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SOME NOTES UPON THE DESIGN AND USE OF SPEED REDUCING GEARS.

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This paper is entitled "Some Notes upon the Design and Use of Speed Reducing Gears," and deals with that class of gear in which a worm and worm wheel are used when it is desired to transmit power from one shaft to another running at a reduced number of revolutions. No attempts have been made to introduce any mathematical or theoretical considerations of the principles involved in the construction of the gear, as most of the standard treatises on machine design deal more or less exhaustively with this side of the question. It may be stated broadly that, theoretically considered, this device has been declared inefficient as a means of transmitting power, due to the excessive friction factor, and it is generally so regarded amongst those who have had but occasional use for it.

Professor Unwin, in dealing with the construction of worm gearing ("Machine Design," Chapter XII., 1901 Edition), after going exhaustively into the relationship existing between the various factors involved, fixes the efficiency of single, double, and treble worm gearing as follows:—Single, 35 per cent.; double, 40 per cent.; treble, 50 per cent—where the radius of the worm equals twice the pitch and μ, the co-efficient of friction, equals .15. (The co-efficient is here taken as on metal and metal surfaces running dry.)

In this the Professor makes no estimate for losses in what may be termed the accessories of the gear—i.e., the bearings taking up the end thrust in worm and worm wheels and the journal usually constituting a part of the gear box.

That a formula, expressing mathematically the relation of the various functions involved in the design of such a gear, may easily become complex, will be readily conceded when the variables which affect the friction factor are realised; we have—

(1) The nature of the material of which the working parts are composed.
(2) The pitch of the worm wheel.
(3) The character of the lubricants used.
(4) The rate and nature of power transfer.

And these quite outside the design of the teeth of the worm wheel and devices for reducing friction due to end and side thrust, all of which may become pronounced causes of frictional losses if not carefully considered and accounted for in the designing of the gear; but it is a pleasing and, the writers believe, unique feature in this class of gearing that it has been found to furnish in practice a much higher efficiency than purely theoretical considerations might appear to warrant. This feature is strikingly illustrated by the results obtained with worm gears in a series of trials which were undertaken with a view to determining its efficiency as variously designed. (Plate XXXVI.)

As the writers are of the opinion that the results thus obtained may prove of some practical interest to members, the designs (Plates XLII. to XLV.) of the various gears under trial are submitted with a tabulated result (see Plate previously referred to) of the trials.

As will be seen from the plans, the designs of the gears, though practically the same, differed somewhat in general detail, due in part to the exigencies of their application, but for the most part to a striving after a more efficient, neat, and compact gear.

The various parts are marked with indicative numbers. A and B are the worm and worm wheel respectively. Sections (Plate XXXVII.) through the wheel are given showing the points of contact, and a uniform sliding action has been aimed at.

The worm and shaft are one steel forging, so made with the object of keeping the diameter of the worm as small as possible, so as to obtain as great an angularity in the thread of the worm as practicable, the frictional losses between the worm and worm wheel being, as is well known, thereby proportionately reduced. The worm shafts are carefully turned and finished, run in gun-metal bearings, and are each provided with a ball thrust bearing, an enlarged view of which is shown on Plate XXXVIII. In the first design this bearing was designed with grooves for a four-point contact, the groove being practically filled with balls; but after some experience it was found that a two-point contact, fewer balls, and a cage gave equal, if not better, results, was easier to machine, and a cheaper job.

The wheel centre is of cast iron, the rim of phosphor bronze. The centre is carefully turned and the rim rough turned upon the outside, finished in the bore, and pressed on the centre and secured with three keys arranged at equal distances round its inner circumference. The whole is then keyed upon the worm wheel spindle and the teeth roughed out upon a tooth milling machine. Finally the teeth are finished with a milling hob, a fac-simile of the worm with which the wheel is intended to work. The reducing gear, as a whole, is made so that the parts are reversible, the worm shaft being designed to turn end for end in its bearings, and may be driven from either end. The worm wheel may also be reversed in a similar manner. Although no provision has been made to reduce the friction due to side thrust upon the worm wheel, as is done in the case of the worm shaft, the writers are of opinion that this might be done with advantage, especially where, as in the case of a treble or quadruple thread, the angularity of the thread is such as to occasion considerable side thrust. It will be seen that, in all four designs, the wheel is under-driven, and although for many purposes the over-driven wheel gear has advantages, it was determined to adopt the design as shown as lending itself to a more efficient lubrication of the working parts.

By reference to the test sheet (Plate XXXVI.), it will be seen that the gears tested were as follow:—

(1) A 40 to 1 double-threaded gear estimated to transmit 3 h.p. (Plate XLII.)

(2) A 30 to 1 double-threaded gear estimated to transmit $7\frac{1}{2}$ h.p. (Plate XLIII.)

(3) A 20 to 1 treble-threaded gear estimated to transmit 10 h.p. (Plate XLIV.)

(4) A 20 to 1 treble-threaded gear estimated to transmit 15 h.p. (Plate XLV.)

By comparing the h.p. transmitted with the efficiency obtained, it will be seen how near these figures approached actual results.

It may be here remarked that, although it is evident a greater h.p. might be transmitted, experience with these gears has shown that it is at the expense of the life of the gear. A gear tried under working conditions designed for $1\frac{1}{4}$ h.p., but submitted to a 2 h.p. duty, showed signs of erosion after a few months' wear, and was practically done after nine months' wear. The P.V. was—

$$\frac{2\times 33000}{.26}=254,800$$

so that a pressure velocity factor—

$$\frac{\text{H.P.}\times 33000}{a}$$

where "a" equals the average area in rubbing contact—of not more than 200,000 is recommended. The following is the method adopted for determining the efficiency of the gears:—

The power was supplied by a direct-coupled 15 h.p. direct-current semi-enclosed motor, working upon a 220-volt circuit at 625 r.p.m. The rated efficiency curve, as furnished by the makers, with the results obtained by trial, are plotted side by side (Plate XXXIX.) The latter tests were carried out with a Sellor's dynamometer and the usual ammeter and voltmeter for recording the input factors (Plate XL.), the formula used for working out the results obtained (shown in Column 8) being—

$$E=\frac{\text{H.P.}\times 3.4}{\text{T.A.}}$$

where 3.4 equals amps. per h.p. hour at 220 volts.
T.A. equals total amps.
Volts constant.
Speed do.

So that for the first test $E=\frac{3.98\times 3.4}{20}=67.66\%$
do. do. second do. $=80.75\%$
do. do. third do. $=81.6\%$
do. do. fourth do. $=89.9\%$

The h.p.'s given in Column 7 of the table were estimated allowing a margin of loss in the reducing gear. For example, in Test No. 1, with a 4 h.p. output by the motor, it was considered probable that 3 h.p. might be obtained from the gear when the dynamometer and electrical readings were taken. The motor, when started, had the load, furnished by the dynamometer upon the 11in. diameter pulley, adjusted until the ammeter registered an output of 20 amperes, the pull upon the dynamometer being then taken. The motor was coupled, through a Raffard coupling, direct to the worm gear, and, for the purpose of obtaining a suitable peripheral speed for the better working of the dynamometer, a pulley of 48in. diameter was keyed upon the worm wheel shaft. Upon the motor obtaining its normal speed, the dynamometer was carefully adjusted until a reading was obtained upon the ammeter similar to that obtained when testing the motor only for efficiency, the dynamometric pull being again taken.

The h.p. for the gear was as follows:—

Formula: $\text{H.P.}=\frac{P\times C\times R}{33{,}000}$ where P=Dynamometric Pull.
C=Circumference of Pulley in feet.
R=No. of r.p.m.

1st test: $\text{H.P.}=\frac{400\times 12.56\times 15.6}{33000}=2.38$

Having obtained the input to the gear box from the motor in the first test, 3.98 (see Column 7), and output, 2.38 (see column 12), then efficiency of box$=\frac{2.38}{3.98}=59.8\%$ and the combined efficiency of the motor and gear box

$$\frac{67.66\times 59.8}{100}=40.45\%$$

In the fourteenth column is shown the efficiency of the gear, as calculated by the use of Unwin's formula:—

$$\eta=\frac{1-\mu\frac{P}{2\eta k}}{\frac{2\pi k}{1+\mu\frac{2\pi k}{P}}}$$

where p=a real pitch of worm
k=the radius of worm at pitch dia.
μ=Coeff. of friction taken

i.e.. for continuously lubricated surfaces, and that actually obtained, as shown in Column No. 13

in tests Nos. 3 and 4, the marked difference being attributable, in the writers' opinion, to a forced lubricating action which takes place in a gear partially or wholly submerged in a lubricating fluid. That this is so, the writers think, will appear when the nature of action of the working faces is considered in connection with the viscosity of the lubricant employed.

During the course of some experiments carried out by Professor Osborne Reynolds with the object of devising some means of reducing the frictional losses due to end thrust in the Parson's Turbine, it was found that, in quick-revolution rubbing surfaces having two or more shallow wedge-like recesses, their thin edges being arranged away from the direction of motion, a species of forced lubrication was set up, due to the viscosity of the oil, the friction between the lubricant and the adjacent standing parts drawing the lubricant in under pressure, the friction being, by this action, materially reduced. It is stated that the efficiency of the surfaces so constructed compares favourably with that of any ball-bearing device for similar purpose. It at least appears feasible that, considering the manner in which the working parts in a worm wheel engage each other, a like action takes place.

It has to be borne in mind that the efficiency results set forth in Column 13 represent the combined efficiency of worm gear and box whilst those in Column 14 are for worm and gear only. It appears to the writers highly probable that a similar difference would have been evidenced between the actual and calculated efficiency of the two first gears tried as between that of the last two, had it not been for the loss outside the gear itself, due to the excessive angularity of the double-threaded worm.

Referring again to the test sheets, it is interesting to compare temperature rises in the oil used in the three tests. In Test No. 1 with a 40 to 1 double-threaded worm, only transmitting approximately 3 h.p., the observed rise was 121 degrees—77 degrees, or 44 degrees F. In the second test with a 30 to 1 double-threaded worm, transmitting 6 h.p., the rise in temperature was 76 degrees—62 degrees, or 14 degrees F., the dynamometric pull on the pulleys being practically the same. Comparing these results with those obtained from the 20 to 1 treble-threaded worm, transmitting 10.62 h.p., the dynamometer reading 550 lbs., it will be seen the temperature rise was only 80 degrees—65 degrees, =15 degrees F.

These figures point to the conclusion, comparing temperatures with efficiencies, that the resistance and consequent losses due to friction are far smaller in treble-threaded worm gear boxes than in those using double gear, which, indeed, is what one might have expected. The grade of oil used was in all cases the same. It may be interesting to note that several grades of oil were tried, with varying results, but the one found most suitable was a heavy cylinder oil having a viscosity of 633 compared with rape oil equal to 100 at a standard temperature of 140 degrees F. The gears ran cooler and quieter with a heavier and more viscous oil, this, in the writers' opinion, being attributable to the forced lubrication action referred to previously.

It may be pointed out that the efficiency curve for the motor emphasises the fact that, the nearer the output approaches the rated output, the higher the efficiency may be expected to be, the ideal being evidently obtainable with the load normal and constant.

As something in connection with the present subject which the writers thought might be of interest, a pattern is shown of a double-threaded worm stuck up in plaster of Paris by means of a device of which the following is a description. A model of this is upon the table :—

A spindle of wood is made, the diameter being somewhat smaller than the bottom of the thread that it is desired to construct. Upon this spindle, near one end, is glued a sheet of paper set out with lines as shown upon Plate XLI., the length of the sheet being equal to the circumference, and the breadth sufficient to furnish a complete thread. The lines upon the paper now constitute an outline enabling a trench to be cut of sufficient depth to act as a master thread.

The spindle is now fixed up in bearings as shown, a peg or key being provided engaging with the thread Upon this being rotated, the spindle travels endways. A chalk-line is made along the line of the proposed screw, and short nails are driven into the spindle to form a base for the worm to be spun upon. Whilst the spindle is being revolved, liquid plaster of Paris is thrown on the shaker, B, being used to form the thread of the desired shape. Although not perfect, it forms a cheap and ready contrivance for making what might otherwise prove an expensive pattern. An examination of the working model will, no doubt, clear up any ambiguity in the writers' description of the device.

In conclusion, it is the writers' belief that, with careful designing, the worm and wheel may be made as efficient as any type of gear designed for a similar purpose.