ENGAGING MATHEMATICS STUDENTS THROUGH UNDERGRADUATE RESEARCH

Layna Groen (Layna.Groen@uts.edu.au)

School of Mathematical Sciences, University of Technology, Sydney NSW 2007, Australia

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ABSTRACT
Science education has a long history of engaging undergraduate students in inquiry-based learning. Every undergraduate student studying a scientific discipline has first hand experience with the application of scientific method and the technology and practice of their discipline (in the first instance, through their laboratory classes). Such classes are primarily educator-led and subject content-focused. Recent years have seen greater inclusion of student-led and research-focused activities across programs, as seen in Zoology in Edwards, Jones, Wapstra, and Richardson (2007) and in Chemistry and Physics in McDonagh, Mears, Kirkup, Ward, Cortie, Ung, Doble, and Beavis (2011), for example. Mathematics is perhaps the only scientific discipline where this inclusion has yet to broadly manifest itself. This paper will describe how undergraduate research is being incorporated into experiences of third-year mathematics students. As suggested by Beckman and Hansel (2009), we will provide our own definition of undergraduate research as 'an inquiry or investigation that is original in scope, of an experimental, a theoretical or a creative nature, undertaken in a rigorous manner by undergraduate students, that is open to public scrutiny, and that makes a contribution to a discipline(s)'. Here then, we seek to engage students with the full range of research activities, from the formulation of the research questions, to the dissemination of results. Further, students are empowered to lead the research in collaboration with each other and with the academic staff, with the product a publication in a peer-reviewed, international journal.

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INTRODUCTION
Science education has a long history of engaging undergraduate students in research and inquiry. Every student studying a scientific discipline has first hand experience with the application of scientific method and the technology and practice of their discipline. These activities commence in their first semester and are conducted on an almost daily basis throughout their degree program. Such activities, while frequently entirely new to students, are carefully designed and implemented by their teaching and technical staff. That is, while there is an emphasis on inquiry and problem-solving, and students are active participants, activities are educator-lead (Healey, 2005). Hence, these activities, while contributing to the advancement of knowledge and skills, do not provide students with the independence, critical analysis, and creativity of the practice of research. Further, such activities are unlikely to constitute advances in the frontiers of a student's discipline. The student knows they are a student and not a scientist or mathematician. All that changes when they undertake research.

The benefits to students in undertaking research are broad in extent and far-reaching in impact. Undergraduate research:

- Develops in students the full range of skills required to undertake research and inquiry, including critical thinking and problem solving in unfamiliar settings (Jewell & Brew, 2010, p10, Beckman & Hensel, p42);
- Empowers students to discover and share new knowledge and endeavour (Beckman & Hensel, p42);
- Engages students with the discipline and their community of practice and allows students to see themselves as part of that community (Jewell & Brew 2010, p10);
- Encourages students to grow in confidence and facility (Beckman & Hensel, p43);
- Improves communication, interpersonal and, time and resource management skills (Beckman & Hensel, p42);
- Prepares students for Honours programs and higher research degrees (Jewell & Brew 2010, p10; Beckman & Hensel, p40).

How then can research be integrated into undergraduate Mathematics teaching and learning in a way that engages and empowers the student?

In this paper, we will examine a case study that incorporates undergraduate research into the third year of a Mathematics program, culminating in a peer-reviewed publication in an international journal.

BACKGROUND

Management Science is one of the mathematical sciences, and it is concerned with assisting and improving managerial decision-making through the application of scientific method. It is interdisciplinary, using concepts and techniques from applied mathematics, statistics, management and information technology. When first designed in 2007 the underlying theme of Quantitative Management Practice (QMP) was (and still is) professional practice of the management scientist. Students undertaking QMP have undertaken at least two other subjects in Management Science prior to enrolling in QMP, and are most commonly in the third, and final, year of their undergraduate mathematics degree. QMP introduces students to real-life problems in all their complexity, and from 2009, the scope was extended to include professional practice in the research domain. That is, the subject examines the two primary domains of professional practice – that of the managerial decision-maker or management consultant; and, that of the management science researcher.

In the first domain we see

> *An integrated exposure to professional practice through dynamic and multifaceted modes of practice-oriented education* (UTS, 2011).

Here, the primary focus is on the first of the subject objectives – to identify, model and solve real-life and realistic problems involving the application of Management Science methodology. Activities are supported by textbooks, software and internet sites (interactive and static) developed in the United States. Local flavour is added by case studies, talks, presentations on careers, and workshops undertaken and designed by the local chapter of the professional society.

In the second domain we see

> *Learning which is research-inspired and integrated, providing academic rigour with cutting edge technology to equip graduates for life-long learning* (UTS, 2011).

Here the primary focus is placed on a single research project designed to foster high-quality undergraduate student-faculty collaborative research. In addition to the classes where the research is undertaken, activities included workshops and discussions on - research methodology in the context of the discipline; the internet for literature reviews, data searches and for collaboration; a computer laboratory on the use of GIS software and software packages for mathematical optimisation; tips and hints on managing research and working with others, and other activities designed to educate students in the nature and execution of research. Notably, the outcome of the research was to be a research paper to be submitted for publication in a peer-reviewed, international journal. The research concerned

> *Professional practice situated in a global workplace* (UTS, 2011).

The subject has run every year since 2007, and class sizes have varied from 18 to a low of 6 to 22 in 2011. QMP will run again next year, and in its current form until at least the next course (program) review in 2013.

IMPLEMENTING UNDERGRADUATE RESEARCH – THE LEARNING DESIGN

Given the nature of the discipline, the unifying theme chosen for the research domain was scientific method. Within this theme, two approaches can be adopted with regard to the role(s) students play – the research can be student-initiated, or faculty-initiated (Beckman & Hensel, 2009, p41). In the case we will consider, students were encouraged to set their own direction within the context of the research topic provided by their lecturer, and were treated more as peers than as research assistants. Students were also encouraged to think of themselves as part of a research team, working towards a common goal (Beckman & Hensel, 2009, p42).

We will now go on to consider each of the main elements of the learning design – the learning activities and the learning environment.

LEARNING ACTIVITIES – THE PRACTICE OF SCIENTIFIC METHOD

The learning activities will be presented and discussed in the order in which they were undertaken, that is, following scientific method.

Identify a problem

What is the extent of the problem and what do we aim to achieve?

The research topic was selected such that students possessed the theoretical and practical background appropriate for undergraduates AND which could be extended to that required to solve the problem AND for which data and literature were available AND which represented a contribution to society AND which could be completed in the available time. The research was to determine the optimal location of detection components of a tsunami warning system in the Mediterranean region given the existing and planned infrastructure of tsunameters and coastal sea-level monitoring stations. We were interested in assessing the NEAMTWS implementation plan (ICG/NEAMTWS-III, 2007) and seeing if additional detectors would improve the proportion of the coastal population receiving a timely warning. Set-covering techniques were used to do this.

The lecturer presented the context of the project, and lead a discussion on the specifying the problem and assigning tasks and resources that might be required to address the problem. The group then divided themselves into the tasks previously identified and commenced work on the first steps of scientific method (the pre-work) – problem definition and research questions, background and rationale, literature review, preliminary data collection and preliminary model(s) and solution methodology(ies) suggested by the literature. Input is requested from all students and some tasks were job-shared. Some class time was allocated for the lecturer to provide guidance and facilitate the execution of the tasks. Further details on these steps follow.

Rationale
Why is this problem important?
Once the problem has been selected, some time needs to be spent on discussing why the problem represents a contribution to the advancement of knowledge and/or its application. Here we discussed the significance of the problem to the people of the Mediterranean region.

Background
What has been done before? And how does it impact on the problem we’re investigating?
Here we place the problem in the context of the literature – how the same or similar problems have been addressed (the methodology and the data), the primary results found, and any shortcomings that can be identified. Here, students require a tutorial/workshop on the practice of internet search techniques for data collection and literature review.

The class was set a time frame for collecting papers/articles/websites. They were asked to provide a brief description, including why the item should be examined by the group. Individual search results and preliminary analyses were classified, critiqued and edited. All items were gathered together in one area of the subject’s BlackBoard® site, and all students were asked to familiarise themselves with the materials. Students worked collaboratively refining each aspect of researching the literature. Students allocated themselves sections to be written up. These formed draft sections for the Introduction, Background, Literature and Methodology sections.

Data and technology
What do we need to address this problem?
Analysis of the problem and the literature naturally lead to selection of a mathematical model and/or methodology. This in turns leads to the selection and collection of relevant data. Key in this component of the application of scientific method to undergraduate activities is the question as to whether the students have the technology toolkit required and does it need to be supplemented. The answer to this question is partly a function of the choice of problem, the nature of the research, the time frame available and the choices made by the students based on their literature reviews and current knowledge and skills. In the current case study, after all students reviewed the collected materials, a facilitated discussion on how we would seek to address the problem was undertaken, that is, we sought to answer ‘What is the preferred methodology?’ and ‘What data needs to be collected to implement this methodology?’

The data required were the communication and response times of detectors, the wave speed, height and range, the population centres of interest, the locations of the coastal sea-level stations and tsunameter buoys (the detectors), and the historical record of tsumani generation sites.

Methodology
How are we going to implement the model?

Here, the model chosen was to maximise the population warning potential over coastal cities of the Mediterranean. The warning potential represents the proportion of the population of the region receiving a timely warning of the arrival of a tsunami:

$$P_u = \left\lfloor \frac{\sum_{v=1}^{V} p_{u,v}(y)}{\sum_{v=1}^{V} P_v} \right\rfloor$$

In the equation above, $p_{u,v}$ is P_v, the population of centre v, if a timely warning can be received. If a timely warning cannot be received then $p_{u,v}$ is zero. P_u, then represents the proportion of the population receiving a timely warning of a tsunami generated at site u. An overall measure of system performance can be made by averaging P_u over all tsunami generation sites, or by calculating a weighted average based on the relative frequency of tsunamis generated from each site and sites in its immediate vicinity.

Implementing the methodology - results

The students implemented the methodology. In the case study, students employed technology they were familiar with to calculate travel times using a spreadsheet. It was then a relatively easy matter to determine whether populations receive a timely warning by using spreadsheet functions. From there it was also a simple step to calculate warning potentials and determine the average and weighted average warning potentials for the systems as described in the relevant implementation plan. Students selected this option in preference to a mathematical program, as the model could be transparently implemented in a spreadsheet environment relative to a ‘black box’ optimiser.

Analysing the results in the context of the aims of the research

Again, discussion was facilitated and guided, rather than lead. Students used the spreadsheets and set-covering techniques to identify the areas where coverage could be improved, suggesting locations where additional detectors could be deployed. Students found that the planned tsunami warning system yielded approximately 90% of population with a 15-minute warning for most tsunami generation points. Performance fell to approximately 60% for a 30 minute warning, though improvement in warning potential could be achieved with one additional tsunameter buoy and two additional sea-level stations in Turkey and Greece.

Disseminating results

The earlier collated material was revisited, and the previously written sections were modified based on the implementation of the methodology and the results found. Small groups of students worked on the Results and Discussion sections, and the paper was in mostly-complete draft form by the end of the semester. The lecturer and a small number of students worked on the paper over the semester break, and submitted the paper to an international journal early in the following year. The paper was accepted subject to minor changes, and was published a few months later (Groen, Joseph, Black, Menictas, Tam, & Gabor, 2010).

LEARNING ENVIRONMENT – SPACE AND RESOURCES

It is perhaps obvious that the learning environment for an undergraduate research project is the environment under which the research would otherwise be conducted. For the management scientist, this includes a research space, computers of appropriate specifications, and access to the relevant data and literature. All of these resources were (and are) available to mathematical sciences students – there are a variety of appropriate spaces throughout the campus, these include access to appropriate computers; the internet and library provide comprehensive access to databases and literature.

ASSESSING LEARNING

Assessment tasks were integrated with the activities undertaken in each domain and were divided equally between the two domains. With regards to the research project, the pre-work was allocated a weighting of 20%, and the execution of the methodology and contribution to the writing of the paper, a weighting of 30%. The written draft sections were criterion-reference assessment, with a student seminar that served both as a formative task and a summative task. The contribution to the draft paper was assessed through a combination of criterion-referenced assessment, peer-review and self-reflection.

CONCLUSION

In conclusion, embedding a research project in a third year undergraduate subject proved to meet many of the benefits to students listed in the Introduction to this paper. In addition, the benefits to the university and community are numerous (Jewell & Brew 2010, p10, 12-13). Undergraduate research:

- Increases the quality and cohort size in Honours and Research Higher Degree programs;
- Provides academic staff with assistance in ongoing research projects;
- Creates new knowledge within the discipline, in the form of scholarly peer-reviewed articles, and conference and seminar presentations;
- Develops the ability of academic staff to carry out higher degree supervision.

Providing undergraduate students with research opportunities is therefore a win-win situation.

REFERENCES

Beckman, M., & Hensel, N. (2009), Making explicit the implicit: Defining undergraduate research. *Council on Undergraduate Research (CUR) Quarterly, 29*(4), 40-44.

Edwards, A., Jones, S. M., Wapstra, E., & Richardson, A. M. M. (2007), Engaging students through authentic research experiences. In K. Placing (Ed.). *Proceedings of the Science Teaching and Learning Research including Threshold Concepts Teaching and Learning Symposium*, (pp. 168-171), Sydney: UniServe Science..

Groen, L., Joseph, A., Black, E., Menictas, M., Tam, W., & Gabor, M. (2010). Optimal location of tsunami warning buoys and sea level monitoring stations in the Mediterranean Sea, science of tsunami hazards, *Tsunami Society International, 29*(2) 78-95.

Healey, M. (2005), Curriculum design and the teaching-research nexus. Linking research and teaching: disciplinary spaces. In R. Barnett (Ed.) *Reshaping the university: new relationships between research, scholarship and teaching*, (pp. 30-42), Maidenhead: McGraw-Hill/Open University Press.

ICG/NEAMTWS-III (2007), Implementation plan for the tsunami early warning and mitigation system in the North-Eastern Atlantic, the Mediterranean and Connected Seas (NEAMTWS), 2007–2011, UNESCO, Retrieved September 9, 2009, from http://ioc3.unesco.org/ioc-24/documents/TS_73_NEAMTWSDraftImPlan.pdf. IOC Technical Series No. 73 Action Plans. I.O.C.T. Series. Bonn, Germany.

Jewell, E., & Brew, A. (2010), Undergraduate research experience programs in Australian Universities, Australian Teaching and Learning Council. Support for the original work was provided by the Australian Learning and Teaching Council Ltd, an initiative of the Australian Government Department of Education, Employment and Workplace Relations. Retrieved July 4, 2011 from http://www.altc.edu.au.

McDonagh, A., Mears, A., Kirkup, L., Ward, R., Cortie, M., Ung, A., Doble, P., & Beavis, A. (2011), Developing a research integrated learning culture in undergraduate teaching and learning. In *Proceedings of the Australian Conference on Science and Mathematics Education*, Sydney: UniServe Science (in press).

University of Technology, Sydney, (2011), The UTS model of learning. Retrieved July 3, 2011 from http://www.uts.edu.au/teachlearn/ltpf/pdfs/uts-model-short-guide.pdf.