Researching the transferability of mathematical skills

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Abstract: Science students are required to use mathematics, at various levels, in all their science subjects. Certain mathematical skills are essential for success as an undergraduate student, and in the student's future career. An ability to transfer the skills learned in mathematics to other disciplines is expected of both undergraduates and graduates. It is important to discover therefore, whether or not students have this ability. An instrument was developed to test the transferability of mathematical concepts across various scientific disciplines. The instrument consists of mathematical problems set in various scenarios. The instrument was trialled with mathematics, microbiology and physics students from higher years. First year students from the different disciplines will be tested using the instrument once it has been refined using the results from the trials. Results of the trial with students from higher years are presented. The research methodology and the process of developing a useful instrument are discussed. The results of this research will have an impact on curriculum design and the teaching and learning of mathematics, both within mathematics and other scientific disciplines.

Introduction

Mathematical skills are essential for a science student's success regardless of the choice of discipline. All science students need to use mathematics while completing their degrees and many will use mathematics in their future careers. The ability of students to transfer their skills in solving mathematical problems to other scientific areas is generally expected, yet anecdotal evidence suggests that many lack this ability. While this has been widely recognised as a problem in higher education for a very long time (Gill, 1999a & b), it is important to discover the extent to which students have this ability. The project reported in this paper has set out to develop and pilot an instrument to do this.

While there is a substantial body of research on the use of mathematics in everyday life (Lave, 1988) and a good deal of interest in the extent to which mathematical skills transfer to problem solving in real world situations (Buckingham, 1997; Carraher et al., 1985; Harrison, 1986; Lemire, 1998; Moy, 1999; Sun 1995), there is very little work which specifically addresses the question of the extent to which students are able to transfer their mathematical skills and knowledge to other higher education subjects. An exception is Gill (1999a & b) who studied the performance of physics and mathematics students. He concluded that the ability to understand graphs is a key to transferability of mathematical skills.

Much of the work which recognises the difficulty students have with transferring their mathematical knowledge, focuses on the design of courses to improve both students' mathematical skills and their ability to transfer it across subjects (Lindsay, 1999; Chia, 1987). There is also a recognition that the type of teaching may be responsible for whether or not students can transfer their mathematical skills and knowledge across different situations.

Some researchers (Lave, 1988; Sun, 1995) go so far as to question the very notion of learning transfer. This suggests that the difficulties students have in transferring mathematical knowledge to other science subjects in higher education may be a facet of a wider problem and therefore to be expected. Nonetheless, the transferability of mathematical knowledge is a persistent cause of concern to, as well as an expectation of, higher education teachers. It is therefore important to investigate the extent to which students are able to transfer their mathematical skills and knowledge to different science subjects.

This project is being carried out by researchers from the Institute for Teaching and Learning, and from departments within the Faculties of Science and Agriculture, namely Physics, Mathematics, Computer Science and Microbiology, at The University of Sydney. The team is developing an instrument to test the transferability of mathematical concepts across the scientific disciplines represented on the team. The instrument consists of mathematical problems set in various scenarios. In this paper we outline our research methodology, discuss issues that have arisen in setting up the instrument, and describe initial trials with higher year students.

Research methodology

Some mathematical skills necessary for success in first year physics, chemistry, computer science and second year microbiology were identified as algebraic manipulations, graphical representation and mathematical modelling with functions of one and two variables. The mathematical content common to all discipline areas and of interest to the researchers was mathematical modelling with functions of one variable, in particular applications of exponentials and logarithms in discipline specific problems.

We developed discipline specific problems based on exponentials and logarithms, the solution of which required a practical understanding of the mathematical rules for manipulating logarithms. The set of problems has gone through several iterative cycles and will constitute the instrument for studying the transferability of mathematical skills in students from a range of diverse science subjects.

Currently there are two problems with physics and two with microbiology contexts, three with a computer science context and five questions from mathematics. One microbiology and one physics problem are very similar in structure (Figures 1 and 2). The other problems are structurally quite different. The mathematics questions have no context. The mathematical understanding required is approximately at the level of 2 Unit Mathematics for the New South Wales Higher School Certificate. (This is the lowest level of mathematics required for entry to Science courses in The University of Sydney.)

Preliminary trials have been conducted using higher year students who have already chosen a discipline in which to specialise. The point of the exercise was to gain feedback from them with regard to clarity of the questions, perceived difficulty, and length of time taken to complete the questions. Since the aim was to study transferability of mathematical skills in a range of different and unfamiliar contexts, all students were given the entire instrument. They completed all the problems in their own time, noting the time taken. In addition the students were asked to comment on the problems in terms of ease of understanding the material. Results from the trial with higher year students are presented in this paper.

The instrument will be refined in response to the feedback from higher year students, prior to trialling with a small group of first or second year students from the different disciplines. Student responses to the problems on the instrument will be categorised in terms of ability to transfer mathematical skills to unfamiliar scenarios. Individual students' responses will be compared with examination results to look for patterns as well as to determine the representativeness of our sample. Student attitudes to the instrument will be evaluated via open-ended questions.

Trials of the instrument by higher year students

As part of the process of refining the instrument, some postgraduate and higher year undergraduate students were asked to attempt the questions comprising the instrument. The students were not selected randomly, but were students the researchers could readily contact. During February 2001 nine students attempted the questions. Five of these were physics postgraduate students, one was a microbiology postgraduate student, one an undergraduate microbiology student, one a graduate in mathematics and computer science, and one a mathematics honours student. We expected that these students would be able to complete the questions without difficulty.

Parallel Questions

- **Mathematics**

A function $y(t)$ increases exponentially for $0 \leq t < 2$, remains constant for $2 \leq t < 3$, and decreases exponentially for $t \geq 3$.
  - Sketch a graph of ln*y* against *t*.

- **Microbiology**

When a culture of bacteria is grown in a container ... a plot of bacterial concentration against time has a typical four-part shape. ... When the cell concentration is converted to a logarithmic scale, the four phases approximate straight lines with slopes that are zero, positive, zero and negative, in that order.
  - Draw a sketch of log(cell concentration) vs time.

- **Physics**

Consider a beam of photons with identical energies all travelling in the same direction head-on into medium-1. The photons that pass through medium-1 enter a second medium of different properties, medium-2. ... A graph of number of photons on a logarithmic scale against penetration distance has straight lines with negative slopes. ...
  - Draw a sketch of log(number of photons) vs distance into the medium.

Figure 1. Parallel questions involving graphing in Microbiology and Physics, with the corresponding Mathematics question

Parallel Questions

- **Microbiology**

Growth in the exponential phase can be described by $N(t) = N_0 2^{kt}$
where $N(t)$ = cell concentration at time *t*,
... Sketch another graph of log concentration vs time with a scale on the axes, such that the lag phase has a duration of 1.0 hr and concentration of 10^3 cells/mL and the exponential phase a generation time of 0.5 hr. ... Draw only the first 2 phases.

- **Physics**

The reduction in photon numbers can be described by $N(x) = N_0 2^{-kx}$
where $N(x)$ = number of photons at distance *x*. ... Medium-1 is 10 mm thick with a half thickness of 0.5 mm. Given that there are 10^{10} photons entering head-on into medium-1, draw a graph of log(number of photons) vs distance with the axes labelled.

Figure 2. Parallel questions involving graphing exponential equations in Microbiology and Physics

The feedback we received was extremely useful, and the questions will be further refined in the light of the students' comments. The time taken by the students to complete the problems was much greater than we had anticipated. Somewhat surprisingly, a majority of these students was unable to successfully complete all the problems. The computer science questions proved particularly difficult for some of the students who had not studied computer science. The mathematics questions, which were of HSC standard, were completed successfully only by the mathematics students. There was no indication that the other students performed better on those questions set in the context of their chosen discipline.

Discussion

Some interesting issues have arisen throughout the process of developing the instrument. Firstly, we had thought that the instrument could be developed around a topic taught in first year mathematics, that was used in first year physics and computer science and in microbiology. It was a little surprising to find that there is apparently no such topic. The instrument is therefore based on mathematics which is taught at HSC level, and used in physics, microbiology and computer science.

One particularly interesting aspect has been the difficulties encountered in trying to write questions set in a particular discipline that are comprehensible to people outside that discipline. The first draft of the instrument included some explanations which were not entirely comprehensible to those of us who had not written the questions. The questions needed to contain enough discipline specific information so that they could be solved using mathematical knowledge only, without any previous knowledge of the particular discipline. Clearly, it is difficult for academics not to make certain assumptions relating to their discipline, when writing background information. In this respect, the contribution from the researcher from the Institute of Teaching and Learning was extremely valuable. With minimal formal background in science, she was able to point out subtleties in the wording of the questions which had been taken for granted or overlooked by the scientists. The fact that the instrument has evolved into a set of questions that are necessarily quite wordy is an issue which will have to be considered, in terms of distinguishing between students' abilities to cope with the mathematics and with the language.

It has also been interesting to note that the mathematician expected mathematics to be used much more precisely than the other scientists were clearly accustomed to doing. While the use of mathematics in an imprecise way may not hinder scientists in their everyday work, it may be confusing for students. Indeed, there is much food for thought with respect to the implications for teaching raised by all these issues.

Conclusion

In summarising the findings of a study on the transferability of mathematical skills to physics problems, Gill (1999a) highlights the importance of communication between educators. Our study has certainly borne this out. We have found that we have learnt much from working together across our disciplinary domains. We have learnt to appreciate better the different ways in which mathematics is used in solving problems in other discipline areas. We have discovered that we differ on the criteria of what constitutes 'correctness', and that we are all over-optimistic about the number of problems that students can solve in a given time. We have noted too the importance of linguistic and interpretive skills in solving mathematical problems.

The ultimate aim of the project is to deepen our understanding of students' ability to transfer mathematical skills to other discipline areas in order to inform the process of developing better quality student learning. While the next stage of the project will be to pilot the instrument that we have developed with our undergraduate students, we have already learnt much to improve our understanding of student experiences and outcomes.

Acknowledgements

This research was supported by an Inter Departmental Education Research Grant through the Science Faculty Education Research group (SciFER). The project has included input from Judy Kay and Tony Greening (Basser Department of Computer Science) and Simon Barrie (Institute for Teaching and Learning).

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