

Bach and Numerology: 'dry mathematical stuff'?

David Rumsey

Carl Philipp Emanuel Bach wrote to his father's biographer, Forkel, on January 13th 1775:

The account of my late father's life [given] in Mizler, dearest friend, was thrown together by the late Agricola and myself in Berlin, and Mizler added just the section from the words 'He joined the Society' to the end. It is not worth much. The departed, like myself or any true musician, was no lover of dry mathematical stuff.

The Mizler to whom Carl Philip Emanuel referred was Lorenz Mizler von Kolof. Mizler was born in Heidenheim/Württemberg in 1711 and, having gained a doctorate in each of Philosophy and Medicine, eventually went on to become a University Lecturer and Magistrate in Leipzig and a court mathematician in Poland before dying in Warsaw at the age of sixty seven. A pupil of J. S. Bach in both harmony and clavichord, author of *Die Anfangsgründe des Generalbasses* (1739), and translator of Fux's *Gradus ad Parnassum* into German (1742), Mizler founded the periodical *Musikalische Bibliothek* in 1736 and two years later expanded it into a small academy called Correspondirende Societät der Musicalischen Wissenschaften (Correspondence Society of Musical Sciences).

Johann Sebastian Bach had become a member of this society in 1747 and deserved better than a dismissive assessment of his interest in mathematics from the son who had lived in Berlin since 1738 and seen far less of his father than Mizler had in the final decade of Bach's life. Mizler's society was specifically dedicated to the connections between science and music, which in the culture of the day included theology, mathematics, medicine, ethnomusicology and a variety of philosophical areas. Thus we find quite a wide range of articles in *Musikalische Bibliothek*, including (in summary and translated):

<i>Author</i>	<i>Translation of Title</i>
Meibom	'Ancient Greek Music'
Wallis	'Comparison between Ancient Music and that of Modern Times'
Meibom	'Aristoxenes' "Harmonics"'

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Semler	'Vocal and Instrumental Music of the Levites from Scriptural Sources'
Psellus	'Essentials of Music from the Greek'
Venzky	'Notes and Notation of Ancient Hebrew Music'
Werckmeister	'The Noble Art of Music, its value, Use and Misuse, based on Holy Scripture'
Beeren	'Musical Discourse, deduced through the Principles of Philosophy'
Dr. Albrecht	'Physical Treatise on the effect of music on the live body'
Werckmeister	'Musical Material, dealing with compositions and temperament'
Weigel	'Concept of a complete Mathematics ...which deals with Music'
Mizler	'Elementary Fundamentals of Thoroughbass, dealt with from a Mathematical Point of View ...'
Schroeter	'The necessity of Mathematics, for the basic learning of Musical Composition'

Many of these contributions are a crisp reminder of the theories of Leibnitz that Music is an 'unconscious exercise in arithmetic'. Articles also included one on African and Japanese music, a fascinating 'Talk about music and its superiority over painting, poetry and dramatic art' by Graefenbahn, and a dispute over Opera with an interesting commentary on the eighth chapter of a Dr. Ludwig's *Critical History of the German Language* entitled 'Attempt to demonstrate that a Singspiel or an Opera cannot be good'. Mizler allowed publication of Agrippa's article 'Of the doubtfulness and Vanity of Learning', then tore it to shreds in a series of footnotes which take more space than the text.¹ Another dispute was over the art of J. S. Bach himself.

Bach's own contribution to Mizler's society was not in the form of written articles, but in the music of his late works: 'Musical Offering', 'Art of Fugue', 'Canonic Variations on *Vom Himmel Hoch*', and a portrait containing a puzzle canon. These late works were all either written for the society, as a requirement of membership, or strongly influenced by the society's musico-scientific-mathematical culture. Carl Philip Emanuel may not have fully understood his father in respect of Mizler's remarks about an interest in mathematics. The evidence has been mounting for the past half century that Johann Sebastian Bach did anything but eschew mathematics, pure or applied, biblical or secular, dry or not. When we take stock of the part it played in his musical life there seems to be scarcely any area of it that he neglected.

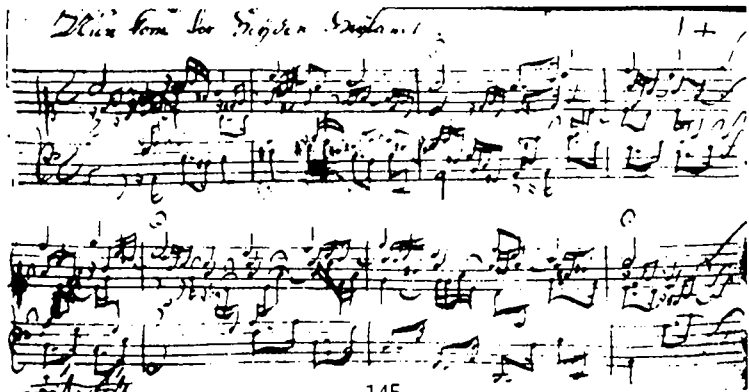
I

In the eighteenth century, music, particularly German music, used a kind of 'language'. This was a philosophy and practice of music well-chronicled by contemporary writers such as Werckmeister, Heinichen, Mattheson, Quantz, and Marpurg. We generally divide this language up into two main components: *Affektenlehre* and *Figurenlehre*. *Affektenlehre*, translated in eighteenth-century English as 'Doctrine of the Affections', 'was the belief held in the 17th and early 18th Centuries, that the principal aim of music is to arouse the passions or affections (love, hate, joy, anger, fear, etc., conceived as rationalised, discrete and relatively static states)'. *Figurenlehre*, translated as 'Doctrine of Figures', is defined as 'any of various attempts made in the 17th and 18th Centuries to codify music according to classes of musical figures thought to be analogous to the figures of rhetoric'.² Furthermore:

... the whole of the Baroque *Affektenlehre* was predicated on [hermeneutics], and all composers of the age attempted to portray words, even ideas, by using musical figures that have a pictorial quality expressible in the very graphic image of the score.³

Clearly, in order to understand fully the music of this era we need to comprehend the symbols of its language. And if that language includes other symbols, such as numerology, then these also become a category of *Figuren*.⁴

Albert Schweitzer's analysis of *Figurenlehre* in Bach's *Orgelbüchlein* of ca. 1717 uses the chorale texts to provide us with essential links to the music associated with them and show precisely how Bach's creative mind worked in this connection.⁵ Thus 'Nun komm, der heiden Heiland'—the first advent chorale dealing with the theology of God coming down to earth—has downward falling motives of an octave all through it.



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The prelude on 'Vom Himmel kam der Engel Schaar' with its ascending and descending scale figures conjures up Jacob's vision: 'a ladder, set up on earth, and the top of it reached to heaven: and behold the angels of God ascending and descending on it' (Genesis 28: 12).



Not one of the forty five preludes of the *Orgelbüchlein* lacks such connections; they all have specific links between their texts and the chosen *Figuren* and *Affekten*. Again, in the *St. Matthew Passion* the scoring of strings playing long-held chords in the recitatives during the actual words of Christ symbolises the sanctity of these words, adding a kind of musical halo.

Bach's musical language went well beyond these pictorial figures, moreover. In the 'Passion' section of *Orgelbüchlein* we observe that the first three employ the musical device of canon. As a symbol of law and obedience canon is ideal, for one musical voice sets out a theme which must be followed by another. Although we have no contemporary textual record that he expressly used imitation or canon for this purpose, he did use these musical techniques consistently whenever law and obedience were concepts: in each of the three organ settings we have of 'Dies sind die heil'gen zehn Gebot', for example, one in *Orgelbüchlein* and two in *Clavier Übung III*, some twenty two years later in 1739.

II

As a practicing Lutheran musician in Saxony during the eighteenth century, Johann Sebastian Bach encountered mathematics in one form

or another in his daily life. As a keyboard player, in the teaching and practice of music of his era, not one day would normally pass without his playing what is termed 'figured bass'. This was a system of providing a bass line with a series of numbers to indicate the harmonies intended to be played above it. As an organist his skill in registration depended on understanding the harmonic series, and using organ stops acoustically to synthesise desired sounds. Numbers on these stops, such as '8", represented foundation tone, '4" the second harmonic, '2" the fourth, and so on. Like simple currency dealings, these were absorbed into his daily life and need not concern us further except as a reminder that behind them lay substantial scientific concepts which had to be properly understood and applied.

We must also mention here the question of keyboard tempering. Bach developed his own system, but did not record it except indirectly, first, in providing two books of twenty four preludes and fugues to demonstrate its efficacy and, second, through choice of key as contributor to *affekt*. The development of any tempering system presupposes a combination of sophisticated musical scientific and mathematical skills in order to handle the acoustic and other problems associated with it.

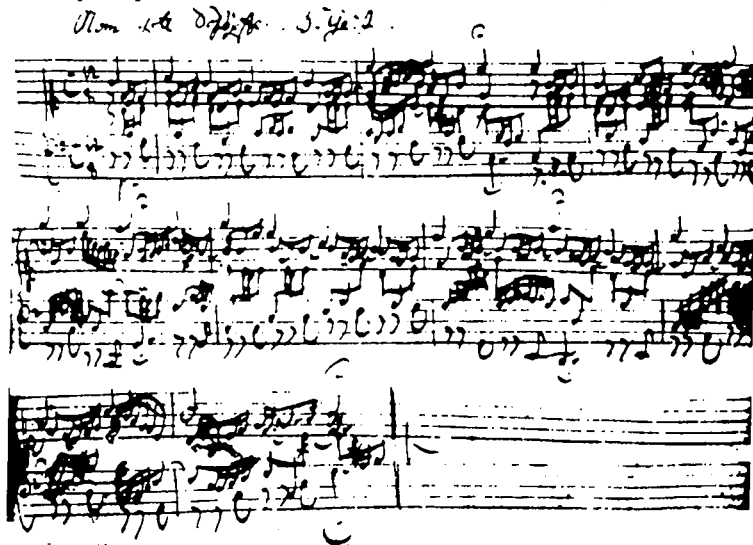
There were other mathematical disciplines and pastimes current in Bach's day that had reached a climax chronologically and geographically precisely during his lifetime and in his region. Such publications as the *Buch der Biblische Mathematicus* of Johann Jakob Schmidt (1736), the preface to Pachelbel's *Hexachordum Apollinis* (1699), and Jakob Adlung's *Anleitung zur musikalischen Gelahrtheit* (1758) are among many that deal explicitly with number symbolism in the general cultural context of their day. Chapter 19 of the *Paradoxal-Discourse* of Andreas Werckmeister (1707) is entitled 'Von der Zahlen geheimen Deutung' ('Of the secret meaning of numbers') and gives a basic listing of number-symbols. Werckmeister, who in a treatise on organ building nine years earlier had already established his credentials in other areas of mathematics, science and music, wrote thus:

As now the musical intervals are nothing other than numbers and proportions and since God created and ordered everything in Numbers, Proportion and Weighting, so also must the musician, indeed every person, be diligent and study how to reproduce such marvellous Order.

From these writings, we extract several important facts: first, that there were divisions of Mathematics which in those days were all respected as separate categories of the same discipline; second, that 'Biblical

Mathematics' was one of these categories; third, that 'proportion' was another; fourth, that there was an established fascination with number systems and with the Cabala, particularly the use of *gematria* or number-substitute alphabets.

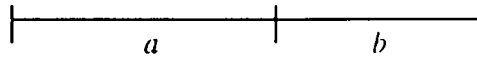
In terms of broad based Biblical mathematics we observe this in many examples of Bach's music. Whether representing gifts of the spirit, finality of finalities, or just deadly sins, the number 7 is found throughout *Orgelbüchlein*: there are seven preludes in the Passion section; also consistently seven pedal quavers in each phrase of 'Komm, Gott, Schöpfer, heiliger Geist' a Whitsunday work and therefore associated with the Holy Spirit. (Moreover, in a piece dealing with the Holy Spirit or third member of the Trinity, each of these seven quavers comes syncopated on the third subdivision of a beat.)



In the *St. Matthew Passion* we find the well-known chorus with eleven responses of 'Bin ich's?' ('Is it I?') to Christ's words 'Truly I say one of you will betray me'. Judas knew who it was—leaving eleven disciples who could validly ask the question. In the structure of the *Clavier Übung III*, a collection dedicated to the Holy Trinity, we find 27 pieces (3 x 3 x 3). The internal division of this collection also shows many significant manifestations of the number 3: three themes in the Prelude with *Figuren* clearly representing Father, Son, and Holy Spirit; a triple fugue at the end; three manifestations of the one theme.

If we take the great *Leipzig C-minor Prelude* (BWV 546) we note that it has two themes and that together these account for some 144 bars exactly. Not only is this a symbolic number itself (12 x 12), but the total bar numbers of two themes of 89 and 55 respectively form the final two

stages of a standard Fibonacci series going from 1 through 144: 1 2 3 5 8 13 21 34 55 89 144. The further we progress along such a series the closer consecutive numbers come to the proportion of the 'Golden Section', the division of a given line so that the greater part is proportioned to the whole as the lesser part is to the greater:



or

$$\frac{a}{a+b} = \frac{b}{a}$$

The solution of the quadratic equation which results yields a ratio of approximately 1 : 0.618. The 'Golden Section' holds a special place in art and architecture, from the ancient Greeks to the present day, as a formula for perfection in proportions.

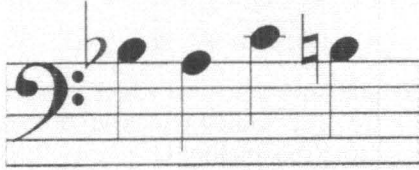
We might dismiss these findings with scepticism. Yet they occur frequently and consistently and, if we take the trouble to understand the culture of Bach's era from his own perspective, it then becomes difficult simply to attribute them to mere coincidence. There is not much proof to confirm or deny that these number manipulations were taking place intentionally, but proof does exist. For instance, there are some fascinating numbers on the manuscript copy of 'Vor deinen Thron tret ich hiermit' in Bach's own handwriting. On one line, in a range of 1-10, the numbers 1-7 are underlined, while on the next, in a range of 1-5, the numbers 1-3 are underlined. Although their meaning remains obscure, the fact that they are significant numbers and that they seem to have had something to do with Bach's compositional processes would be hard to deny.

1. 2. 3. 4. 5. 6. 7. 8. 9. 10.
 - - - - -
 1. 2. 3. 4. 5.

Let us divert our attention for a moment to musical cryptography. This is the science—possibly also art—of burying a message or symbol of some kind in a piece of music. We have clear evidence that it was a common enough practice around the Baroque and other eras.

Johann Jacob Froberger (1616-1667), for example, in composing a harpsichord piece on the death of Frederick III, concludes with a rising scale and three repeated top 'F's, appending a drawing symbolising Frederick's ascension through clouds up to heaven. The three 'F's represent Frederick III cryptographically.

Bach encrypted his own name in his music by using the note sequence *Bb-A-C-B#*, which in German notation is designated *B-A-C-H*. We have express confirmation that this was intentional in a contemporary reference by a relation and close friend of the composer⁶:



This brings us one step closer to *gematria*, Cabala alphabets, and the substitution of numbers for letters and words. It is the most vexing and speculative area of mathematics and numerology associated with Bach research, one which divides the musicological community into insecure believers and ardent disbelievers. Since the 1940s a series of publications has appeared, initiated mainly by German musicologist Friedrich Smend. In particular his *J. S. Bach bei seinem Namen gerufen* published a long sought solution to the puzzle canon left by Bach in the E. G. Haußmann portrait submitted as his entry obligation to Mizler's Correspondence Society of Musical Sciences.⁷



Using a second portrait where the canon was held upside down and lacked the composer's name, Smend found that there were some 480 different solutions to the puzzle, from which followed four basic solutions, each determined by inversions and mirrored retrogrades, clef exchange (each producing 120 variants), and so on. That such a canon could be composed merely by chance would be a virtual impossibility statistically, although it might be difficult to calculate it as an actual mathematical probability.



In the course of their work, Smend and his colleagues noticed that the number 14 was constantly appearing in various guises. In the Mizler puzzle canon, Smend draws attention to:

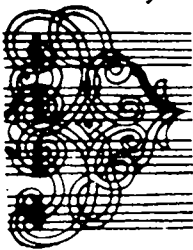
- 14 buttons on Bach's coat in the first portrait;
- exactly 14 buttons in the second (although other details were changed);
- Bach was the 14th member of Mizler's Society;
- Bach seems to have waited to become the 14th member;
- 14 notes between the repeat barlines in the top two voices of the given puzzle canon (the third voice was borrowed from Handel);
- the three canons of the first basic form occur as canons at the 4th, 5th, and 5th respectively: $4 + 5 + 5 = 14$;
- the three canons of the third basic form occur as canons at the 2nd, 3rd, and 9th = 14 again.

Smend, working together with two colleagues, deduced from this and the frequent occurrences of the number 14 in other works by Bach that it was a form of cryptography. He concluded that Bach was using a Cabalistic alphabet to derive the number 14 by adding together the letter equivalents:

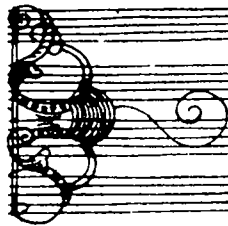
$$B (2) + A (1) + C (3) + H (8) = B A C H (14)$$

From this flowed a number of published books of calculations and conclusions, some a little hastily conceived and some improperly substantiated: Smend's own *J. S. Bach Kirchen Kantaten* (collated 1966), for example; Kees van Houten and Marinus Kasbergen, *Bach en het Getal* (1985); Pierre Vidal, *Bach: Les psaumes dans l'œuvre d'orgue*; Ludwig Prautzsch, *Vor deinen Thron tret ich hiermit* (1980); most recently, Ruth Tatlow, *Bach and the Riddle of the Number Alphabet* (1991). All except the Tatlow and Vidal's strange essay and many articles launched a veritable battery of figures, ideas and exposés of Bach's numerology. By their sheer bulk and reasonable consistency, Bach's pre-occupation with mathematics, symbolic numbers, cryptography, and religious beliefs can hardly be totally dismissed. (Apart from anything else, Bach was musically fringing on a quite popular pastime of literary culture: the creation of paragrams or poems and dedications whose lines added up to identical mathematical totals when a Cabala alphabet was applied to them.)

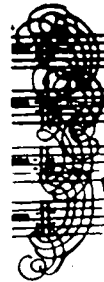
In his *Vor deinen Thron tret ich hiermit*—a treatment of Bach's last great works, 'Musical Offering', 'Art of Fugue', 'Canonic Variations on Vom Himmel Hoch' and the Chorale, 'Vor deinen Thron tret ich hiermit'—Ludwig Prautzsch pushes the whole system to its limits and gives a much extended list of Werckmeister's and other number symbols, regrettably without always being too careful to quote his authorities. Prautzsch and others also include scrolls and seals drawn or designed by Bach to add a support or credibility of their own. From the 'Art of Fugue', for example, he sees the letters J S B drawn into the following in various ways:



Contrapunctus 2



Contrapunctus 6



Contrapunctus 10



For Prautzsch the most important of these is the 14 lines of the J S B insignia, bottom right. 'The letters J and S are found twice each, once the right way around and also mirror-reversed, while the B appears only once in smaller form, connected with the underside of the S. This gives exactly 14 letters'. We note, in passing, the 3, 5 and 7 and crown and mirror symbols of Bach's seal, bottom left.

By the 1990s, we find an impressive variety of significant numbers being used by investigators in this field, from Biblical and pagan numerology to Cabala-derived alphabets and writers such as Werckmeister; from St. Augustine to common number superstition, and many other sources. A basic compilation of some of these numbers would include:

- 1 God, Unity.
- 2 Man, Eternal Word (God's Son and the second person in Trinity).
- 3 Trinity.
- 4 Earth, World.
- 5 Mankind (Werckmeister; later, Johann Jacob Schmidt in his *Biblische Mathematicus* of 1736 notes that in Revelations it is associated with weakness or Fall of Man: the fifth epistle goes to the dead community of Sardes; the fifth trombone call is for the star fallen from heaven; the fifth anger of God; and so on).
- 6 Completion (with 7 and 8): 6 is $1 \times 2 \times 3$ and $1 + 2 + 3$, the only even perfect number and the number of days God worked at creation (thus 6 *Trio* sonatas, 6 *Brandenburg* concertos, 6 *Schübler* chorales, suites etc.).
- 7 Holy Spirit; the Sleep of God; the finality of finalities; 7 gifts of the spirit, and so on. (St Augustine also comments on 7 in like vein.)
- 10 Commandments.
- 11 Exceeding the Commandments (Prautzsch; the most grievous breaking of commandments having been the crucifixion of Christ).
- 12 Completeness, again: 3×4 (related to $7 = 3 + 4$); eternal 12 Portals of the Holy City (Revelations); after Gregory the Great = Symbol of the Church (Trinity of God declared to the 4 ends of the earth).
- 13 Severe misfortune; Death (as it is today).
- 14 B + A + C + H.
- 17 Completeness of Holiness ($10 + 7$).
- 19 God and the LAW; throne of God in judgement ($10 + 9$).
- 24 Elders in heavenly Jerusalem (Revelation 4: 4).
- 29 S + D + G (Soli Deo Gloria—with which Bach habitually signed off his works) and/because = J + S + B.
- 33 Earthly life of Christ.
- 40 Penance as ordained by God (40 days rain before the flood; 40 years wandering in the desert; 40 days fast on Mount Sinai; 40 days Jesus in the desert).

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- 41 J + S + B + A + C + H; mirror image of 14.
 43 C + R + E + D + O (hence, the number of notes of the final pedal entry in the 'Credo' chorale of *Clavier Übung* III).
 46 Symbol of the Temple (46 years to build).
 47 H + E + R + R.
 48 I + N + R + I or B x A x C x H.
 53 S + O + H + N.
 59 G + O + T + T and G + L + O + R + I + A.
 60 G + F + H + A + E + N + D + E + L.
 61 I + S + R + A + E + L.
 70 J + E + S + U + S.
 71 K + R + I + P + P + E.
 73 Z + E + B + A + O + T + H.
 75 B + E + T + H + L + E + H + E + M.
 77 Hint at the Human Nature of Christ; the generations of Adam.
 83 I + M + M + A + N + U + E + L.
 92 L + C + M + I + Z + L + E + R.
 106 S + C + H + R + O + E + T + E + R.
 112 C + H + R + I + S + T + U + S.
 144 J + O + H + A + N + N + S + E + B + A + S + T + I + A + N; the basic number of 144,000 elected in the eternal city (Revelations 14: 1).
 153 Number of fish (Lake of Tiberias) according to John 21: 11; sum of all numbers between 1 and 17 (has connotations of Mission, conversion of souls).
 158 J + O + H + A + N + N + S + E + B + A + S + T + I + A + N + B + A + C + H

Prautzsch also claims that mirroring is significant (without apparently giving an authority): because 35 = S O H N or the son of God, for example, the reversal of 35—53—becomes the number of the death of the son of God. So with 14 = B A C H, 41 = death of B A C H.

By the time we arrive at this point we end up with just about every conceivable number that a normal piece of music of Bach's era could contain. By including Psalms, as they often do (and that not without justification), these scholars can immediately elaborate on any number between 1 and 150. Here we find the statistical probability of a match becoming altogether too certain, approaching 100%. If a Psalm does not suit, then there are Bach numbers, Biblical numbers, Cabalistic equivalents of Biblical words, Fibonacci numbers, Bach's friends' names, other composers' names—all of which can be reversed as necessity demands to denote the opposite. This is, at the same time, the essential premise and the major flaw in studies of Bach's numerology.

When Ruth Tatlow's book, *Bach and the Riddle of the Number Alphabet*, was published in 1991 it was widely touted as disproving the

whole world of Bach and numerology.⁸ In fact she only deals with the work of Smend—attacking him mercilessly at the start, but ending up quietly saying that he could be right. Her book is a most useful contribution to the discussion and certainly makes strong and convincing calls for us to behave ourselves academically by offering more proof and less speculation. Yet not only does she admit that this is difficult with a subject which has ambivalence and secrecy as its very objectives, she also paints a marvellous and well-researched picture of the culture of numerology and associated activity in Bach's day, proving not only its existence, but confirming the very widespread and secure place it held across a number of art forms including music. She thus establishes motive and opportunity, techniques and traditions, for Bach and his contemporaries to be active in numerology to the point where its absence rather than its presence might be seen as the more remarkable.

My own view is that at least one supporting proof in a non-numerological area, such as *Affektenlehre* or *Figurenlehre*, is needed to enhance credibility for numerological incidence. In an unpublished paper given at the Sydney Conservatorium, Brett Leighton cites aspects of numerology in the *Canonic Variations*:

The cantus firmus appears in 14 bars = B A C H ... The total number of bars in the first two canons is 41 = J S B A C H... Together the second and third movements contain 79 bars = T E L E M A N N ... The lengths of the middle three movements are 23, 56, and 27 bars respectively, totally 106 bars. 106 = S C H R O E T T E R ... The number of bars in the last two canons is 69 = 3 x 23 = C H G R A U N. The total number of bars in movements two, four, and five is 92 = L C M I Z L E R ... Canons I and V total 60 bars = G F H A E N D E L .

He then goes on to discuss the many instances of cryptography utilising the tones B-A-C-H. Clearly we must be careful here, but the fact that the names mentioned were members of Mizler's Society of Musical Sciences and that the work itself was completed as a requirement of membership of the Society goes a long way towards transforming possibility into probability. The fact that it also contains cryptography is added support.

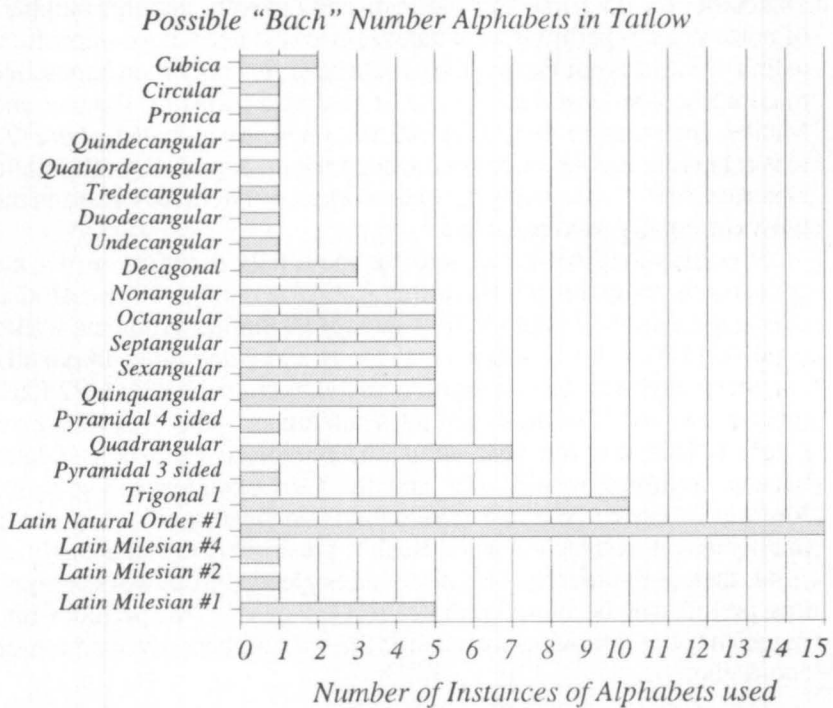
My own 1991 unpublished monograph on the organ *Passacaglia* (BWV 582) relates *Figurenlehre* and *Affektenlehre* with known Biblical number symbols as well as Cabala-alphabet numerology in a three-way—Figuren/ Affekten; Biblical Mathematics; Cabalistic Numerology—mutually-supportive hypothesis. Important *Figuren* of *Orgelbüchlein* are clearly present in the *Passacaglia*. These include imitation (obedience), 'Nun komm der Heiden Heiland' (Advent),

'Vom Himmel kam der Engel Schaar' (Christmas), 'Round World' motives, and a dramatic halt at the Neapolitan 6th chord near the end. (Variants of this harmony were often used by Bach as *Figur/Affekt* to interpret the word 'gekreuziget' or similar concepts in Cantatas, Passions and other works with text. The harmonies at the end of 'O Mensch, bewein' dein' Sünde groß' in *Orgelbüchlein* may also be instanced here.)

From the point of view of Biblical numerology, the *Passacaglia* consists of 10 reasonably strict passacaglia variations (suggesting commandments), 6 free variations which 'lose' the subject (suggesting wandering away from the law, 6 also having some 'imperfect' connotations here), 5 variations with Advent, Christmas and World *Figuren* (the 'Nun komm, der Heiden Heiland' motive is absolutely central, being the 16th of the 33 thematic appearances, in which 5 is the symbol for 'man'), and a double fugue where the 5 minor, 2 major and 5 minor fugal entries form a triptych representing the centrality of added new commandments amongst the re-affirmation of the old 'Law'. Joined to these two already complementary dimensions we have its supporting numerology: the *Passacaglia* theme intervals tally $43 = C R E D O$, having 7 crotchets, the 14th variation is the only one which puts in 'wayward' passing notes (thus Bach possibly encrypting a reference to himself as a sinner, or breaker of God's Laws in the 'wandering' section), there are 158 bars (J O H A N N S E B A S T I A N B A C H) from the beginning of the Advent section through to the Crucifixion chord and this notable Neapolitan harmony occurs at a multiple of 13 (death) bars, whether we measure it from the beginning of the passacaglia ($2 \times 11 \times 13 = 286$), or from the beginning of the fugue ($3 \times 3 \times 13 = 117$). Being exactly 293 bars long we note that the total of $2 + 9 + 3$ is again 14. The whole work might thus be interpreted as a representation of the Old and New Covenant, with Bach putting his personal identification in it, either as belief ($43/C R E D O$ in the opening theme), identification as sinner (14th variation), identification with Christ's life and ministry (158 bars) or again the total bar numbers (digits tallying 14).

However, Ruth Tatlow draws attention to the fact that there were many different Cabala alphabets available to Bach and that we need to show that he actually used the particular one chosen by Smend. She does not deal with writers such as Prautzsch, nor mention Bach's contemporaries like Pachelbel, who demonstrably used Trigonal alphabets which give totally different number-equivalents for most of their letters.⁹ This might have secured her case against Smend a little

more tightly, but what she does give is a most useful survey of nearly 50 number alphabets which were used and a record of their use, as the following table summarises:



I have excluded from this chart other alphabets given by Tatlow, such as Hebrew and Greek Milesian alphabets, since these involved non-European calligraphy. Also removed is any reference to alphabets which lack letters that Bach needed for his own language. Additionally, figures have been calculated without including the users of these alphabets, such as Tolstoy and Smend, who came substantially later than the era of Bach.

From this chart it is evident that the most commonly mentioned alphabet was that which Tatlow names 'Latin Natural Order #1', which is precisely the one chosen by Smend. There are many other reasons why the hand fits the glove here. For example, had Bach used any other alphabet the letters after 'K' would generally have had higher values. In particular 'S', instead of being 18, might have had a range from 60 through to 5,832 or even more.¹⁰ This would certainly have created problems with the number 29—which is so frequently found

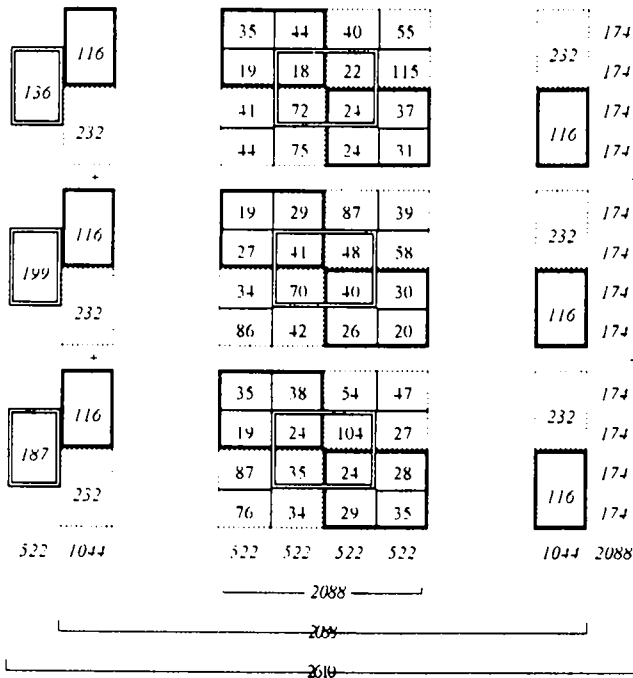
linked to the sum of either J + S + B (Johann Sebastian Bach) or S + D + G (*Soli deo Gloria*). In the Latin Milesian variant #5, 'Z' is given a value of 5000, which would have produced all manner of difficulties for Bach in words like Zion and Zebaoth, since the numbers of notes, bars, repetitions, or whatever else was used in a composition to link in with this number symbol would have generally been impossible to compose—at least until the era of extended forms by Wagner and Mahler, for example. In Cubica, 'Z' is 13,824. An alphabet where 'Z' was relatively low in value was needed for manageability. The Latin Natural Order #1 was really the only workable system for Bach and the most commonly used in the era.

Even this may not be enough for the totally sceptical, so let me advance the argument a little further using the number 29. Recall that 29 involves an 'S' in both S D G and J S B. Similar problems would result with 41 = J + S + B + A + C + H and many other frequently occurring numbers if the value of 'S' is too high. In the year 1722 Bach produced volume I of his 48 preludes and fugues—the *Well Tempered Clavier*. This was the year before he moved to Leipzig and later became involved with Mizler and the Correspondence Society of Musical Sciences. We can reasonably assume that, if we prove a mathematical connection with Bach's music in 1722 and again in 1747, then our numerological discoveries gleaned from works around this period will be more credible. Book I of the '48' preludes and fugues has the following scheme for the bar numbers of each prelude and fugue:

Prelude	1	35	Prelude	13	30
Fugue	1	27	Fugue	13	35
Prelude	2	38	Prelude	14	24
Fugue	2	31	Fugue	14	40
Prelude	3	104	Prelude	15	19
Fugue	3	55	Fugue	15	86
Prelude	4	39	Prelude	16	19
Fugue	4	115	Fugue	16	34
Prelude	5	35	Prelude	17	44
Fugue	5	27	Fugue	17	35
Prelude	6	26	Prelude	18	29
Fugue	6	44	Fugue	18	41
Prelude	7	70	Prelude	19	24
Fugue	7	37	Fugue	19	54
Prelude	8	40	Prelude	20	28
Fugue	8	87	Fugue	20	87
Prelude	9	24	Prelude	21	20
Fugue	9	29	Fugue	21	48

Prelude	10	41	Prelude	22	24
Fugue	10	42	Fugue	22	75
Prelude	11	18	Prelude	23	19
Fugue	11	72	Fugue	23	34
Prelude	12	22	Prelude	24	47
Fugue	12	58	Fugue	24	76

In a document from an anonymous source¹¹ these 48 numbers can be arranged so they form three magic squares according to the following plan (central block):



It is noteworthy, first, that the mathematical probability of these numbers occurring by chance is astronomically small, at least 1 in 48²¹, or 1 in 5 trillion trillion trillion and, second, that the construction of such a triple magic square is a relatively simple matter (it could be handled by a student doing Year 10 maths in the Australian school system).¹² What, then, would the probability be of all these numbers bearing a relationship to each other that coincides with 29, a well-known Bach number derived from Tatlow's Latin Natural Order alphabet, variant # 1?

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116	= 4 x 29	= 2 x 2 x 29	= 2 ² x 29
174	= 6 x 29	= 2 x 3 x 29	
232	= 8 x 29	= 2 x 2 x 2 x 29	= 2 ³ x 29
522	= 18 x 29	= 2 x 3 x 3 x 29	= 2 x 3 ² x 29
1044	= 36 x 29	= 2 x 2 x 3 x 3 x 29	= 2 ² x 2 ² x 29
2088	= 72 x 29	= 2 x 2 x 2 x 3 x 3 x 29	= 2 ³ x 3 ² x 29
2610	= 90 x 29		= 2 x 3 ² x 5 x 29
116 x 6	= 696	= 24 x 29	= 2 ³ x 3 x 29
232 x 6	= 1392	= 48 x 29	= 2 ⁴ x 3 x 29
174 x 12	= 2088	= 72 x 29	= 2 ³ x 3 ² x 29

In the light of this alone we can only conclude that the alphabet adopted by Smend was the right one,¹³ but further endorsement is possible. With the now unarguable certainty that the number 29 was of particular significance to Bach, we will return to the available number alphabets and eliminate any remaining which Bach would have found impractical.

From this we see that, for eighteenth-century Bach *gematria*, we will need all letters in use in the Germany of that epoch: A to Z and unmlauted vowels such as ä, ö, ü (noting however that the latter may always be represented as ae, oe, ue). Hebrew and Greek *gematria* can be excluded since Bach did not use their characters. The more the value of any individual letter exceeds 29 then the less likely it becomes that Bach would have found it possible to achieve the number 29—although clearly that number might be derived from a long word with many letters of small value. The most likely 29-word might be Bach's own name, but we can eliminate that, for all but the Latin Milesian, variant #6 (which has only consonants, no unmlauts, none possible, even by combining vowels, and B-C H totals 9) and the trigonal and other related series of alphabets give more than 29 for B A C H (H being in the range of 36 to 512). The remaining 10 give B A C H as 14. Furthermore none of the alphabets has the exact number 29 representing a letter at all. So we must conclude that we are involved with at least two letters to sum 29. Of these remaining 10 alphabets we find that the range of available letters with which we can sum the number 29 becomes restricted. The following table details this situation as found in the number-alphabets given by Tatlow in her Appendix 1:

GLOSSARY:

Alphabet: is the name given in Tatlow;

Letters available up to equivalent 29: gives an indication of the ease or difficulty with which a *quia numeris aequivalet* can be formed by showing the range of single letters of value less than 29 (no single letter

has a value of 29 in any of these systems);

Problems: identifies alphabets without all the necessary letters for Bach's language(s) and era;

Highest: shows the highest value letter as an indication of the difficulty of utilising it in a musical context where bar-numbers, numbers of notes in a chord, number of notes in a piece, number of pieces in a collection etc. are the most likely applications;

B A C H: shows the sum of the number-equivalents to the letters B + A + C + H;

S D G / J S B: shows the sums of S D G and J S B separately and identifies where these are 29 as well as where they differ and are therefore not paragrammatically linked.

Alphabet	Letters available up to equivalent 29	Problems	Highest	BACH	SDG JSB
Latin Milesian	A to L	No W	Z = 500	14	101 101
Latin Milesian #1	A to L		Z = 600	14	101 101
Latin Milesian #2	A to L		Z = 150	14	101 101
Latin Milesian #3	A to L	Tolstoy 1865-69	Z = 160	14	101 101
Latin Milesian #4	A to M		Z = 150	14	101 101
Latin Milesian #5	A to L	No W	Z = 5000	14	101 101
Latin Milesian #6	A to N	No Vowels	W = 160 Z = 100	9	68 69
Latin Natural Order	A to Z	No W	Z = 23	14	29 29
Latin Natural Order #1	A to Z		Z = 24	14	29 29
Latin Natural Order #2	A to Z	O = 0	Z = 23	14	28 29

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Alphabet	Letters available up to equivalent 29	Problems	Highest	BACH	SDG JSB
Latin Natural Order #3	A to Z	No K, W	Z = 22	14	28 28
Trigonal	A to G	No W	Z = 276 J = 300 β = 325	46	209 219
Trigonal #1	A to G		Z = 300 umlaut = 325 or more	46	209 219
Pyramidal	A to D		V = 3654	135	1244 1309
Quadrangular/ Zensica	A to E		Z = 576 umlaut = 625 or more	78	389 409
Pyramidal (4-sided)	A to C		V = 6930 umlaut = 5525 or more	224	2279 2399
Quinquangular/ pentagonal	A to D		Z = 852	108	569 599
Sexangular/ hexagonal	A to D		Z = 1128	142	749 789
Septangular/ heptagonal	A to C		Z = 1404	174	929 979
Octangular/ Octagonal	A to C		Z = 1680	1206	1109 1294
Nonangular/ enneagonal	A to C		Z = 1956	238	1289 1359
Decagonal	A to C		Z = 2232	270	1469 1549
Undecangular	A and B		Z = 2508	302	1649 1739
Duodecangular	A and B		Z = 2784	332	1829 1929

Alphabet	Letters available up to equivalent 29	Problems	Highest	BACH	SDG JSB
Tredecangular	A and B		Z = 3060 2119	366	2009
Quatuordecangular	A and B		Z = 3336	398	2189 2309
Quindecangular	A and B		Z = 3612	430	2369 2499
Pronica A to D			Z = 600	91	418 438
Circular A to E			V = 156 umlaut = 144 or more	61	156 156
Cubica A to C			V = 20683 umlaut = 15625 or more	548	6239 6569

The only reasonable conclusion we can possibly draw from this is that the Latin Natural Order #1 is the alphabet which gives us the conditions we seek, namely:

- all 24 letters A-Z available (the equivalence of I, J being common to all alphabets and U, V to all but 4 where V is a separate, extremely high value);
- all 24 letters available as potential summands of '29' ;
- highest values still workable, thus Z = 24 rather than 5,000;
- umlauts obtained by adding 'E' rather than unworkable high-values;
- no missing letters or zero-value letters;
- if previous assumptions are correct we look for B + A + C + H summing to 14;
- *gematria* available to Bach (not those first used later);
- if a paragram is possible, as it is with J + S + B = S + D + G = 29, then this will greatly strengthen the case for a particular scheme.

Apart from the proven strong incidence of Tatlow's Latin Natural Order #1 in the era of Bach (see Chart 1 above), we must now add that it is the only alphabet which fulfils all relevant conditions. Thus Smend, for whatever reasons he did so that have not come down to us, probably chose well when he selected what Tatlow describes as Latin Natural Order #1:

A=1	D=4	G=7	K=10	N=13	Q=16	T=19	X=22
B=2	E=5	H=8	L=11	O=14	R=17	UV=20	Y=23
C=3	F=6	IJ=9	M=12	P=15	S=18	W=21	Z=24

* * * * *

It cannot be doubted that Johann Sebastian Bach found mathematics a most fascinating and exciting tool, one which determined the course of his creative activity, intertwined itself in his professional life, expressed his faith and connections with the Bible, and fully validated his membership of Mizler's Correspondence Society of Musical Sciences.¹⁴ We find it hard to reconcile his son's comment about 'dry mathematical stuff' with our findings. Prautzsch makes a fine summary of it:

We encounter here a deep strata of Bach's music that ... concerned itself with the physical and numerical foundations of music its ... Ratios and Proportions, means and medium. Music is in this sense cosmology; in it is mirrored the Godly order of creation, from which numerical laws in intervals, harmonies, rhythms and forms are derived.

How far we can travel down the road of credible numerology in specific works must always be conjectural. But by the same token we cannot avoid acknowledging that it has full and critical importance in the *Figuren* of this music. We may also perceive these mathematical and numerological components in Bach's creativity as a final flourishing of music as part of the old *quadrivium*—where it was allied with astronomy, mathematics, and geometry in the seven liberal arts.

Notes

- 1 *Neu eröffnete Musikalische Bibliothek* - unchanged reprint of the original Leipzig edition 1739-54, Hilversum, 1966.
- 2 See *The New Harvard Dictionary of Music*, ed. Don Randel.
- 3 Paul Henry Lang, *George Frederic Handel*, London, 1967, p. 624.
- 4 A more expansive treatment of this subject is given in David Rumsey 'The Three Doctrines of the 1933 Methodist Hymn Book' in *The Sun of Our Choices* (McGill Studies in Religion), ed. Arvind Sharma, Atlanta, Georgia.
- 5 Albert Schweitzer, *J. S. Bach le musicien Poète*, Leipzig, 1905.
- 6 Johann Gottfried Walther, *Musicalisches Lexicon*, facsimile edition, Kassel, 1953.
- 7 Friedrich Smend, *J. S. Bach bei seinem Namen gerufen*, Bärenreiter, 1950.

- 8 Ruth Tatlow, *Bach and the Riddle of the Number Alphabet*, Cambridge, 1991.
- 9 Oddly enough she quotes *J. S. Bach bei seinem Namen gerufen* in the text, but does not mention it in the bibliography.
- 10 Brett Leighton is now at Linz Conservatorium, Austria.
- 11 Personal communication of the Melbourne organist John O'Donnell.
- 12 'Cubica' of Schwarzbach 1630, Harsdörfer 1651
- 13 The document reached the author in Vienna in about 1965, possibly originating with a Dutch collaborator of Smend.
- 14 My gratitude to Dr. Paul White, Australian Catholic University, for looking into these questions with me.
- 15 In a recent communication, John O'Donnell confirms that Book II consists of another magic square format with 29 once again as its number-focus.
- 16 Tatlow gives $H = 9$ for the Quincangular, but this is incorrect. It should read 90.
- 17 One notes that much of Smend's research in this connection took place during the Second World War and he suffered losses and disturbances to both his home and professional libraries as a result of bombing and associated activity (Tatlow).
- 18 See Kellner, 'Was Bach a Mathematician?', in *The English Harpsichord Magazine*, 2 (1978).
- 19 Ludwig Prautzsch, *Vor deinen Thron tret ich hiermit*, Hänssler, 1980.