DESIGNING EXTENSION QUESTIONS: MATHEMATICAL COMMUNICATION AND CREATIVITY TO ASSIST WITH SECONDARY-TERTIARY TRANSITION

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INTRODUCTION AND CONTEXT

The transition from secondary to tertiary studies (abbreviated STT for secondary-tertiary transition) in mathematics comes with many challenges for learners and teachers. In a recent systematic literature review on STT, Di Martino, Gregorio, & Iannone (2023, p. 25) point to four main research themes: “the mathematical gap between secondary school and university, the potential of technology to facilitate STT, and other factors correlated to academic success and to academic failure in STT.” They further draw attention to an identified shift in the research on STT from focusing predominantly on cognitive factors affecting STT to a more holistic approach that also includes affective factors.

The work of Tall, and his three worlds of mathematics (Tall 2008), helps to identify and articulate the significant cognitive demands on learners of mathematics as they move from symbolic calculations of secondary studies to more formal mathematics at university. Beyond the many cognitive demands, recent studies also point towards factors in the affective domain that impact a successful STT. For example, Hernandez-Martinez et al. (2011) identify changes in learner’s identities and relation to mathematics during STT while the work of Geisler & Rolka (2021) point to how negative emotions and perceptions in relation to mathematics can negatively impact the STT process.

One skill that has an impact on both cognitive and affective domains of learners is mathematical communication. On mathematical writing, Teledahl et al. (2024) draw attention to competing conceptualisations of mathematical writing having both formal and discursive elements. In the formal model, mathematical writing is seen as a formal set of rules to be learnt – often aligning with how students learn to present calculations and proofs. However, in the discursive model, there is recognition that writing can be dependent on social context, with meaning being constructed by parties. A focus on mathematical writing skills, and mathematical communication more generally allows students to develop their understanding in both cognitive and affective domains that are important during STT.

During the STT process, learners are often required to shift their perceptions of mathematics. During secondary school, mathematics is often presented as a set of techniques and algorithms to follow, which in turn determine a solution. As learners move into tertiary studies, an important realisation is that mathematics often allows for creativity and has space for individual styles. Mathematical creativity has been conceptualised in different ways in the literature. Leverson (2013) outlines possible conceptualisations of creativity as non-algorithmic decision making, divergent and flexible thinking, and unusual or insightful solutions to problems. When the term creativity in mathematics is used, the last of these conceptualisations is a common representation by those less familiar with mathematics, where creativity comes in small bursts of ingenuity, and is the sole domain of mathematical geniuses. This is not an accurate depiction of creativity and, instead, creativity can be cultivated in everyone. For example, Silver (1997, p. 76) argues that research indicates that “creative activity results from an inclination to think and behave creatively”. Therefore, rather than it being a unique quality of certain individuals, we can instead foster self-perceptions of learners as creative agents by designing tasks that allow for creativity.

The purpose of this paper is to share our assessment design practices in a first-year undergraduate mathematics program, aimed at addressing some of the issues around STT discussed above. In particular, we focus on the design of optional bonus/extension questions for students in this program, aimed at addressing the STT issue for this cohort. This paper will present our assessment design criteria, describe their purpose in relation to supporting STT, and provide example questions that map to the criteria. We note that we have not conducted a formal evaluation of the effectiveness of our approach, but we provide a suggestion for marking criteria that make our approach low-cost in terms of time, relative to any potential benefit. We hope that sharing our question-design criteria, together with some explicit examples, provides resources and ideas for other teachers of learners going through the STT phase.

The program is a sequence of two mathematics subjects across two semesters – a linear algebra subject in the first semester, and a calculus subject in the second. These are at the first-year undergraduate level. In this program, students must pass linear algebra to move onto calculus. Enrolments are approximately 100 students each year. While the subjects are at the first-year university level, the program is taken by advanced final-year secondary school students as an extension to their secondary school studies. The program is taught in classrooms in various School Centres by secondary school teachers, with support from the subject coordinators, who are academics at a large research-intensive Australian University. The coordinators provide course materials and administer the assessment, which is graded centrally at the university. The assessment in each semester is in the form of three written assignments worth 25% in total, and a final exam worth 75%. All three assignments are thought of as a mix of summative and formative assessment: they contribute to the final grade and students receive feedback on their work shortly after they complete these tasks. Full solutions to each assignment (including marking schemes) are made available to students a few days after the submission deadline. The authors have been the subject coordinators of this program from 2020 to the present time.

Being secondary school students taking a first-year university mathematics subjects, these are students at the cusp of the transition from secondary to tertiary studies. Therefore, interventions that assist in this transition play a pivotal role to their success, both in the program, and when they continue into tertiary studies fully.

QUESTION DESIGN AND ASSESSMENT

To address some of the above-mentioned issues that students face in their transition from secondary to tertiary mathematics, we designed each of the take-home assignments to include a bonus/extension question, which is the focus of this article. The tasks were designed to meet the following criteria:

- Situating and extending: The tasks should situate the subject's content knowledge within the broader mathematics discipline and provide opportunities to extend understanding beyond a syllabus. For example, tasks often relate what students are currently learning with more advanced topics or contemporary mathematics. This criterion aims to deepen understanding of the mathematics that the students are learning, further developing their relationship with mathematics in new contexts. This supports the STT phase in continuing to develop students' cognitive ability as well as providing directions and opportunities to connect understanding to future mathematics topics. We note that this criterion was particularly important for our cohort, due to the "extension" nature of the program.

- Independence and self-efficacy: The tasks should allow students to develop their independence, research skills and identity in relation to mathematics. For example, providing opportunities to teach themselves or others, and providing guidelines, but not scripts for completing tasks. This criterion taps into the affective factors of STT concerned with learner's identities, emotions and perceptions in relation to mathematics.

- Writing and communication: The tasks should give students realistic opportunities to develop and extend their mathematical communication skills, which has the potential to support STT both at the cognitive and the affective level. For example, tasks may involve writing for specific audiences, engagement with non-standard texts, and emphasis on conveying ideas rather than only solutions.

- Creativity and exploration: The tasks should include elements that allow for creativity, a skill that supports STT by encouraging students to shift and develop their perceptions of mathematics and of themselves as creative agents. For example, using open-ended problems, problems that do not have an algorithmic solution, or problems that allow students to make choices and create their own meanings.

These bonus questions were optional. Students could still obtain full marks for the assignments even if they decided not to respond to these questions. Those students who attempted them could obtain a small number of marks (about 5% of the total of the given assignment), which went towards making up for any lost marks for the compulsory questions in that assignment only. When grading, the bonus questions were assessed on two criteria:

- Did the student demonstrate good understanding of the task and provide a correct response?

- Did the student communicate their response well, according to any prompts given in the question?

One of the benefits of assigning these tasks as optional bonus/extension questions is that it allowed for more freedom in assessment, and feedback, as it removed marks as a focal point. Generally, as the questions only had potential to contribute a small number of marks, the assessment criteria were graded on a Yes/No scale, with “Yes” meaning the student received the mark for the criteria. Regardless of the grade, students received written feedback on their work.

EXAMPLE QUESTIONS

To illustrate how our approach to question design can be implemented in practical terms, we share here some examples of questions we have asked students. For conciseness in our presentation, we briefly describe the questions and their purpose here, though the full questions can be found in Appendix A for any interested readers.

Example 1, from Linear Algebra 2025
This question introduces students to a developing field in mathematics called Topological Data Analysis (TDA). The question first gives background on the theory, including links to different resources. Then, students are prompted to use techniques learned in Linear Algebra in this setting, and to interpret and validate their results in this new context.

Example 2, from Linear Algebra 2022
This question introduces the definition of a linear differential equation with constant coefficients (which they have not yet seen, since Calculus comes after Linear Algebra in this program’s sequencing) and asks students to use the language that they have developed in linear algebra, to the new calculus context. Students are encouraged to do research on the topic and make independent decisions regarding the level of detail that will contribute to good quality writing in their exposition of the answer.

Example 3, from Calculus 2023
This question asks the students to write a proof of a calculus result that extends the second order Ordinary Differential Equations (ODEs) content knowledge taught in the subject. While the proof writing is similar to a proof they would have seen during the calculus course, rather than just reproducing a modified proof, the students are asked to engage in a creative way that also develops their communication skills, by playing the role of a lecturer writing notes for their students.

Example 4, from Calculus 2020
This question asks the students to do some research on a technique for solving ODEs that is not covered in the subject: variation of parameters. They are prompted to independently learn and familiarise themselves with the technique and key definitions, and to then apply that newly learnt knowledge and terminology to show a given result.

Each of these questions was designed to address the four design criteria outlined in the previous section to some degree. Individual questions sometimes were designed to focus more on a subset of those, depending on factors such as the topic that was covered or the focus of bonus questions in previous assignments. Table 1 provides a brief (and not necessarily exhaustive) summary of elements in the sample questions that address each of the desired criteria.

Table 1. A table showing some of the properties from each example question that satisfy the task-design criteria.

Criteria	Example 1	Example 2	Example 3	Example 4
Situating and extending	Connects what students learn to TDA, a currently developing field of research.	Connects linear algebra to calculus.	Extend understanding of a technique in class by studying a new but similar problem.	Learning a new technique for solving differential equations, generalising some techniques from class.
Independence and self-efficacy	Independent research and using learned techniques in a new context	Using learned techniques adapting language to a new context	Independent development of notes to explain a newly learned proof.	Independent learning of a new technique.
Writing and communication	Engaging with written literature, learning new notation and explaining results in a new context.	Explaining a concept in calculus using the language of algebra.	Creating notes with specific environment and audience in mind.	Learning new notation and terminology and learning to explain a new computation.
Creativity and exploration	Open-ended problem. Involves exploration and interpretation of result.	Open-ended response. Has scope for decision making on how to describe result.	Open-ended creation of notes.	Open-ended approach to learning a new technique, and then using it.

REFLECTION AND DISCUSSION

In this paper, we have shared our current practice in designing bonus/extension problems for a cohort of secondary school students undertaking a university extension course. The design of the questions gives an example of how the literature on STT, mathematical communication and creative mathematics can be applied in a practical setting. While we have aimed to explain our approach and its relevance and alignment with the literature, we remark that this article is not intended to be a research article evaluating our specific approach. Our goal, as stated, was to share our practice and reasoning for it. While including bonus/extension questions in assessment for extra marks is not a novel idea, our contribution here has been by proposing specific criteria for these questions aimed at supporting the STT phase.

Having included these types of problems in our courses for over 5 years, we believe that it is a worthwhile effort to think of ways to help students make that “jump” from the way mathematics is taught at schools to the way mathematicians engage with mathematics. Helping students become independent learners and gain self-efficacy goes beyond a specific discipline, but it is essential in tertiary studies more generally. While the types of bonus/extension questions we describe here are not the only way, we have found that they are a useful, and perhaps surprisingly low-cost (in terms of time and resources on teachers) way to rework traditional assessment design. While sometimes coming up with a good setup for a task can take some work, we hope that our design criteria and examples can provide guidance to any educators wanting to implement such tasks.

In terms of demand on marking and evaluating, we have found the two questions guiding the marking criteria useful, and not onerous. While students have room to be creative and to write their own responses, they recognise that they must provide a complete response to achieve marks for the question. An experienced teacher can generally recognise whether the criteria have been satisfied after a reading of the work, without getting too bogged down in tedious calculations. Our view has been that the potential gains outweigh the minimal effort required to implement these practices.

Lastly, we remark that, while our focus in this article has been on designing bonus/extension tasks for learners in the STT phase, the criteria for task design that we've outlined (situating/extending, independency/self-efficacy, writing/communication, creativity/exploration) could be adapted to all levels of mathematics education.

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APPENDIX A

This appendix includes a complete version of the example questions described in the Example Questions section. We have modified some of the text slightly from what was given in our courses to avoid references to other parts of the course that would be unclear to readers of this article.

Example 1, from 2025 Linear Algebra

Topological Data Analysis (TDA) [the question links an article on TDA] is a new approach to data analysis, arising from the field of algebraic topology, which aims to understand the "shape" of data. A key component of understanding TDA is understanding homology [the question links an article on homology]. While one can study the homology of high dimensional spaces, this question looks at the homology in dimension one, by studying the homology of a graph [the question links an article on graphs].

A graph is a set of vertices V, and a set of edges E, where edges are just two-element subsets of V. A directed graph is a graph with arrows on the edges, which determine an intial and terminal edge for each vertex. In this case, instead of a two-element subset, an edge can be thought of as an ordered pair of vertices (x, y), with x being the initial vertex, and y, being the terminal vertex.

Given an ordering on the vertices $V: (v_1, \dots, v_m)$ and edges $E: (e_1, \dots, e_m)$ of a finite directed graph, the incidence matrix of the directed graph is a matrix A with entries:

$$a_{i,j} = \begin{cases} -1 & \text{if } v_i \text{ is an initial vertex of } e_j \\ 1 & \text{if } v_i \text{ is an terminal vertex of } e_j \\ 0 & \text{otherwise} \end{cases}$$

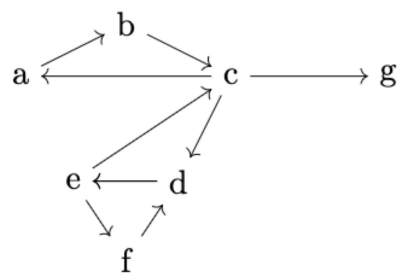
The incidence matrix can be thought of as a linear transformation from D:$R(E) \to R(V)$, where

- $R(V)$ is the real vector space with basis $\{v_1, \dots, v_m\}$, the vertices of the graph.
- $R(E)$ is the real vector space with basis $\{e_1, \dots, e_n\}$, the edges of the graph.

For a directed graph G, we define the homology of the graph as

$$H_1(G) = \ker(D).$$

Consider the directed graph G given by:



This graph has

- Vertices: $\{a, b, c, d, e, f, g\}$
- Edges: $\{(a, b), (b, c), (c, a), (c, d), (c, g), (d, e), (e, c), (e, f), (f, g)\}$

The task is the following.

a) Write down the incidence matrix of G. Use the ordering of edges and vertices as listed above as the basis for $R(E)$ and $R(V)$
b) Compute $H_1(G)$ by finding a basis for the kernel of D. What is the dimension $\dim\big(H_1(G)\big)$?
c) Interpret the results, by relating $H_1(G)$ to the number of cycles in the graph. Explore with further computations whether this interpretation is valid.

Example 2, from 2023 Linear Algebra
A linear differential equation with constant coefficients is an equation of the form

$$a_n \frac{d^n y}{dx^n} + a_{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_1 \frac{dy}{dx} + a_0 y = r(x)$$

Where $a_i \in R$ for $i \in \{0, \ldots, n\}$.

a) Define appropriate vector spaces, and a linear transformation L so that the left hand side of the equation can be written as $L(y)$. Check that the L you define is a linear transformation.
b) Show that any two solutions $y_1(x)$ and $y_2(x)$ to the differential equation differ by an element of $\ker(L)$

You may need to do some research and look up the definitions to do this question. If it will make your work clearer, you should include these definitions in your work.

Example 3, from 2023 Calculus
Suppose you have a homogeneous second order ODE with constant coefficients which has an associated characteristic equation of the form $\lambda^2 - c^2 = 0$ where $c > 0$. Prove that the general solution for the ODE can be written as a linear combination of hyperbolic functions.
In this task, assume you are a lecturer writing notes for your students with full explanations and justifications.

Example 4, from 2020 Calculus
In this Calculus subject, you have only explored first order linear differential equations and second order constant coefficients differential equations.
However, differential equations are a whole area of study themselves. There are lots of techniques available that help us solve different kinds of differential equations. One such technique is the method of variation of parameters.

a) Do some research on the method of variation of parameters. You will probably encounter references to your future best friend for the beginning of any differential equations course: the Wronskian. This part does not require answers, just for you to familiarise yourself with the concepts.
b) Let $a, b \in R$. Use the method of variation of parameters to show that the general solution to the second order ODE

$$y''(x) + (a + b)y'(x) + aby = F(x),$$

can be written as

$$y(x) = C_1 e^{-ax} + C_2 e^{-bx} + \frac{1}{b-a}\int_{x_0}^{x} \left(e^{-a(x-t)} - e^{-b(x-t)}\right) F(t)\ dt,$$

Where x_0 is some initial point.

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