# DISTINCT TARGETING OF MULTIPLE MATHEMATICAL PROFICIENCIES IN FIRST-YEAR SERVICE TEACHING

Leon Poladian (leon.poladian@sydney,edu.au)

School of Mathematics and Statistics, University of Sydney, Camperdown NSW 2006, Australia

**KEYWORDS:** online learning, procedural fluency, math anxiety, self regulation, motivation

## ABSTRACT
This paper provides a rationale and an approach for service teaching of mathematics to students in the life sciences. Mathematics students need to develop several distinct proficiencies simultaneously. These include procedural fluency, strategic competence, adaptive reasoning, conceptual understanding and a productive disposition towards the subject. These distinctions can be made explicit to students and they are given differentiated learning outcomes, activities and assessments. Procedural fluency requires repetition and immediate and personalised feedback; this is provided through online formative assessment tasks. Valuable tutorial time can then be devoted to strategic competence and adaptive reasoning with extended problems drawn from different domains. Conceptual understanding is aided by drawing analogies between concrete examples from different disciplines and subsequently abstracting key concepts. A productive disposition is inculcated by embedding all activities in an authentic context and using contemporary and thought provoking applications.

Proceedings of the Australian Conference on Science and Mathematics Education, University of Melbourne, Sept 28th to Sept 30th, 2011, pages 139-144, ISBN Number 978-0-9871834-0-8.

## BACKGROUND AND INTRODUCTION
Teaching compulsory mathematics units to large classes of students from the life sciences presents several challenges. Most of these students do not have an intrinsic interest in mathematics and even those who successfully complete service units maintain a negative attitude towards the subject. Student surveys regularly indicate that many students in our service courses do not appreciate the value or purpose of these units, nor their relationship to their degree. Recurring problems identified in the literature include maths anxiety (Richardson & Suinn, 1972, Tobias, 1993, Ashcraft & Kirk, 2001, Cates & Rhymer, 2003), compartmentalisation or failure to transfer skills between subjects (Quinell & Wong, 2007, Britton, New, & Sharma, 2007, Roberts, Sharma, Britton, & New, 2009), and a lack of self-regulation (Hannula, 2006, Hodges, 2009).

It is compulsory for all students in the Science Faculty at the University of Sydney to take 12 credit points of mathematics (usually in first year). Students who have only studied 2 unit mathematics at the HSC typically choose MATH1013 “Mathematical Modelling” as one of their second semester units. This 1st year service unit has over 600 students mostly majoring in Biology, Psychology or Medical Science. This unit typically has 20 tutorial groups and almost a dozen different tutors. For almost all of these students it will be their last formal mathematics unit. The timing of assessment tasks must be centrally coordinated and staggered to smooth out the demand on shared resources. With over 6 different first year courses and 60 different tutors in our School, a degree of uniformity must be imposed on the rubric of the quizzes and assignments. Unfortunately, there is also compelling research evidence (Trigwell, Prosser, & Waterhouse, 1999) that such a highly regimented environment promotes surface approaches to learning and deters students from developing self-regulation and constructing healthy study orchestrations.

The following section introduces the theoretical framework and philosophy behind the changes made to MATH1013 in 2009 and 2010 and then the subsequent sections deal with contrasting the distinct proficiencies and how each is tackled.

## THEORETICAL SCAFFOLDING
The choice of the word scaffolding is deliberate to indicate that the chosen theoretical framework supports without constraining. Mathematical proficiency is more complex than the extremist positions held by some proponents within the so-called “math wars” debate that has raged for the last three decades. Wu (1999) has coined the phrase “a bogus dichotomy in mathematics education” to describe the position that “a demand for precision and basic skills runs counter to the acquisition of conceptual understanding” or that we must choose between “facts vs. higher order thinking”. In recent

years, common sense and a growing body of evidence-based pedagogy has advocated a more mature approach. The National Research Council (2001) has noted

> One of the most serious and persistent problems facing school mathematics in the United States is the tendency to concentrate on one strand of proficiency to the exclusion of the rest. For too long, students have been the victims of crosscurrents in mathematics instruction, as advocates of one learning goal or another have attempted to control the mathematics to be taught and tested. We believe that this narrow and unstable treatment of mathematics is, in part, responsible for the inadequate performance that U.S. students display on national and international assessments. Our first recommendation is that these crosscurrents be resolved into an integrated, balanced treatment of all strands of mathematical proficiency at every point in teaching and learning. (p.11)

Although the above NRC report refers to school education and specifically the USA, the evidence-based and nuanced approach to mathematical proficiency elucidated by Kilpatrick (2001) who chaired the Learning Studies Committee for the NRC, also provides an extremely useful, pragmatic and powerful scaffolding for the teaching and learning of mathematics at the tertiary level. The National Research Council's "five strands of mathematical proficiency" are described as follows:

- *Procedural fluency* - Carry out an appropriate known procedure accurately and efficiently.
- *Strategic competence* - Formulate, represent and then solve a problem. This includes tackling both routine problems and transfer of techniques to non-routine problems.
- *Conceptual understanding* - Integrated and functional grasp of ideas within and across their contexts. A student not only knows isolated facts and procedures but also knows why certain mathematical ideas are important and the contexts in which it is useful.
- *Adaptive reasoning* - capacity for logical thought, reflection, explanation, and justification.
- *Productive disposition* - Habitual inclination to see maths as valuable and a belief in diligence and one's own efficacy. Students who are engaged in mathematics *believe* they can solve problems as well as learn concepts and procedures even if it requires effort. They view mathematical proficiency as an important part of their future.

This last goal was articulated extremely eloquently by one of my students in response to a survey question about graduate attributes:

> *The way the course was approached gave just the right amount of teaching to make it possible for me to succeed on my own afterwards.* (MATH1013 Student Feedback, 2010)

## PRODUCTIVE DISPOSITION, RELEVANCE AND MOTIVATION

Although productive disposition is listed last in the NRC list of proficiencies, in the context of service teaching it deserves special attention and in some way drives all the other changes. The mathematical techniques taught in MATH1013 were unchanged, however, the order of presentation was made thematic. The content was presented in twelve single-topic single-week chapters with titles such as "Resource-Limited Growth" or "Interactions" that emphasised the concepts and contexts above the techniques. Furthermore, all significant examples in the notes are authentic material from ecology, resource management, pharmacology, epidemiology, economics and sociology. More traditional material and examples familiar to physics and engineering students (such as pendulums and electrical circuits) but not relevant to the life sciences were completely purged from the course.

Pintrich (2003) reviews research on student motivation and includes several motivational generalisations and design principles including providing content material and tasks that are relevant, personally meaningful and interesting to students, and displaying and modelling interest and involvement in the content and activities. I chose to deliberately include politically controversial topics such as Peak Oil, sustainable harvesting and the spread of rumours in social networks; especially since all of these topics can be discussed and understood using only simple first-year university calculus and algebra. I also talk about how excited I feel that mathematics is contributing to these important issues. This was a deliberate attempt to demonstrate to life science students that the course is *contemporary* and *relevant* to their majors and degrees, and that interesting issues are within the grasp of their current mathematical skill level.

A particularly interesting challenge is choosing examples relevant to each of the original disciplines without having to teach biology to the psychology students or vice versa. I also knew that it was crucial that the accompanying tutorial material could still be used by casual tutors who are enthusiastic about maths but who do not have a background in these other disciplines.

Suitable examples which can bring the real world into the lecture theatre include high-profile events attracting a lot of media attention. For example, in September 2009, at the height of the national discussion on the H1N1 epidemic, while the students watched I downloaded real data from the web and, using nothing more than a spreadsheet and asking the students to follow along with their handheld calculators, we verified that the equations they were studying that very week could describe the observed data extremely accurately. I was then able to easily foreshadow the limitations and elaborations to that model that we would explore a few weeks later. As a further example, the 2010 assignment was to calculate and compare the recovery times for fish populations devastated, for example, by the Gulf of Mexico oil spill using two different but current models of sustainable harvesting.

The third and fourth student survey questions in Table 1 reveal that motivation and perception of relevance have doubled since the introduction of these changes. Comments from MATH1013 students in 2009 support the quantitative survey results about changes in attitude and motivation. Some typical remarks were:

- *Its relevance and how it relates to my other units of study Geography and Political economy you would be surprised.*
- *This subject is all about modelling patterns/trends found in the sciences; it made perfect sense that you would need to understand this to study science.*
- *I didn't at first, but biological examples helped show the relevance to medical science.*
- *I hate maths but it's compulsory for science. In saying this, I didn't mind this subject and there were parts that were relevant to biology (my major).*

**Table 5: Student evaluation results**

The survey instrument used to obtain the results below asked for student responses to statements on a 5-point Likert scale. The figures in **bold** show the percentage of students that **agreed or strongly agreed** with the statement and the bracketed figures in *italics* show the corresponding percentages for *disagreed or strongly disagreed*. Both surveys were held in the final week of semester.

| Student evaluation results for MATH1013<br>**Agreement** vs. (*Disagreement*) | Before<br>(2008) | After<br>(2009) |
|---|---|---|
| The learning outcomes and expected standards of this unit were clear | **38%** (*23%*) | **83%** (*5%*) |
| This unit of study helped me develop valuable graduate attributes | **16%** (*39%*) | **41%** (*10%*) |
| I was motivated to engage with the learning activities in this unit of study | **26%** (*38%*) | **62%** (*8%*) |
| I can see the relevance of this unit of study to my degree | **28%** (*43%*) | **51%** (*25%*) |
| The lecturer (or lecturers) explained the material well | **49%** (*26%*) | **93%** (*1%*) |
| Overall I was satisfied with the quality of this unit of study | **43%** (*24%*) | **85%** (*1%*) |

## CONCEPTUAL UNDERSTANDING AND STRATEGIC COMPETENCE VS. PROCEDURAL FLUENCY

Mathematical concepts that are important and relevant in service teaching can be described using *generic scientific vocabulary* that is not necessarily *mathematical vocabulary.* Consider for example the concepts of *general and particular solutions*. One does not need specialised language to state that a problem might have a general solution (that encompasses all possible solutions) or that one might prefer or be satisfied with only one particular solution: emphasising that the words *general* and *particular* retain their everyday meanings. Students can be shown examples of general and particular solutions for a variety of problems from different disciplines. Furthermore, the generic idea that *additional information* is required to take something *general* and reduce it to something *particular* occurs through all disciplines of science, as does the related idea of using extra information to determine previously unknown parameters. All of the crucial concepts in this course are comparable to this example and are all presented using everyday language and in the context of one or more authentic examples. Also important are articulation of what a concept is, what a technique for solving a specific problem is, and what the strategies for classifying or identifying the relevant concepts and techniques. The visual presentation of the objectives and outcomes in the lecture notes on a topic-by-topic basis is thus made very distinctive as follows.

Objectives associated with conceptual understanding or strategic competence are displayed at the *beginning* of each chapter in a distinctive font, described using *plain everyday language* and reiterated verbally at the *beginning* of each week of lecturers. Some examples are:

- Extracting useful information from a model with solving it exactly,
- Interpreting parts of models, and using simple models as building blocks,
- Combining many simple tasks and skills to complete a complex task,
- Understanding the limitations of methods and when and why they fail.

Outcomes associated with techniques or procedural fluency are listed *after* the more generic learning objectives, using a different font and also a much more *specialised and conventional mathematical vocabulary*. These are reiterated verbally in lectures at the *end* of each week and are summarised at the *end* of each chapter using a tabular and more traditionally mathematical representation and are often accompanied by recapitulation of common equations and formulas. Some examples are:

- Writing down equilibrium conditions for differential equations,
- Using sketches to locate approximate solutions to equations,
- Solving the characteristic equation for linear $2^{nd}$ order recurrence relations.

The first student survey question in Table 1 shows that student satisfaction with the clarity of the learning outcomes doubled as a result of these changes. Comments from students included:

- *The lecture notes were set out clearly so you could follow and divide up course into different areas.*
- *This course is much better compared to last year. It has been greatly improved.*
- *The lecture notes are very clear, interesting and comprehensive.*

In 2010 the distinction between different strands of mathematical competency was extended to the rubric of the tutorial sheets. Each tutorial set is printed on a doubled sided A4 sheet. The first page is labelled “Preparatory” and consists of simple tasks in *conventional mathematical language* presented in an abstract and context-free setting. These questions are intended to develop procedural fluency in specific micro-skills. Also as a direct response to requests from the 2009 cohort, short answers to these preparatory questions are provided in advance with detailed answers provided after the tutorials. Students and tutors are made aware of the expectation that this preparatory page should be attempted before attending tutorials. The second page is labelled “Tutorial”. This page has fewer but longer questions. Each of these questions has an authentic context and is asked in plain language with vocabulary appropriate to the life sciences. These questions require application, combination or articulation of the simpler skills from the first page and are intended to develop strategic competence. Tutors are advised to concentrate on these harder questions in the tutorial.

## PROCEDURAL FLUENCY AND SELF-REGULATION

Despite the stigma associated with rote-learning and the possibility that memorisation of basic techniques may lead to students adopting a surface approach to learning; repetition and practice are nevertheless useful for developing *procedural fluency* (Angus & Watson, 2009). Unfortunately repetition is not the most valuable use of the limited face-to-face time in tutorials; and different students will require different amounts of practice. Although the rubric of the tutorial sheet encourages students to practice on their own, they still need immediate and personalised feedback, especially if they make errors. Thus, in 2010 a new online *formative* assessment environment was trialled.

MapleTA™ was specifically developed for mathematics teaching. It is one of very few online tools that goes beyond multiple choice and simple numerical answers. It also allows for free student responses in a conventional mathematically acceptable format. It is sufficiently sophisticated to implement any mathematical procedure that can be programmed in the underlying Maple computer algebra system. It can control the diversity and difficulty of the random questions it generates. The students can try the questions online, or print the quiz and take it away and log-on later to grade it.

There were no marks associated with the online formative assessment tool (MapleTA™) and students were also told that, since it was brand new, they would use it at their own risk with zero training or support. Nevertheless, 200 out of 600 students tried it and half of this group responded to a voluntary informal survey about how they used this tool. The comments reveal a variety of productive ways to use this resource:

- *I always do the questions online. If I feel I'm already comfortable with material I won't bother answering it, particularly if the questions is* [sic] *identical or very similar to questions previously generated.*
- *I grab the question, work it out on paper. Then write answer in answer box. Great way to time myself and how well I can retrieve the answers!*

- *I do it online over and over again until I hit 85% minimum.*
- *When I am totally blank about how to do a question I will just press grade and try to decipher it that way. If I feel I am on a bit of a role* [sic] *I will complete the quiz. I generally do them over and over again, I feel that maths is fairly similar to playing guitar, it's about repetition and muscle memory.*

In 2011 I intend to encourage all students to try the online practise quizzes and I will demonstrate specific online questions in lectures. The MapleTA tool will also be used to randomly generate half of the questions for the in-tutorial quizzes (which are assessable) leaving only a smaller proportion of questions that need to be hand designed. Thus, it is expected that regular use of this online tool will open up time in the tutorials to work on problems that required higher level conceptual and cognitive skills and advice from tutors. As a result, students will get more opportunity to tackle more realistic problems in an environment where human feedback is available.

## ADAPTIVE REASONING AND QUALITATIVE MATHEMATICS

Adaptive reasoning in higher mathematics is associated with tasks such as developing a conjecture, searching for a counterexample, arguing the appropriateness of a definition, or checking a proof for logical flaws. The archetypical literary presentation of adaptive reasoning is *Proofs and Refutations* (Lakatos, 1976) about a group of students using Socratic dialogue to discuss and generalise Euler's formula for polyhedra. Although this level of mathematics is not necessary for students in service classes, they will still be expected to understand the scientific method and the role of the corresponding types of adaptive reasoning in each of their home disciplines. Thus the new element that was introduced into MATH1013 to stimulate adaptive reasoning was a focus on advanced qualitative reasoning. Some of the models explored in the course cannot be solved exactly, and it is important for these students to both have the confidence to approach such problems and the necessary mode of reasoning to extract some useful information. Examples included deciding if a model predicts stable steady-state behaviour *without* actually calculating the solution to the equations, and use of graphical techniques to classify the number of distinct regimes or behaviours a model predicts, again *without* explicitly solving equations.

Another strategy is to expose students to apparently reasonable models that lead to scientific impossibilities and consider where the flaw lies. For example, the simple assumption that the growth rate of an organism is proportional to its surface area will lead to the conclusion that it becomes infinitely large within a finite amount of time. This example is simple enough to study in detail with first year service classes and yet provides a rich opportunity to combine logic, scientific method and mathematics. Nevertheless, since this particular problem operates on multiple levels, less secure students (i.e. those whose reasoning is not as adaptive) can still engage with the problem on a purely mathematical level, while those with a genuine interest in the scientific method are given an opportunity to see how this type of reasoning occurs within mathematics.

## DISCUSSION AND CONCLUSION

Kilpatrick's framework of the five interwoven strands of mathematical proficiency provides useful scaffolding within which to redesign a mathematical service course. Each strand leads to a different type of activity or choice of content. According to the surveys, students appreciate having the structure and purpose of the different components of a course made explicit. The survey results shown earlier naturally represent only the affective aspects of how the students engage with the course but provide evidence of a substantial shift in student perception and satisfaction.
Future research with this unit of study will look at how students are using opportunities for self-guided practice and study and what they retain and transfer to their other disciplines. The ability to transfer skills and knowledge to others area can also use the same nuanced approach by looking at a variety of proficiencies rather than simple retention of a formula, procedure or concept from one year to the next.

## REFERENCES

Angus, S., &Watson, J. (2009). Does regular online testing enhance student learning in the numerical sciences? Robust evidence from a large data set. *British Journal of Educational Technology, 40*(2), 255–272.

Ashcraft, M. H., & Kirk, E. P. (2001). The relationships among working memory, math anxiety, and performance. *Journal of Experimental Psychology: General, 130*, 224-237.

Britton, S., New, P., & Sharma. M. D (2007). Investigating students' ability to transfer mathematics. In (A. Brew & J. Sachs) *Transforming a university. The scholarship of teaching and learning in practice,* (pp. 127-140), Sydney: Sydney University Press.

Cates, G. L. A., & Rhymer, K. N. (2003). Examining the Relationship Between Mathematics Anxiety and Mathematics Performance: An Instructional Hierarchy Perspective*, Journal of Behavioral Education, 12*(1), 23-34*.*

Hannula, M. S. (2006). Motivation in Mathematics: Goals Reflected in Emotions, *Educational Studies in Mathematics, 63,* 165-178*.*

Hodges, C. B. (2009). Self-Regulation of Learners in an Asynchronous University Math Course*, The Quarterly Review of Distance Education, 10*(2), 233-237.

Kilpatrick, J. (2001). Understanding Mathematical Literacy: The Contribution of Research, *Educational Studies in Mathematics*, *47*(1), 101-116.

Lakatos, I. (1976). *Proofs and Refutations: The Logic of Mathematical Discovery*, Cambridge: Cambridge University Press

National Research Council. (2001). *Adding it up: Helping children learn mathematics.* J.Kilpatrick, J. Swafford, and B.Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

Pintrich, P. (2003). A Motivational Science Perspective on the Role of Student Motivation in Learning and Teaching Contexts*, Journal of Educational Psychology*, *95*(4), 667–686.

Quinell, R., & Wong, E. (2007). Using intervention strategies to engage tertiary biology students in their development of numeric skills. In K. Placing (Ed.) *Proceedings of the Science Teaching and Learning Research Including Threshold Concepts Symposium,* (pp. 70-74), Sydney, NSW: UniServe Science.

Richardson, F. C. and Suinn, R. M. (1972). The Mathematics Anxiety Rating Scale, *Journal of Counseling Psychology, 19,* 551-554.

Roberts, A.L., Sharma, M.D., Britton, S., & New, P.B. (2009). Identification and Use of Theoretical Frameworks for a Qualitative Understanding of Mathematics Transfer, *CAL-laborate International*, *17*, 1 – 14.

Tobias, S. (1993). *Overcoming Math Anxiety.* New York: W. W. Norton & Company.

Trigwell, K., Prosser, M., & Waterhouse, F. (1999). Relations between Teachers' Approaches to Teaching and Students' Approaches to Learning. *Higher Education, 37,* 57-70.

Wu, H. (1999). Basic Skills versus Conceptual Understanding *American Educator* (Fall Issue). Retrieved August 23, 2011, from http://www.aft.org/pdfs/americaneducator/fall1999/wu.pdf.