is replaced by a phyllode with a much lesser number, and consequently a much more effective chlorophyll bearing organ for a xerophytic type. If, then, as is probably the case, this genus found its way from northern lands to Australia, it probably entered as a mesophyte, found conditions suitable to its growth, and modified its foliage later in response to changing climatic conditions. Having developed a method of water conservation which proved so satisfactory, it then was able to occupy many of the most arid regions of Australia.

We might, then, add a third group of plants to the two with which we started—namely a group consisting of modified mesophytic types acting as a connecting link between the Indo-Malayan types and the true endemic species.

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ON OBTAINING RESULTS FROM EXPERIMENTS.

By Lt.-Col. R. C. Simpson, V.D.,

Lecturer in Electrical Engineering, Sydney Technical College.

In most experiments two or more quantities have to be measured and a series of readings is obtained for each of them. Curves are then drawn showing how the various values so obtained vary under various changes of conditions and, very often, results have to be worked out from these curves, and it may be that further curves are then plotted.

In measuring the quantities, certain possibilities of error are always present ; among these are :

(*a*) The personal errors of the observers. Some people can read instruments more accurately than others. Untrained observers will read to the nearest division on the scale, and not worry with any greater degree of accuracy than that gives. Others, more expert, will read sometimes to the nearest division and sometimes to half a division. While a trained man will give his reading as he estimates it to one-tenth of a division, or if these are very small, to one-fifth. But, on most instruments, one-tenth can be estimated. Sub-standard instruments always have a knife-edge pointer with a mirror behind it, and care must be taken that, when taking a reading, the pointer hides its own reflection, so that parallax error may be avoided.

In such a set of readings, as taken by the average careful man, some readings will be given as whole numbers, with no figure shown on the right of the decimal point, while others will have one more figure, which is incorrect and the result of false teaching in arithmetic.

In this connection, some consideration must be given to the real meaning of figures. Say that a reading is given as 29·5. This means that, as accurately as it can be read, the real value is somewhere between 29·45 and 29·55, and so it is given as 29·5. And, in this reading, the accuracy is about one in 295, or one-third of one per cent. But there may be another reading in the same series of 34. Simply given as 34 without any figure on the right of the decimal point; which means that the value is somewhere between 33·5 and 34·5, and the accuracy is only one in 34 or three per cent. If this value, as is probably the case, is meant to be 34·0 as nearly as can be read, it should be read and recorded as 34·0, which means that it is between 33·95 and 34·05. Incidentally, in figures the symbol "0" is "nought," and not "oh." Experimental results are not telephone numbers. It is to be hoped that the spread of automatic telephony will restore the word nought to its proper use.

(*b*) The errors of the instruments. How large these are depends, of course, on the particular instruments used; but in any exact work these errors should be known and allowed for where necessary.

(*c*) Other errors which may affect the results. These may be of many kinds. For example, in a case where the speed of a machine is supposed to be constant, it may vary from time to time. Or the temperature of the air or of something else may have an effect which ought to be allowed for. In some cases the pressure of the atmosphere as read by the barometer may be important, and so on.

All these types of errors occur, and their effects may add together and make large errors, or they may more or less compensate for each other, but in any case they have the effect that any individual reading is likely to be incorrect to some extent, and it is not possible to say, by merely looking at a table of results, which are the most nearly correct.

In the case of a quantity which ought to be constant over the whole range of readings, its real value can generally be found by taking the average of all of them, but even in this case there may be some readings which are distinctly different from the others, and it is then difficult to know whether to take them into account or not.

In all cases where there is any such uncertainty, as well as when it is desired to be able to see at a glance how one quantity varies when another one alters, it is desirable to plot curves connecting the various quantities, and the first thing to do in this connection is to determine the scales to be used.

It can be laid down as a general rule that points can be marked more accurately on squared paper than they can be read on an instrument, especially if a needle point is used to mark them, and so it is, in general, inadvisable to make the scales such that the distance between two adjacent lines on the paper is much larger than that between two divisions on the scale of the instrument. It can even be a little smaller, because of the greater accuracy of plotting. If, as is not unusual, the divisions on the instrument are not all of the same size, a compromise can be effected by making the whole length of the scale on the paper about the same length as that on the instrument. This generally gives a very convenient size for a diagram.

To make the scales on the paper much larger than those on the instrument on which the readings were taken, as is often done in an effort after a degree of accuracy which is really unattainable, is very inadvisable, as it exaggerates the errors, and is likely to result in a set of points so spread out that the real shape of the curve they should form cannot be seen.

Even if none of the readings are small, so that all of them are far removed from zero, it is, in most cases, advisable to take all the scales in the diagram down to zero. Very often the curves which have to be obtained must, by the nature of things, go through zero, which is very useful as a guide when drawing curves, as it means another point well removed from the others and quite definitely known. There is, however, a danger to be avoided here, as all curves do not go to zero, and care must be taken not to distort them into doing so when they should not.

But it is not always essential to put in the whole of the scales. A case where it is not necessary is where the readings obtained are values of something which should be constant, and the object of drawing the curve is to determine their mean value but, even then, it is not advisable to use larger scales on the paper than on the instruments.

When plotting a curve all the points on it which are available should first be marked and then a smooth curve drawn among the points in such a way that they are evenly distributed on each side of it. If any are distinctly out of the general line, they can usually be neglected.

All points should be clearly marked, those for different curves being distinguished, those of one curve being circles, another crosses, etc.

When drawing in the curves, beware of French curves, whose use results in curves being drawn to suit them rather than the points. When the curves have been drawn, if not before, it is usually necessary to work out some results, which means doing arithmetic and, in doing this, care must be exercised to work out the results to the full degree of accuracy that is really known but, at the same time, to avoid wasting time in writing down imaginary figures.

Probably the best way of stating the accuracy in any particular case is as a percentage or to give it as, say, 1 in 100 or 1 in 1,000.

And it must be remembered that the accuracy of the result cannot be any better than that of the set of readings which is of the lowest degree of accuracy. Thus, if three values are known to one part in 1,000, it is of no use if another which has to be used with them in working out the results is only known to one part in 50.

There are two things commonly taught in arithmetic which must be discarded in working out experimental results ; one of these has been already mentioned, that is, that a nought, when it is the last figure on the right of the decimal point, must not be omitted if it is known, and the other is that in working out results from experiments answers must not be expected to "come out". In ordinary school arithmetic questions are generally so arranged that the answers are whole numbers but, for that to happen with the results of experiments is pure accident, and can only happen occasionally.

Another thing which must be avoided is the insertion of imaginary figures. What is meant by this is best illustrated by an example.

Taking an actual case. In a certain experiment the following readings were taken: Volts 100·0, Amperes 10·65, Watts 880·, and it is desired to find what is called the Power Factor, which is equal to the watts divided by the product of the volts and the amperes.

The result by arithmetic done in the ordinary way would be:

```
10·65×100·0=1065.
1065|880|0|0·82629
     852|0
     ---|-
      28|00
      21|30
     ---|---
       6|700
       6|390
       -|---
        |3100
        |2130
        |----
        | 970
```

First, as the watts are short by one figure on the product a 0 is added to them. Then, after the first subtraction, another 0 is brought down, and so on, and the result is worked out to as many figures as may be thought fit.

Now how many of these figures are really known?

Consider first the original readings. The volts are given as 100·0, which means that they are somewhere between 99·95 and 100·05, but the last 0 is not really known to be 0, but may easily be, for example, ·98 or ·02, which would be hard to read on the instrument.

Similarly, the current is given as 10·65, and the 5 might easily be 4 or 6. And, in the case of the watts, 880 on the instrument used might easily be 879 or 881. In fact, in all cases the accuracy is to within about 3 in 1,000.

Then, in doing the long division, a 0 is added to the figures for the watts. As the 0 already there is not really known, there is no justification whatever for assuming that the next figure is 0. It is just as likely to be any of the other nine digits. And the same applies to each of the others which are added in successive stages of the division.

It is thus quite accurate to say that any figures on the right of the full vertical line put into the long division sum are quite imaginary, for they depend upon the 0's which were added. This means that only the first three figures of the quotient are really known, the others only have the values shown because noughts were added in an arbitrary way when any other figures would have been just as likely to be correct.

In fact, if the long division was worked out as follows it would be accurate to exactly the same extent as by the method given above.

```
1065|8804|0·82673
     8520
     ----
      2847
      2130
      ----
       7173
       6390
       ----
        7835
        7455
        ----
         380
```

The only difference between the two cases is that the noughts which were arbitrarily inserted in the first have been replaced in the second by other figures which are just as likely to be correct.

It will, however, be seen that in both cases the first three figures of the answer are the same, 0·826, which is because those three figures are really known within the limits of accuracy of the experiment. (Actually, the 6 is rather doubtful.)

Such a sum as this should be worked out by the contracted system, in which, instead of adding imaginary figures to the dividend in successive steps, figures are cut off the divisor instead, and the process becomes :

1065|880|0·826
852

106| 28
21

1| 7
6

Here only the figures that are really known are written down, and the result shows the same answer again, as far as it is known.

This example also illustrates the necessity for writing down noughts when they are known. The quantity 880 watts could also have been written 0·880 Kilowatts, a Kilowatt being 1,000 watts. And by the ordinary methods would have been written as 0·88 Kilowatts, when the contracted long division sum would have been made one step shorter, and the answer would have only contained the two figures 0·82. Of course, in this case, as the watts are expressed in Kilowatts, the volt-amperes would also have to be divided by 1,000, and would read 1·065.

The same rules about not putting down imaginary figures apply equally to multiplication. This did not show up in multiplying 10·65 by 100·0, but supposing that the two numbers are 53·7 and 47·9. The sum, done in the ordinary way, will be :

53·7
47·9
483·3
3759x
2148xx
2572·23

In this there are three imaginary figures, or rather figures which are assumed to be 0. These are not shown, but are indicated by x. They are just as likely to be any other figure, so the last two figures of the answer can only be described as imaginary values.

Contracted multiplication should be used, the idea being to begin to multiply with the first figure instead of the last, taking care to insert the decimal point in its right place. Then in the succeeding lines, no figures are put in which go further to the right than the line first worked out. In the second line, the first figure on the right of the decimal point is put down as 6 instead of 5, because the next figure, if put down, would be larger than 5, namely 9, giving a result which is obviously nearer the real value.

If there are several figures to be multiplied together and then the result divided by the product of several others, the ordinary procedure would be to do the two multiplications first, and then a long division sum, resulting in a great waste of time and the writing down of many unknown figures.

Thus, suppose in working out a result the following expression is obtained :

$$\frac{143 \times 509 \times 793 \times 349}{514 \times 627 \times 421 \times 128}$$

the usual method would be to multiply together all the numerators first, then all the denominators, and then divide one by the other, that is, 20144311759 divided by 17366916864, which is a very large long division sum.

A far better way is to rearrange the original expression in the form of a number of separate fractions, each of which is near unity, that is, write it in the form :

$$\frac{149}{128} \times \frac{349}{514} \times \frac{793}{627} \times \frac{509}{421}$$

and then find the value of each of these fractions by contracted division.

```
128|142|1·11
    128
    ---
 13| 14
     13
     --
  1|  1

514|349|0·68
    308
    ---
 51| 41
     41

627|793|1·26
    627
    ---
 63|166
    126
    ---
  6| 40
     36
```

```
421|509|1·21
    421
    ---
 42| 88
     84
    ---
  4|  4
      4
```

When, instead of the formidable expression previously arrived at, we get simply $1{\cdot}11\times0{\cdot}68\times1{\cdot}26\times1{\cdot}21$, and an easy contracted sum in multiplication gives the answer.

It will not usually happen that the numbers are such that a series of fractions can be written down that are all near unity. Taking another example of a much more probable sort :

$$\frac{6{\cdot}28\times5500\times50{\cdot}0\times128}{514\times0{\cdot}00237\times535\times2{\cdot}69}$$

These values, which have been selected at random, should be rearranged so that each of them has one figure on the left of the decimal point, and is multiplied when necessary by 10 to the power of something, when the expression becomes :

$$\frac{6{\cdot}28\times5{\cdot}500\times10^3\times5{\cdot}00\times10\times1{\cdot}28\times10^2}{5{\cdot}14\times10^2\times2{\cdot}37\times10^{-3}\times5{\cdot}35\times10^2\times2{\cdot}69}$$

Of course, it is not necessary to write down all these separate tens, the various powers can be added up and the result written :

$$\frac{6{\cdot}28\times5{\cdot}500\times5{\cdot}00\times1{\cdot}28}{5{\cdot}14\times2{\cdot}37\times5{\cdot}35\times2{\cdot}69}\times\frac{10^6}{10}$$

or simply $\times10^5$.

Then the working out can be done as before, and there is no room for any doubt as to the position of the decimal point, the answer in this case being $1{\cdot}31\times10^5$. And this is the best form in which to leave it as the alternative, namely 131,000, involves the insertion of three noughts which are not known, while the first way shows the known figures clearly and gives no imaginary ones.

In ordinary arithmetic it is usual to say that problems should be worked out to a certain number of places of decimals. This is a meaningless limit to put on the degree of accuracy required, and should never be used. Obviously, two places of decimals in such a quantity as 27·45 is quite a different degree of accuracy from two in a quantity such as 1456·87. Further, the use of this limit may lead to absurdities, for example, if it was necessary to multiply ·025 by ·025 and this limit was used, the answer would be ·00.

Another limit often used and rather better is to work always to the same number of " significant figures ".

The best way of explaining what significant figures are is to explain what figures are not significant. The only figures which are not significant are noughts which come directly on the right of the decimal point, and only then if there is not any figure other than nought on the left of the point. Thus in the expression 12·237 all the figures are significant, as also they are in 12·007 and 12·070, but in 0·01247 the only significant figures are 1247.

But the number of significant figures does not really correspond to the degree of accuracy, for 99 has only two figures, and 101 has three, and yet the accuracy is 1 in 100 in each case, and there is a great difference in accuracy between 23 and 99, and yet each has two significant figures.

In recording very exact calculations it is usual to write the last figure, which is not generally really known exactly, smaller than the others and rather below them, thus 1432_8. Which means that 1432 is exactly known, and that the 8 is more likely to be right than 7 or 9.

Most ordinary experimental results can be worked out with quite sufficient accuracy with a ten inch slide rule. The accuracy of this is nearly the same all along its scale. On the lower scales near the left-hand end, it is divided to one in 100, and at the other end to one in 99, or sometimes ·5 in 99. It is easy to estimate with a considerable degree of accuracy to ·1 in 100, which is as near as most ordinary instruments read.

For greater accuracy than this, when it is worth while, either a longer rule must be used, or recourse had to logarithms, if the results are not worked out on paper, which means wasting time. Spiral slide rules can be obtained which read with actual divisions to one in 1,000 and to one in 10,000. An estimation between the divisions will give still another figure. And, owing to it being divided logarithmically, the number of figures read on a slide rule is equivalent to its degree of accuracy.