In the Prussian regulations for reinforced concrete in building construction, the following is worthy of notice in regard to working stresses:—

In the members subjected to bending, the compressive stress in the concrete shall not exceed one-fifth of its ultimate resistance; the tensile and compressive stresses in the steel shall not exceed 17,000 pounds per square inch.

The following loads shall be provided for:—

a. For structural parts subjected to moderate impact the sum of the live and dead loads.

b. For parts subject to higher impact or widely varying loads; the dead loads added to one and one half times the live load.

c. For parts subject to heavy shocks; the dead load added to twice the live load.

In columns the concrete shall not be stressed above one-tenth of its breaking strength, and in computing the steel reinforcing for column flexure a factor of safety of five shall be provided.

The shearing stress in the concrete shall not exceed 64 pounds per square inch.

If greater shearing resistance is shown, the shearing stress shall not exceed one-fifth of the ultimate resistance.

The adhesive stresses shall not exceed the allowable shearing stress.

The coefficient of elasticity of the steel shall be taken as fifteen times that of the concrete, unless another ratio be shown.

For the computation of columns for flexure Euler's formula shall be used when the height exceeds eighteen times the least diameter.

The equation for locating the neutral axis in a rectangular beam when expressed by the symbols adopted in this paper for the sake of comparison is:—

$$x = \frac{E_s p}{E_c}\left\{\sqrt{1 - \frac{2 u E_c}{p E_s}} - 1\right\}$$

For obtaining the maximum intensity of stress on the extreme fibres in compression:—

$$c = \frac{2M}{bh^2 x}\left(u - \frac{x}{3}\right)$$

and for the maximum intensity of stress in the metal reinforcement:—

$$f = \frac{M}{p\,b\,h^2\left(u - \frac{x}{3}\right)}$$

Fore tee-formed sections, such as shown in Fig. 24, the equations remain the same if the neutral axis lies in the flange or at the junction of the flange with the web.

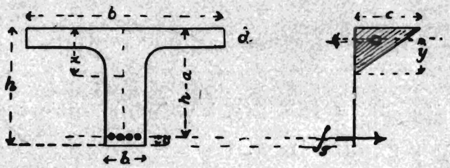


Fig. 24.

If the neutral axis passes through the web, the slight compressive stresses in the web may be neglected, and we have approximately

$$x = \frac{(h-a)\,n f_s + \frac{1}{2}\,bd^2}{b\,d + n f_s}$$

$$y + x - \frac{d}{2} + \frac{d^2}{6\,(2x - d)}$$

The maximum stress in the steel is:—

$$p_s = \frac{M}{f_s(h - a - x + y)}$$

The maximum stress in the concrete is:—

$$c = p_s \frac{x}{n(h-a-x)}$$

In applying the foregoing results to the practical design of reinforced concrete beams, we may consider the curve of stress on the compression side of the neutral axis, to be represented approximately by a straight line, and for the sake of safety we may neglect the tensile strength of the concrete.

The substitution of a straight line for the parabola will give the compressive stress at the extreme fibre about 10 per cent. too great for working stresses, and about 25 per cent. too great for the breaking stress. From these considerations the following working stresses and approximate equations are recommended:—

The extreme fibre stress in compression = 1000 pounds per square inch.

The shearing stress in compression = 100 pounds per square inch.

The direct compression stress = 500 pounds per square inch.

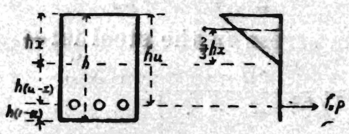
The tensile stress in the steel reinforcement = 16000 pounds per square inch.

The compressive stress in the steel reinforcement = 12000 per square inch.

The shearing stress in the steel reinforcement = 10000 pounds per square inch.

No tensile resistance must be taken for the concrete. The ratio:—

$$\frac{E_s}{E_c} = 15$$



The equations become:—

$$\frac{cx}{2} = fp \quad (1)$$

$$\frac{c}{f} = \frac{E_c}{E_s}\left(\frac{x}{u-x}\right) \quad (2)$$

$$\frac{M}{bh^2} = fp\left(\frac{3u-x}{3}\right) \quad (3)$$

$$x = \sqrt{\frac{p^2E^2}{E_c^2} + 2up\frac{E_s}{E_c}} - p\frac{E_s}{E_c} \quad (4)$$

To find the area of the steel reinforcement in a beam 10 inches wide by 20 inches deep to carry a distributed load of 500 pounds per foot run on a span of 24 feet—

Take $hu = 18$ in order to allow sufficient concrete below the steel bars, so that $u = 0.9$.

The bending moment $= \frac{12{,}000 \times 24 \times 12}{8} = 432000$ in. lb.

$$\therefore \frac{M}{bh^2} = \frac{432000}{4000} = 108$$

$$\frac{M}{bh^2} = 16000\left(\frac{3u-x}{3}\right)p = 108$$

Take $x = 0.4$ as a first approximation,

Then $p = \frac{108}{12266} = .0088$

And the area of steel $= 200 \times .0088 = 1.76$ sq. in.

The value of x for $p = .0088$ from eq. (4) is .37.

Hence $\frac{M}{bh^2} = 16000\left(\frac{2{\cdot}7 - {\cdot}37}{3}\right).0088 = 109$

We may use four bars each ¾ in. diameter. The compression on the extreme fibre will be approximately:—

$$c = 32000\ \frac{{\cdot}0088}{{\cdot}37} = 761 \text{ lbs. per sq. in.}$$

To find the safe load applied to the centre of a beam 10×10 in. when supported at points 10 feet apart; the area of the reinforcement is 1.8 sq. in.

In this case $p = 0.018$ and $p^2 = 0.000324, u = 0.85, \frac{E_s}{E_c} = 15$

$$x = \sqrt{225 \times 0.000324 + 25.5 \times 0.018} - 0.27 = 0.45$$

$$\therefore \frac{M}{bh^2} = 16000 \times 0.018 \left(\frac{2.55 - 0.45}{3}\right) = 201.6$$

$\therefore M = 201600$ inch pounds

and $W = 6720$ pounds.

also $c = 1280$ lbs. per sq. in.

The actual bending moment which produced fracture was 604800 inch pounds, so that the factor of safety is:—

$$\frac{604800}{201600} = 3$$

In this example the percentage of reinforcement is high, and the compressive stress developed in the concrete exceeds the safe limits recommended. A large number of beams, however, reinforced to the same extent, and tested to destruction, did not fail by compression of the concrete.

Prof. Talbot has derived equations which give results corresponding more nearly with those obtained in actual experiments by considering the ratio of the deformation in the extreme fibre in compression under a given stress, to the deformation at the stress producing fracture; thus, if the ratio is denoted by 9, we have the following formula for the determination of the neutral axis:—

$$x = \sqrt{\frac{2\frac{E_s}{E_c}p}{1 - \frac{q}{3}} + \frac{\left(\frac{E_s}{E_c}p\right)^2}{\left(1 - \frac{q}{3}\right)}} - \frac{\frac{E_s}{E_c}p}{1 - \frac{q}{3}}$$

This gives the position of the neutral axis after the tensile stress in the concrete become negligible and before the concrete reaches its utimate strength. The value of x will vary somewhat for the range of g considered.

For $q = 1$ we have:—

$$x = \sqrt{3p\frac{E_s}{E_c} + \frac{9}{4}\left(\frac{E_s}{E_c}p\right)^2} - \frac{3}{2}p\frac{E_s}{E_c}$$

which gives the position of the neutral axis when the concrete is at the limit of its compressive strength. If $q = 0$, we obtain equation (4) based upon the stress strain deformation assumed to be a straight line.

If $q = \frac{1}{4}$ we have:—

$$x = \sqrt{\frac{24}{11}p\frac{E_s}{E_c} + \frac{144}{121}\left(p\frac{E_s}{E_c}\right)^2} - \frac{12}{11}p\frac{E_s}{E_c}$$

which gives the position of the neutral axis for deformations corresponding closely with those developed under working stresses. The compressive stress in the extreme fibre becomes:—

$$c = \frac{2fp}{x}\left(\frac{1-\frac{q}{2}}{1-\frac{q}{3}}\right)$$

This equation also gives different values for different values of the ratio q thus:—

If $q = 1$ we have

$$\frac{c}{\frac{2\cdot pf}{x}} = \frac{3}{4}$$

If $q = \frac{3}{4}$; $\dfrac{c}{\frac{2pf}{x}} = \dfrac{5}{6}$

If $q = \frac{1}{2}$; $\dfrac{c}{\frac{2pf}{x}} = \dfrac{9}{16}$

If $q = \frac{1}{4}$; $\dfrac{c}{\frac{2pf}{x}} = \dfrac{21}{22}$

If $q = 0$; $\dfrac{c}{\frac{2pf}{x}} = 1$

If the beam is reinforced so that the extreme fibre stress in compression is within the safe working stress for the concrete used, the moment of resistance may be expressed in terms of the percentage of steel reinforcement.

$$\frac{M}{bh^2} = fp\left(\frac{3u - x}{3}\right) \quad (3)$$

MOMENT OF RESISTANCE OF T SECTIONS.

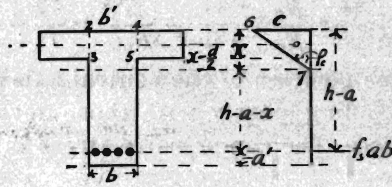


Fig. 27.

Let P_c = the total compressive strength in concrete

„ P_c' = the compressive strain in the stem of the T

„ P_c'' = the compressive stress in the flange of the „

„ P_s = the tensile stress in the steel

„ S = the shearing stress in the concrete in pounds per square inch

„ a = the sectional area of the steel per inch width of the beam

then—

$$ab\,f_s = P_c = P_s$$

$$ab = \frac{P_c}{f_s} = \frac{P_s}{f_s}$$

Let S_h = the total shear between the rib and flange along their plane of union

„ S_v = the total vertical shear on the two planes 2-3 and 4-5

Assume that $S = \frac{c}{8}$ and also $P_c'' = S_h = S_v$ These assumptions are on the safe side.

It is clear that for equal strength in shear b should be equal to $2\,d$

If l = the span o the beam in feet

$$S_h = \frac{S}{2} \times b \times \frac{l}{2} \times 12 = 3b\,Sl$$

$$S_v = \frac{S}{2} \times 2d \times \frac{l}{2} \times 12 = 6\,d\,S\,l$$

$$\frac{h-a'-x}{x} = \frac{f_s E_c}{c\,E_s}$$

Since the stress-strain curve is a straight line in compression 6—7, and p_c = the intensity of compressive stress on the plane 3—5 we have

$$\frac{p_c}{c} = \frac{x-d}{x} \therefore p_c = c\frac{(x-d)}{x}$$

$$\therefore P_c' = \frac{b}{2}(x-d)^2 \frac{c}{x}$$

$$P_c'' = \frac{b'd}{2}(p_c + c) = \frac{b'cd}{2x}(2x-d)$$

$$S_h = 3bsl = P_c'' = \frac{b'cd}{2x}(2x-d)$$

$$s = \frac{c}{8}$$

$$\therefore S_h = \frac{3}{8}bcl = \frac{b'cd}{2x}(2x-d)$$

$$\therefore b' = \frac{6blx}{8d\,(2x-d)}$$

$$\therefore P_c'' = \frac{6blx}{8d\,(2x-d)} \times \frac{cd(2x-d)}{2x} = \frac{3}{8}blc$$

$$P_c = P_c' + P_c'' = \frac{bc}{2x}(x-d)^2 + \frac{3blc}{8}$$

$$\therefore M = \frac{2P_c'}{3}(x-d) + P_c''\left(x - \frac{d}{2}\right) + P_s(h-a'-x)$$

$$M = \frac{bc}{3x}(x-d)^3 + \frac{b'cd}{4x}(2x-d)^2 + f_s ab\,(h-a'-x)$$

Example—Let h = 36 inches

b' = 36 inches.

b = 12 inches.

a' = 3 inches

d = 6 inches

$$\frac{E_s}{E_c} = 15$$

$$c = 500$$

$$f = 16000$$

$$s = 50$$

$$ab = 7{\cdot}5 \text{ square inches}$$

$$\frac{h - a' - x}{x} = \frac{f_s\, E_c}{c\, E_s}$$

$$\therefore \frac{33 - x}{x} = \frac{16000}{500 \times 15} \therefore x = 10{\cdot}53$$

$$M = \frac{12 \times 500}{31{\cdot}59}(4{\cdot}53)^3 + \frac{36 \times 500 \times 6}{42{\cdot}12}(15{\cdot}06)^2$$

$$+ 16000 \times 7{\cdot}5\ (22{\cdot}47)$$

$$= 3{,}295{,}594 \text{ inch pounds.}$$

Floor beams and girders of reinforced concrete are nearly always continuous over the supports. If they had everywhere an equal resistance to flexure the stresses could be determined by the usual formulae for calculations of strength of materials. In other words, if L were the common length of the spans, and w the uniformly distributed load per foot run, the bending moment would be:—

$\frac{WL^2}{12}$ on supports.

$\frac{WL^2}{24}$ at the middle of the span.

But in most constructions the load bears only upon certain spans, and therefore the moment at a support may be much lower than $\frac{1}{12}wL^2$ and much higher in the centre than $\frac{1}{24}$ wL^2. Considere adopts as a safe minimum at the supports —— so that the resistance at the centre must be based upon a bending movement of $\frac{1}{2}$ wL^2

DESIGN OF REINFORCED CONCRETE COLUMNS.

The experiments described on various kinds of reinforcement show a decided advantage in ultimate crushing resistance in the case of hooped concrete, but in regard to stiffness the hooped concrete is less than in concrete reinforced with longitudinal rods, and the concrete without reinforcement is stiffer than when reinforced, although its ultimate strength is considerably less. Te spirals forming the hoops cannot act until the effect of the longitudinal stress causes the concrete to swell laterally and press against the hoops. In the case of the reinforcement by longitudinal steel rods the column cannot be deformed without corresponding longitudinal deformations in the steel rods. The disadvantages of the longitudinal reinforcement due to the initial stress developed in consequence of the shrinkage of the concrete have been considerd, also the increased ultimate resistance of hooped concrete compared with longitudinal reinforcement by means of steel rods, but the practical advantages of the longitudinal reinforcement in connecting the colums to beams, as well as the increased stiffness obtained, suggest that for square columns of concrete the best reinforcement would be angle bars at the corners connected together on all four sides by lattice bars, the latter acting in a similar manner to the steel spirals.

For circular columns either the angle bar should be arranged at six or eight points in an inner circle without hooping, or they may be arranged inside a spiral of steel wire. In the case of the circular column hooped by means of a spiral, and of the square column with lattice bars connecting the angles at the corners, the safe working stress may be derived from the formula already given, thus:—

$$\text{Safe stress} = \frac{C}{A}\left(A + \frac{E_s a}{E_c}\right)$$

The values of C, and of $\frac{E_s}{E_c}$ must be chosen for the particular kind of concrete used, and the quantity of the material used in the lattice bars or hoops. If the effect of the lattice bars or spiral is to increase the strength of the concrete to 4000 pounds per sq. in. and the ratio of the co-efficients of the concrete and steel is 15, then dividing the safe working stress by V, and the areas of the concrete and steel reinforcement by A and a respectively, we have:—

$$V = \frac{800}{A}\left(a + 15a\right)$$

In a similar manner the working stress may be obtained for other qualities of concrete.