance between the ordinates of the parabola = 1, the equation may be written:—

$$y_2 - y_1 = \frac{4a}{l}\left\{1 - \left(\frac{x_2 + x_1}{l}\right)\right\}$$

The shearing stress between one foot from the ends and the ends of the span is:—

$$y_2 - y_1 = \frac{4a}{l}\left(1 - \frac{1}{l}\right) = \frac{4a}{l^2}(l - l)$$

and the area of rods inclined at 45 deg. is $= \frac{4a}{l^2}\ (l - 1)$ Section 45 deg.

Between 2 feet and 1 foot:—

$$y_2 - y_1 = \frac{4a}{l}\left(1 - \frac{3}{l}\right) = \frac{4a}{l^2}(l - 3)$$

the area of rods inclined at 45 deg. is:—

$$\text{Area} = \frac{4a}{l^2}(l - 3) \text{ Sec. 45 deg.}$$

It should be clearly understood that the actual stress upon the inclined rod is the difference between the total stress given by the above equations and the shearing resistance of the concrete. Hence generally no reinforcement will be needed near the centre where the small shearing stress may be left to the concrete.

As an example we may find the area of the reinforcement to resist the shearing stresses in a concrete beam 10 inches by 10 inches cross section, reinforced horizontally by 3 rods ⅞ inch in diameter if the span is 10 feet and the load uniformly distributed.

The area of 3 rods ⅞ inches diameter = 3 × 0·6 1.8 square inches. For the first foot from the ends the area required if arranged vertically is

$$\frac{4 \times 1.8}{10}\left(1 - \frac{1}{10}\right) = 0.648 \text{ square inches.}$$

and if inclined at 45 deg.:—

$$0{\cdot}648 \times 1.414 = {\cdot}0916 \text{ square inches.}$$

For the portion included between $x_2 = 2$ and $x_1 = 1$:—

$$\frac{4 \times 1.8}{10}\left(1-\frac{3}{10}\right) = 0\ 504 \text{ square inch verticaly}$$

$= 0{\cdot}504 \times 1{\cdot}414 = 0{\cdot}713$ square inch inclined at 45 deg.

For the portion included between $x_2 = 3$ and $x_1 = 2$:—

$$\frac{4 \times 1{\cdot}8}{10}\left(1+\frac{5}{10}\right) = 0{\cdot}36 \text{ square inch vertically}$$

$= 0{\cdot}36 \times 1{\cdot}414 = 0{:}509$ square inch inclined at 45 deg.

For the portion included between $x_2 = 4$ and $x_1 = 3$:—

$$\frac{4 \times 1\ 8}{10}\left(1+\frac{7}{10}\right) = 0{:}216 \text{ square inch vertically}$$

$= 0.216 \times 1.414 = 0{:}305$ square inch inclined at 45 deg.

We may provide steel stirrups having the area calculated above, or inclined bars having an area equal to the vertical stirrup multiplied by 1:414. The bars arranged to take the shearing stresses should be rigidly connected to the horizontal reinforcement in all cases. Mr. Julius Kahn accompishes condition by the use of bars of the form shown in Fig. 19.

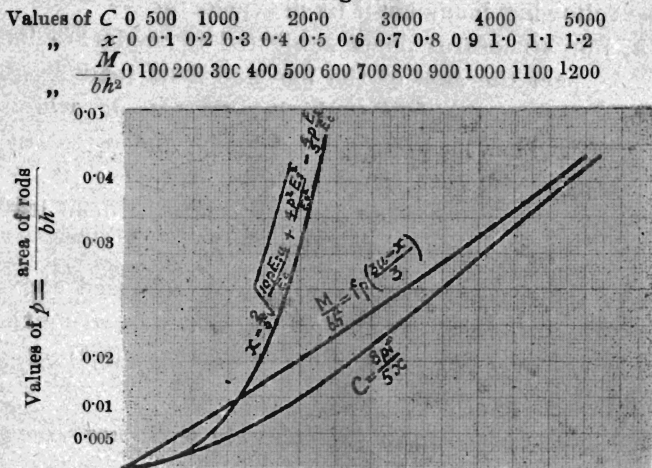
Mr. A. L. Johnson has proposed and largely used corrugated bars, consisting of rolled steel bars having a ribbed surface, to reinforce concrete beams which exceed a span of 15 feet; for spans between 8 and 15 feet expanded metal is used to reinforce the concrete. Both the corrugated bars and the expanded metal furnish a mechanical bond, quite independent of the bond due to adhesion between the steel and the concrete. M. Considere has shown that the adhesive resistance tends to yield to a soliciting force, and there can be no doubt that some structures are exposed to vibrations and shocks which must tend to break the ordinary adhesion bond between steel and concrete.

Fig. 20 shows the arrangement of corrugated bars in Johnson's reinforcement for deep beams.

Fig. 21 shows the arrangement of the Kahn bars in a similar beam, in which the reinforcement is inclined to the vertical, with varying upward curvature approximating to the lines of principal tensile stress:—

The diagram Figs. 22 and 23 show the values of $\frac{M}{bh^2}$, x and c for the concrete used in the three beams, but with different values of p. In 23 the values of f $= 42000$ lbs., $u = 0{\cdot}9$ and $\frac{E_s}{E_c} = 12$ have been substituted in the equation given in Fig. 22.

Fig. 22.



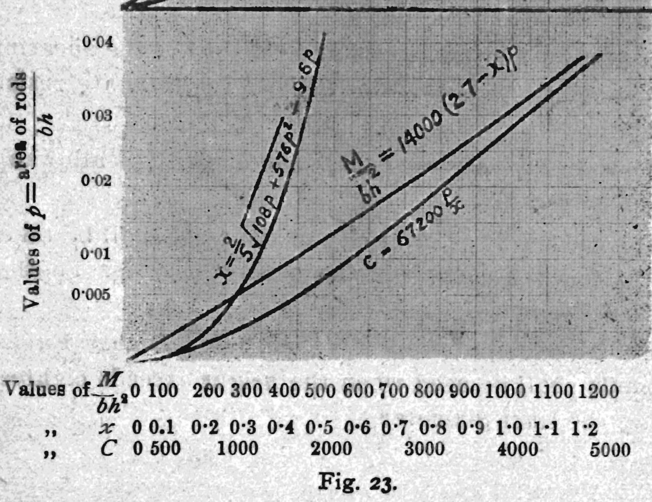


Fig. 23.