Reply to H. M. Gastineau-Hills

David Rumsey

No musician of the eighteenth century could escape questions of tempering. In relation to Bach, his criticism of Gottfried Silbermann's organ tempering proves active involvement in the issues—presumably he had alternatives—and he is reported to have tuned his thirds 'sharp', suggesting his own system(s). Barnes (*Early Music*, April 1979) postulates a Bach tempering by statistical analysis of the *Well-Tempered Clavier*, producing a system which is unique and fulfills all known 'Bach' parameters. Barnes has had no significant refutation, the tempering works well (is now standard in most electronic tuners). Whilst this might not be the last word on the matter (indeed Bach must have used other temperings) it constitutes a fairly unassailable case for a unique Bach tempering.

The figure of 48^{21} was derived from there being 21 numbers which could be arbitrarily selected from 48 (the rest were all determined by the system). The sense in which 'perfect numbers' is reported in some German publications, to which I have had recourse in this study, show 'perfection' in this sense to be derived from the fact that both product and sum of 1, 2 and 3 yield 6. (This definition fails to work with the additional 'perfect numbers' given by Gastineau-Hills). The contention that 48 numbers should be arranged in some 'logical' order within the squares is a *non-sequitur*—why should they be? Gastineau-Hills appears to be saying that more than one order of these numbers is possible anyway. He should clarify whether tempering and *Affekt* are related or not—the latter could be difficult to sustain in today's musicological climate. His ninth paragraph could purport to be a statement of mine: it is not. It would be most useful to know the probability he alludes to here rather than the vague 'not very small' (surely an 'unchecked intuitive estimate' against which Gastineau-Hills himself later warns).

The likelihood of attaining results you want by either counting until you get concurrence, or having such a multiplicity of available numbers that you will get them anyway, and the dangers of reading too much into Fibonacci series, and all that Gastineau-Hills is so rightly dubious about, is already fully conceded in my article. Indeed it is a premise: only with additional proofs (relevant theology, affiliations, references by other writers, social practices, and so on) can numerological incidences even begin to be validated. The point at which they can then be deemed 'proven' is also conceded as arguable in my article.

As to mathematical comments, Gastineau-Hills mostly supports my case: magic squares have an improbability of random occurrence yet *the construction of them is relatively easy.* This is my central argument (else they would have been out of Bach's reach and patience).

The only remaining question is the probability of 48 random numbers forming the bar-counts of the *Well-Tempered Clavier* I. As I see it this will be calculated on the following:

1) the probability of 29 being the number on which the others are conditional;
2) the probability of any 48 random numbers conditioned by 29 being arrangeable in this way (which seems to be the too broad basis of Gastineau-Hills's argument);
3) the probability then that these numbers will fall within the range of keyboard Preludes and Fugues written in mid eighteenth-century Saxony—specifically of exactly matching the bar-counts of the *Well-Tempered Clavier* I;
4) (might we now also add?) the probability of a second set of Preludes and Fugues having 29 as basis to their number-structure.