Optimal Auctions of Procurement Contracts

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Abstract

We consider tenders/auctions for the procurement of items that do not exist at the time of the tender. The cost of production is subject to ex-post shocks, i.e. cost overruns, which cannot be contracted away or insured at the time of tender. The contractors may default due to the cost overruns once the project is underway. We consider a simple contract that specifies the payment in case of default and the award that is paid upon successful project completion. This contract is allocated at the tender and the award part is determined by competitive bidding. We characterize bidding behavior of contractors in standard tenders and derive the implications for the buyer's expected cost minimization.

1 Introduction

The volume of goods and services procured by the governments and the private sector is enormous. Government procurement alone represents 17.4% of GDP on average for OECD countries.[1] The means used for the procurement differ, but in all the cases cost savings is emphasized. When the item may be procured from a number of potential contractors and it is realistic to expect that those range in their costs of production, competitive bidding is recommended as a tender method for government procurement.[2] Developers and homeowners solicit quotes from several contractors when building or renovating houses. Schools choose catering service providers via similar tenders.

It may seem that procurement tender can be analyzed along the lines of a common auction, simply reversing the roles of buyers and sellers. The crucial distinction between the auction and the procurement tender is that the item being procured at the tender typically does not exist yet — a new building, a new type of aircraft, delivery of school lunches in the future. The production cost therefore cannot be known with high precision at the time of the tender and may well increase at the production

[1] From "Government at a Glance 2009," OECD, http://dx.doi.org/10.1787/724227300453.

[2] See i.e. the norms of Federal Acquisition Regulation in the USA, www.acquisition.gov/far/.

stage. Of course, the contractors preparing their bids factor in the possibility of such cost overruns.

In a situation of high uncertainty over the future cost of the project the use of cost plus contracts is advocated. They indeed perform well when the costs and cost overruns are verifiable, however in a realistic scenario the contractor knows a lot more than the buyer about the actual costs of the project.[3] Auctioning off cost plus contracts in such situation does not help much, such tender would select the contractor that is willing to accept the lowest profit margin, not the one that will deliver the product at the lowest cost.

We therefore concentrate on the tender where the fixed price contract is allocated. This form of the contract is most commonly used in the government defence procurement, see Fox (1974), McAfee and McMillan (1986), and in the public infrastructure procurement, see Bajari et.al (2008). With such contracts the buyer suffers from ex-post risks that are realized if the winning contractor defaults once the project is underway. In a typical auction setting such ex-post uncertainly does not affect the seller, however, in procurement setting this uncertainty is the norm.[4,5]

This paper introduces a few twists in the standard model of procurement via competitive bidding. The cost of the project is subject to the ex-post shock once the contractor makes a significant irrecoverable initial investment.[6,7] Such "significant investment" is normalized to be equal to the estimate of the production cost which is private information of the contractor. Thus our production technology is two-stage. In the common procurement setting contractor's i estimate of the production cost is identical to the total cost if contractor i is given the job. In our model it is the cost of the first stage of production. After the first stage is completed, the contractor makes another draw that now determines the cost of completion of the entire project. As we show, in equilibrium, the winner optimally completes the first stage but not necessarily the second. She may be unwilling to finish the project given the award that awaits and declare a default. Such event of default is verifiable and we allow the (otherwise fixed price determined at the tender) contract to specify the default

[3] Some cost components are, of course, observable and contractible: e.g., the costs of materials and of some labor and capital. However, a significant component of the costs (or cost savings) depend on the much less tangible managerial talent, the quality of internal organization and monitoring, and other concurrent projects of the contractor, etc.

[4] Systematic studies of cost overruns in public infrastructure projects report that in 9 out of 10 projects the cost will exceed the original quote with the average overrun at 28% of the winning bid (see Flyvbjerg et.al. (2002)).

[5] More than 80,000 contractors filed for bankruptcy leaving behind unfinished private and public construction projects with liabilities exceeding $21 billion in the United States during 1990-1997 (Dun & Bradstreet Business Failure Record).

[6] Ashley and Workman (1986) in a survey of contractors and buyers in USA building industry report that project engineering must be 40-60% complete to establish a reasonable estimate for the cost.

[7] This assumption distinguishes our approach from most of the literature that deals with ex-post uncertainty of the auctioned object, see Section 2.

payment.[8] In case of default the buyer captures the positive effect of the initial investment but has to complete the project at extra cost. The contract is allocated at the tender, we consider some common tenders, i.e. the lowest and the second-lowest bid tenders and general incentive compatible tender.

Importantly, our setting allows to reduce the problem of the expected cost minimization to the problem of finding the tender that minimizes the probability of default. In previous studies of procurement tenders such equivalence does not follow. We find that in our symmetric independent private values setting the expected cost equivalence does not follow. We show that (given the level of default payment) the lowest bid tender generates the lowest expected cost among all incentive compatible tenders when the distribution of the ex-post shocks is log-concave. Specifically, with such distribution given the level of default payment, the lowest bid tender minimizes the probability of the winner's default. With appropriately chosen default payment the lowest bid tender implements the optimal auction in our setting and reduces the probability of default to zero.

Our findings help to shed some light on the relative prevalence of the lowest-bid tenders in procurement. The sealed bid second price or open ascending auctions are commonly used to buy goods ready to use, but are virtually never used in procurement, see Carpineti et.al.(2006). In Italy, for example, the format of the procurement tender is regulated by law, and the law states that the winner of the tender can only be paid what he bids. Also in Italy between 1998 and 2006 the required tender mechanism was the average-bid tender, see more on such tender in Section 5. In 2006 a legislative reform allowed to choose between the average-bid and the lowest-bid tenders, see Decarolis (2009). Our results suggest an explanation for this reform.

The rest of the paper is organized as follows. Section 2 positions the paper among the literature. Section 3 outlines the model. Section 4 describes the equilibria in the lowest and the second lowest bid tenders. Section 5 deals with the expected cost minimization and contains our main results. Section 6 introduces the default payment and extends the main findings to the richer setting. Section 7 contains directions for future research.

2 Literature

In this Section we concentrate on the literature that deals with auction settings, $n \geq 2$ bidders and 1 buyer, ex-post shocks to the values and possibility of default.

The closest to our paper is Waehrer (1995). Waehrer and further Board (2007) consider an auction model where after the auction but before the settlement a verifiable shock to all the values is realized.[9] The winner can default on her bid and lose

[8] In contract law such payment is called liquidated damages, in procurement the corresponding term is surety bonds, see www.sio.org for more details.

[9] Contracts with auction prices contingent on the value of the shock are assumed out.

her bond. Further the winner and the seller may negotiate a new price. Several renegotiation regimes are considered. “Strong seller” in Waehrer (1995) or “full recovery” in Board (2007) are equivalent to our assumptions that i) the buyer fully captures the effect of the initial investment and ii) net of the bond the winner’s payoff is reduced to zero in case of default. Waehrer shows that the seller’s payoff is decreasing in the size of the bond, which is the exact opposite of our Proposition 3. Board (2007) shows that the seller prefers the second price auction to the first price auction, which contradicts our Propositions 1.

With higher bond the probability of default increases in equilibrium both in Waehrer’s model and this paper. In addition, Board (2007) shows that the probability of default is higher after the second price auction, just like in this paper. In our model, however, lower probability of default is good news for the buyer. Even though she extracts all the rent from the winner after a default, the buyer faces adaptation costs if the default occurs. In contrast, in Waehrer (1995) and Board (2007) lower probability of default is bad news for the seller, which in Board’s own account raises questions about the full recovery assumption in their model.

Chillemi and Mezzetti (2013) study the design of the optimal procurement tender in the setting where the contractor can quit after the cost overrun because of a more favourable outside option. The opportunity cost of this outside option is private information of the contractor, but it is determined by the same draw as the cost of working on the current project. Then the contractor who is the most efficient on the current project is also the most likely to default and pursue the outside option. It turns out that the optimal tender in such setting is a lottery—the buyer randomizes over the identity of the contractor for her project.

Rhodes-Kropf and Viswanathan (2000) consider a twist to Waehrer’s model where they allow the bidders to bid in securities whose value is derived from the future revenue of the firm being auctioned. They consider a number of financial instruments and show that in many cases non-cash auctions lead to higher expected revenue than cash auctions. Eso and White (2004) derive the equilibrium bidding strategies in Waehrer’s setting where the bidders are risk averse and their values are interdependent.

Parlane (2003) is an attempt to recast Waehrer’s model in procurement setting. The key difference between Parlane and this paper is the timing of events that lead to default and precisely what happens in case of a default. In our setting the cost overrun is realized after a substantial investment by the winning contractor.[10] In Parlane the shock hits after the winner is selected, *before* she makes any investment. Further, if the winners defaults, the buyer starts from scratch. Parlane provides partial analysis of the optimal procurement scheme, in her framework the actual value of the adaptation cost becomes important.

Our model is complementary to the one studied in auction setting by Zheng

[10]This reflects the common point in construction industry literature that the cost of a complex project cannot be known with good precision until the project is well underway.

(2001), and adopted to procurement by Calveras et.al. (2004), Burguet et.al. (2012) and Decarolis (2009). In their model the bidders face common cost of the project that is realized after the tender, the private information is about the amount of the assets they stand to lose in case of default. Both Zheng's and our models explain the "abnormally low tenders" that concern the regulators. In Zheng (2001) equilibrium the contractor with the lowest assets wins the tender and is the most likely to default. The equilibrium in this paper has a similar property, the contractor with the lowest private cost wins, but since she submits the lowest bid, the probability that the award induces her to cover the cost overrun is also the lowest.

In Spulber (1990) the potential amount of cost overrun is privately known to the contractors *before* the tender. When the contract is signed the cost overrun occurs exogenously. Several contract enforcement regimes are considered. Liquidated damages is the one equivalent to the surety bonds in this paper. In Spulber (1990) such penalty for default leads to a pooling equilibrium at the bidding stage and the buyer suffers from default whenever the cost overrun occurs. In contrast, in this paper the equilibrium in the bidding stage is separating, the winner later optimally absorbs some of the cost overruns, and the probability of the winner's default is endogenous and responds to the choice of the tender format and to the size of the surety bond.

The (binary) probability of default is given exogenously as part of the contractor's type in Chen et.al. (2010). The contractors bid is two dimensions, on the cost and the amount of the liquidated damages in case they default. They buyer uses a scoring rule to select the winner and pays the winner the cost part of her bid. Chen et.al. derive the class of scoring rules that lead to the efficient allocation and characterize the ones that minimize the buyer's expected cost.

Several papers deal with the model where the private information is two dimensional, both the value and the budget is known only to the bidder. Rhodes-Kropf and Viswanathan (2005) consider the model with ex-post shocks to the values and possibility of default. They bidders are allowed to access various forms of financial instruments to finance their bids: borrow at exogenous interest rate, issue equity or debt. The main finding is that even though access to financial markets allows to reduce the problem to a single-index auction, with competitive financial markets that auction is inefficient. In Birulin and Izmalkov (2013) the budget constraints are "hard". After the cost overrun the bidders may be unwilling, like in this paper, or unable to finish the project, when the budget is insufficient. The main finding is that the buyer improves her expected cost by splitting the tender award into ex-ante part payable after the tender with the balance payable upon completion of the project.

Laffont and Tirole (1986, 87) and McAfee and McMillan (1986) combine moral hazard with adverse selection. In their model the costs are subject to ex-post shocks that can be mitigated by the winner's higher effort. The buyer auctions off a menu of contracts. McAfee and McMillan consider n risk averse contractors that bid for a linear incentive contract that factors in both the cost (assumed observable ex-post) and the winning bid and derive the optimal contract in this class. Laffont and Tirole

(1986) with one agent and Laffont and Tirole (1987) with n risk neutral contractors derive the optimal selling procedure for an incentive contract. The optimal contract is shown to be linear in ex-post cost with the burden of the cost overrun shared between the winner and the buyer.[11] The winner of the tender in Laffont and Tirole (1987) optimally exerts the same amount of effort as the sole agent in Laffont and Tirole (1986) (and so moral hazard is separated from adverse selection), but enjoys lower informational rent.

The contracts in these papers combine the elements of the fixed price and cost plus contracts. Such contracts are infeasible in our setting as both the original estimate and the final value of the cost remain the private information of the winner.[12] In addition in our model the contractors are protected by limited liability therefore cost sharing schemes even when feasible provide limited incentives.

Piccione and Tan (1996) consider a model in which ex-ante symmetric potential contractors invest in the cost reduction technology and then compete for the procurement contract. The cost can be further reduced by exerting effort with no exogenous shocks. If the buyer is able to commit to the procurement mechanism before the investment stage the first best solution can be implemented by either lowest or second lowest bid tender under quite general conditions. If the buyer chooses the mechanism after the initial investment stage the level of investment is suboptimal. Arozamena and Cantillon (2004) consider a similar model, however, the contractors are ex-ante heterogeneous, investment is only made by one of them and the level of investment is observable which brings asymmetry to the competition. If investment affects which contractor is the most efficient then the lowest bid tender will induce less investment than the second lowest bid tender.

In the setting where the total cost of the project depends on the privately known type (without cost overrun) and on the duration of works Lewis and Bajari (2011a) analyze the tender where the contractors bid on the cost and the number of days to complete. They show that such scoring rule improves expected welfare relatively to the tender where the bids are only on the cost. Asker and Cantillon (2008) in similar setting show that scoring auctions dominate price-only auctions, beauty contests, and menu auctions. Asker and Cantillon (2010) further characterizes the optimal procurement mechanism where both the fixed and the marginal costs of production are private information of the seller. Lewis and Bajari (2011b) deal with the problem where adaptation after cost overrun does not require renegotiation and can be done solely by the contractor. They study efficient contract design where the incentives are provided by deadlines and penalties for breach.

[11] Che (1993) shows that a scoring auction with a scoring rule linear in price implements the optimal scheme.

[12] See also Tirole (1986) on why the cost may be unverifiable even after completion.

3 The Model

The buyer, e.g. a homeowner or a government agency, wants to realize a project that once completed it values at V. We assume V high enough so that the buyer always procures the project. There are N contractors/firms that have a capacity to complete the project. The buyer conducts a tender and selects the winner. A winning contractor has to make an initial irrecoverable investment representing all the capital, labor, and managerial resources necessary for completion of the project, and also faces an ex-post risk of a cost overrun that cannot be resolved at the time of the initial investment. The cost of the initial investment for contractor i, c_i, is his private information. For every contractor c_i is a random draw from $[0, C]$ with smooth c.d.f. F, that is we consider an independent private value setting. In practice some cost components of procurement projects are observable and contractible: e.g., the costs of the materials and of some labor and capital. However, a significant component of the costs (or cost savings) can be attributed to the managerial talent, the quality of internal organization and monitoring, and other concurrent projects of the contractor, etc. These are not directly observable or contractible and c_i is assumed to be their summary statistics.

The contractor i that wins the tender may make an initial investment of c_i in the project.[13] Once she makes the initial investment she faces ex-post risk.[14] Before completion, but after the initial investment of c_i, the winner may have to incur extra cost z. This z is a random variable that represents the cost overrun, it is assumed to be distributed with smooth c.d.f. F_z on $[0, Z]$ possibly with a mass point of $F_z(0)$ at 0. The cost overruns can arise due to a management oversight, an adverse shock to input costs, or some other unforeseen contingencies. It is essential that the cost overrun is realized only after significant investment, c_i, into the project.[15] We assume that these overruns are also not verifiable and so cannot be contracted upon and have the same effect on the cost of any potential contractor.[16] The same symmetry with respect to

[13] In case she does not make such investment her payoff is normalized to 0.

[14] As we show, in equilibrium the winner expects a positive payoff from the continuation of the game therefore he has incentives to invest c_i.

[15] Our model is similar in that respect to Riordan and Sappington (1986), Tirole (1986), and Chillemi and Mezzetti (2013) but differs from Spulber (1990), Waehrer (1995), Zheng (2001), Parlane (2003), Board (2007) and Burhuet et.al. (2012). Riordan and Sappington (1986) consider the tender for monopoly franchise, where the bidders observe their marginal cost, their private information, only after they invest the fixed cost, assumed to be the same for all firms. In our setting the initial required investment is private information but the cost overrun is common. Tirole (1986) studies the contracting problem where the firm chooses the initial investment that later leads to cost reduction. In contrast here the required initial investment is determined by the production technology but is only known only to the contractor.

[16] Before the construction starts the actual cost of the project is unknown to any contractor, and they will in general not agree with their estimates of the cost. However, once the preparatory work has been done the degree of assymetry of the information "subsides". The project becomes one of the "standard" types. It remains true that the professional contractor has better information than the buyer, but the professionals agree among themselves on the cost estimates once they recognize

cost overrun is assumed both in the classic papers of McAfee and McMillan (1986) and Laffont and Tirole (1987), and in the most recent Burguet et.al. (2012) and Chillemi and Mezzetti (2013). The event that the project is completed is verifiable. The award is paid only after successful completion.[17]

The contractor may decide not to cover these extra costs in which case he is not able to complete the project. The event of the contractor's default is of course verifiable and the contract specifies the payment S in case of default. This S is the level of liquidated damages, see i.e. Waehrer (1995) or surety bonds, see i.e. Calveras et.al. (2004). The two are equivalent in our formulation. The value of S is ex-ante set by the buyer or an external party, e.g. industry regulator or code of practice and is known to the contractors at the time of the bidding. If the contractor quits after the initial investment we assume that the buyer captures the positive effect of the investment. We also assume that if the contractor quits, the buyer is not obliged to pay the award. These may be extreme assumptions favouring the buyer.[18] In practice, in construction industry after the disputes that result from cost overruns, the contractor receives partial compensation for the work done, however, the regulators that determine this compensation quite often side with the buyer. Effectively, in most circumstances it is expected that the cost overrun is anticipated by the contractor and factored in the original bid at the tender.

After the original contractor quits if the buyer decides to complete the project she invites another contractor, j. This new contractor examines the state of the project and provides the buyer with the cost estimate of $z + a_j$. Variable $a \geq 0$ stands for the extra direct costs that result from the original contractor being replaced with a new one. They may include adaptation costs, the set up costs, etc. For the buyer the realization of a_j is a random variable, independent from z and c. The buyer may run a new tender, selecting the contractor with the lowest a_j, or she can merely choose a replacement professional from the pool. Importantly, it is assumed that the buyer does not renegotiate with the original contractor, despite $a \geq 0$.[19,20]

We consider a number of policies: modifications of the tender rules, varying the

the standard type.

[17]Birulin and Izmalkov (2013) consider a similar setting where the award can be split into the part that is paid ex-ante and the rest paid ex-post. In the setting considered here it is *optimal* to pay the entire award ex-post.

[18]The equivalent of these assumptions is standard in the literature on auctions with ex-post shocks and limited liability, see Rhodes-Kropf and Viswanathan (2000) and (2005), Zheng (2001), DeMarzo et.al. (2005), "strong seller" in Waehrer (1995), "full recovery" in Board (2007).

[19]What we try to capture with our stylized model is an extensive relationship with the contractor where many cost overruns are possible down the track. If the buyer demonstrates she is open to renegotiation it effectively turns the fixed price contract into a cost plus one which changes the underlying bidding game. In our environment with unverifiable costs the buyer is probably better off not renegotiating and facing potential adaptation costs, however, we do not consider renegotiation explicitly.

[20]In many settings where the buyer is a government agency the renegotiated is restricted or prohibited by law, see Gil and Oudot (2009).

amount of default payment, all of which are determined ex-ante. Since the buyer commits to a particular policy she is only concerned with $\mathsf{E}\min\{a\} \equiv \alpha > 0$. This α may also include the costs that affect the buyer but are not payable to the new contractor, i.e. the cost of delays, the cost of running a new tender, negative emotional effects, etc. The exact value of α is unimportant for our analysis, only the fact that $\alpha > 0$. As we show in our model the minimization of the total expected cost of the buyer is equivalent to minimization of the probability that the original contractor quits the project. Regardless of the particulars of the model the latter negatively impacts the buyer and the policies that avert such events or decrease their chances are important to consider.

The timing of the game can be represented as follows.

1. The buyer announces that she wants to procure the project and declares the rules of the tender and associated policies. Each contractor i learns c_i.

2. The tender is being conducted, the contractors submit bids, the buyer selects the winner, say j, the size of the award is determined.

3. The initial investment of c_j is undertaken by the winner.

4. The shock—the value of the extra investment required—is realized. If $z = 0$ the project is completed and the award is paid. If $z > 0$, the winner either

   (a) makes extra investment z and receives the award, the ex-post payment prescribed for the event "the project is completed" or

   (b) abandons the project in which case the ex-post payments prescribed for such event are executed and the buyer pays $z + a$ to the third party to complete the project.

The details of the particular policies considered are given in the corresponding Sections. Finally, the objective of the buyer is to minimize the expected cost of the project, inclusive or all the payments the buyer faces. The objective of each contractor is to maximize its profits. Both the buyer and the contractors are risk-neutral and have the same value for money.

4 Common Tenders

In this Section for simplicity of the exposition we assume that F_z is continuous on $[0, \infty)$ and has no mass point at 0. We also concentrate the two common tender formats, the lowest and the second lowest bid tenders. Section 3 treats general allocation mechanisms and general distribution of the shocks F_z.

4.1 The Lowest Bid Tender

Assume that the award is determined at the lowest bid tender, where all N contractors submit sealed bids, the lowest bidder wins and the award is equal to the winner's bid. This award is paid only after the cost overruns are covered and the project is completed. Assume there exists a symmetric equilibrium $b = b(c)$ with increasing $b(c)$ and consider bidder i with cost c, who pretends to have cost x and bids $b(x)$. At the bidding stage the contractor, naturally, anticipates further shocks to the cost of the project. The winner will optimally cover any cost overrun $z \leq b$ and receive the payoff $b - z$ in such event. Therefore after winning with bid b and investing c the contractor expects the continuation payoff of $\Phi(b) = \int_0^b [b - z]\, dF_z$. Integrating by parts note that

$$\Phi(b) = \int_0^b [b - z]\, dF_z = bF_z(b) - \int_0^b z dF_z = \int_0^b F_z(x)\, dx. \tag{1}$$

Hence, the continuation payoff of a contractor that wins with bid b is $\Phi(b) = \int_0^b F_z(x)\, dx$, strictly increasing and continuous with $\Phi'(b) = F_z(b)$. Consider $\Phi(b(x))$, the continuation payoff that a contractor with true cost c expects to receive if he wins the tender with bid $b(x)$, which happens with probability $(1 - F(x))^{N-1}$, and invests c. Expected payoff of such contractor is then

$$\Pi^I(c, x) = (\Phi(b(x)) - c)(1 - F(x))^{N-1}. \tag{2}$$

By the standard argument in a symmetric equilibrium each contractor maximizes her expected payoff (2) with bid $b^{(1)}(c)$ that solves

$$\Phi\left(b^{(1)}(c)\right) = \mathsf{E}\left[Y^1 | Y^1 > c\right]. \tag{3}$$

Note that upon winning the tender the contractor with cost c expects positive continuation payoff from the investment of c, hence she makes such investment in equilibrium.

4.2 The Second Lowest Bid Tender

In the second lowest bid tender if bidder i with cost c bids b the lowest bid of the others $\beta_1 \geq b$ determines whether the contractor will cover the cost overrun in case $z \leq \beta_1$ or quit the project. Upon winning and after investing c bidder i expects the continuation payoff of

$$\Phi(\beta_1) = \int_0^{\beta_1} z dF_z$$

Since $\beta_1 \geq b$, and $\Phi(\beta_1)$ is increasing in β_1 with $\Phi(0) = 0$ contractor i with cost c maximizes her expected payoff with bid b that solves $\Phi(b(c)) - c = 0$, hence optimally chooses b such that

$$\Phi\left(b^{(2)}(c)\right) = c. \tag{4}$$

The optimal bid ensures zero profit if bidder i happens to win at her *own* bid. Note that will full commitment the optimal bid is $b(c) = c + \mathsf{E}z$. With limited commitment the bids are below this full commitment level, since $b^{(2)}(0) = 0$ and $\Phi'(b) = F_z(b)$ is increasing in b, see Figure 1.

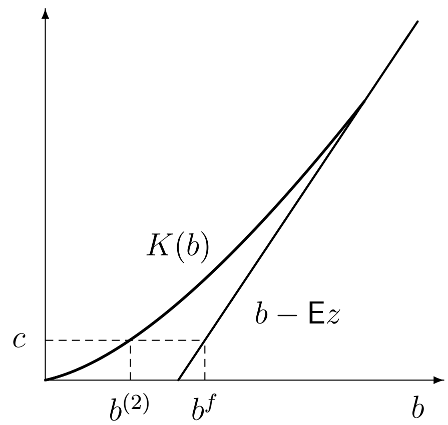

Figure 1: Bidding strategy with limited and full liability.

With $b < Z$ the contractor optimally expects not to always bear the cost overrun and accordingly shades her bid relative to the one with full commitment, b^f in Figure 1. The winner, the contractor with the lowest c, shades her bid the most. Similar result holds in the lowest bid tender and, more generally, in any incentive compatible tender. This may explain the abnormally low, below the expected cost, bids at the procurement tenders that quite concern the regulators.

4.3 Expected Cost Comparison

Consider the buyer who runs either the lowest or the second lowest bid tender. Such tender selects the winner and determines the award, b that the winner will receive upon the completion of the project. Denote with $G^{(A)}(b)$ the distribution of the award in tender A. $G^{(1)}(b)$ in the lowest bid tender is linked to the distribution of the lowest cost, c_1, by (3) and $G^{(2)}(b)$ in the second lowest bid tender is linked to the distribution of the second lowest cost, c_2, by (4). After tender A given the equilibrium behaviour of the winner the ex-ante expected cost of the buyer

$$\mathsf{E}^A C = \int_0^\infty \left[bF_z(b) + \int_b^\infty (z + \alpha)\, dF_z \right] dG^{(A)}(b).$$

In this expression the first term corresponds to the event $z \leq b$ where the winning contractor covers the cost overrun, finishes the project by herself and receives b. The second term corresponds to the event $z > b$, where the winner optimally quits after investing c, and receives no payment. The buyer replaces the original contractor with another one who faces the same cost overrun z. Extra cost $\alpha > 0$ represents the ex-ante expectation of the adaptation costs as discussed in the description of the model.

Adding and subtracting z in every realization (b, z) rewrite the expected cost as

$$\mathsf{E}^A C = \mathsf{E}z + \int_0^\infty \left[bF_z(b) - \int_0^b z dF_z + \alpha \int_b^\infty dF_z \right] dG^{(A)}(b).$$

With the above definition (1) of $\Phi(b)$ such expected cost of the buyer simplifies to

$$\mathsf{E}^A C = \mathsf{E}z + \int_0^\infty \left[\Phi(b) + \alpha \Pr[z \geq b] \right] dG^{(A)}(b).$$

In the lowest bid tender the winner's bid satisfies $\Phi\left(b^{(1)}(c_1)\right) = \mathsf{E}\left[Y^1 | Y^1 > c_1\right]$, and by the law of repeated expectations $\int_0^\infty \Phi\left(b^{(1)}\right) dG^{(1)}(b) = \mathsf{E}_{c_1} \mathsf{E}\left[Y^1 | Y^1 > c_1\right] = \mathsf{E}c_2$. Similarly in the second lowest bid tender $\Phi\left(b^{(2)}(c_2)\right) = c_2$, and $\int_0^\infty \Phi\left(b^{(2)}\right) dG^{(2)}(b) = \mathsf{E}c_2$. Hence in both the lowest and the second lowest bid tenders the expected cost reduces to

$$\mathsf{E}^A C = \mathsf{E}z + \mathsf{E}c_2 + \alpha \int_0^\infty \Pr[z \geq b] \, dG^{(A)}(b).$$

We now concentrate on the last term in the above expression. Recall that Φ is strictly increasing and introduce variable $t = \Phi(b)$. Now consider random variable t^1 that is function Φ transformation of the *equilibrium award* in the lowest bid tender. Denote with $\widehat{G^1}(t^1)$ the c.d.f. of such t^1. Also consider t^2—the Φ transformation of the *equilibrium award* in the second lowest bid tender, and the corresponding $\widehat{G^2}(t^2)$. By the standard argument (see i.e. Krishna (2002), p.23) one can show that $\widehat{G^2}(t^2)$ is a *mean preserving spread* of $\widehat{G^1}(t^1)$.

Using $b = \Phi^{-1}(t)$ observe that the probability of the winner's *solvency* conditional on the equilibrium award b

$$\Pr[z \leq b] = \Pr\left[z \leq \Phi^{-1}(t)\right] = F_z\left(\Phi^{-1}(t)\right)$$

Introduce $U(t) = F_z\left(\Phi^{-1}(t)\right)$. From the buyer's perspective the ex-ante expected probability of the winner's solvency

$$\int \Pr[z \leq b] \, dG^{(j)}(b) = \int U(t) \, d\widehat{G^j}(t).$$

If $\widehat{G^2}(t^2)$ is a mean preserving spread of $\widehat{G^1}(t^1)$ and $U(t)$ is concave then $\int U(t) \, d\widehat{G^2}(t) \leq \int U(t) \, d\widehat{G^1}(t)$. If $U(t)$ is strictly concave at some b the inequality is strict. That is if $U(t)$ is concave the buyer's expected cost is higher after the second lowest bid tender.

Similarly if $U(t)$ is convex the expected cost is higher after the lowest bid tender. Since $b = \Phi^{-1}(t)$ with $\Phi'(b) = F_z(b)$

$$\frac{\partial F_z\left(\Phi^{-1}(t)\right)}{\partial t} = \frac{f_z\left(\Phi^{-1}(t)\right)}{\Phi'(b)} = \frac{f_z(b)}{F_z(b)} \equiv \sigma_z(b)$$

Thus if the reverse hazard rate $\sigma_z(b)$ is decreasing then $U(t)$ is concave, if $\sigma_z(b)$ is increasing then $U(t)$ is convex. Since $\sigma_z(x) = \dfrac{d}{dx}\ln F_z(x)$, for any F_z that is *log-concave* the reverse hazard rate $\sigma_z(b)$ is decreasing.

Proposition 1 *Suppose F_z is log-concave. Then the probability of default is lower after the lowest bid tender and the buyer prefers the lowest bid procurement tender to the second lowest bid tender.*

It is instructive to compare the above Proposition to a classic revenue ranking result with risk averse bidders, Holt (1980).[21] A risk averse contractor in the lowest bid tender with full commitment maximizes $H[b-c]\cdot\Pr[win|b]$, where H is concave. It follows that risk aversion makes contractors bid more agressively, that is bid lower, in the lowest bid tender, whereas their bidding behaviour in the second lowest bid tender is unaffected. Since the buyer minimizes the expected cost he prefers the lowest bid tender. Proposition 1 shows an identical result but the driving forces are quite different. In fact, a naive application of Holt's result would lead to the incorrect ranking of the two tenders. Since limited liability restricts the downward loss of the contractor, his "utility" H is convex, not concave, so he is a risk lower. Reinterpreting the logic of Holt's argument, a risk loving contractor bids more in the lowest bid tender, so the buyer prefers the second lowest bid tender in contrast to what Proposition 1 states.

Our model differs from Holt's on two counts. First, in this paper in the lowest bid tender the contractor maximizes $[\Phi(b)-c]\cdot\Pr[win|b]$, so in contrast to Holt, the objective function is quasi linear in the true type. Second, the functional form of Φ in our model arises endogenously. The result of Proposition 1, albeit similar to Holt's is driven by entirely different principles. Both in the lowest and the second lowest bid tenders limited liability changes the bidding behavior but the buyer's dealing with the original winning contractor results in the same expected cost in both tender formats. The difference in the expected cost stems from the need to invoke a replacement after the original contractor defaults and the probability of that is higher in the second lowest bid tender since the outcome of this tender is more random from the winning contractor's perspective, as Proposition 2 further elaborates.

More recently, Board (2007) compares the first and the second price auctions in the setting where the bidder may default upon learning the value of the item after the auction. Several possible continuations are considered depending on what

[21] Incidentally Holt's original result is also in the context of procurement tenders.

the seller "captures" after the winner's default. In our procurement approach the buyer does not pay after the default and also fully captures the effects of the initial investment of c_i. It is hard to provide a realistic scenario after which the buyer capures more surplus. In the corresponding "full recovery" continuation in Board (2007) the seller captures the winning bidder's value after the default. Then it is shown that the expected revenue *increases* in default probability which makes the second price auction revenue superior.

5 Optimal Mechanism

In this Section we consider more general class of the distributions of the shocks. We still restrict attention to full supports and deal with the distributions that are smooth in the interior of the support but we allowe for mass point at 0 (which corresponds to the case of no cost overrun) and deal with arbitrary Z, the upper bound of the shock distribution support. We also consider general allocation mechanisms. From now on our mechanism is an allocation rule is a vector $q_i(c_i, \mathbf{c}_{-i})$ for each i that satisfies $\sum_i q_i(c_i, \mathbf{c}_{-i}) = 1$. The allocation rule determines which of the contractors will be doing the works. The allocation rule in itself may be a probability distribution given the realized profile of the costs but it eventually selects one contractor, who is called the winner. Extending the conventional allocative mechanism there are three types of transfers for each i that correspond to the only three verifiable events: i) contractor i loses the tender, ii) contractor i wins the tender but fails to complete the project, he then pays S to the buyer, and iii) winner i completes the project, he then receives b from the buyer. We return to the optimal transfers for the losers later. For now we concentrate on the incentives that S and b provide for the winner.

Suppose contractor i has been selected at the tender and the resulting contingent transfers are $b_i(\mathbf{c})$ and $S_i(\mathbf{c})$. We further write simply b and S where this creates no confusion. For given z the winner's optimal *continuation payoff* is $\phi(b, S) = \max\{b - z, -S\}$. It is instructive to write this as

$$\phi(b, S) = (b - z) \cdot I\{z \le b + S\} - S \cdot I\{z > b + S\}, \tag{5}$$

where I is the indicator function. Consider also the expected continuation payoff $\Phi(b, s) = \mathsf{E}_z \phi(b, S)$. If $b + S \ge Z$ the contractor will finish the project regardless of the cost overruns so that $\Phi(b, S) = b - \mathsf{E}z$. If $b + S < Z$ the winner will finish only if $z \le b + S$ and receive $b - z$ in these realizations, he will optimally default and pay S if $z > b + S$. Then $\Phi(b, S) = bF_z(0) + \int_0^{b+S}(b - z)\, dF_z - S\int_{b+S}^{Z} dF_z$. Observe that $\Phi(b, S) = (b + S) F_z(b + S) - \int_0^{b+S} z dF_z - S = \int_0^{b+S} F_z(x)\, dx - S$, after the integration by parts. Therefore

$$\Phi(b, S) = \begin{cases} \int_0^{b+S} F_z(x)\, dx - S & \text{if } b + S < Z \\ b - \mathsf{E}z & \text{if } b + S \ge Z \end{cases}. \tag{6}$$

Note that for $b + S < Z$

$$\partial \frac{\Phi(b,S)}{\partial b} = F_z(b+S) > 0 \text{ and } \partial \frac{\Phi(b,S)}{\partial S} = F_z(b+S) - 1 \leq 0, \tag{7}$$

so that $\Phi(b,S)$ is increasing in b and decreasing in S. Moreover $\Phi(b,S)$ is continuously differentiable (in both b and S) at $b + S = Z$. Note also that $\Phi(0,S) = \int_0^S (F_z(x) - 1)\, dx < 0$ for $S < Z$ and $\Phi(0,S) = -\mathsf{E}z$ for $S \geq Z$.

Now turn to the implications of the above S and b for the buyer's expected cost $\mathsf{E}^W C$. If after the tender, winner i behaves optimally in the continuation the expected cost of the buyer conditional on i winning the tender can be written as follows

$$\mathsf{E}^W C|i = \mathsf{E}\left[b_i \cdot I\{z \leq b_i + S_i\}\right] + \mathsf{E}\left[(z + \alpha - S_i) \cdot I\{z > b_i + S_i\}\right].$$

The expectation in $\mathsf{E}^W C|i$ is with respect to the distribution of the costs $\mathbf{c}$ and shocks z and functions $b_i(\mathbf{c})$ and $S_i(\mathbf{c})$ are, of course, endogenous to the choice of the allocative mechanism. The first term in $\mathsf{E}^W C|i$ corresponds to the event $b_i + S_i \geq z$, where winner i covers the cost overrun, finishes the project by herself and receives b_i. The second term corresponds to $z > b_i + S_i$, where i defaults, pays S_i to the buyer, who then faces cost $z + \alpha$ of finishing the project. Adding and subtracting z in every realization $\mathsf{E}^W C|i$ can be written as

$$\mathsf{E}^W C|i = \mathsf{E}z + \mathsf{E}\left[(b_i - z) \cdot I\{z \leq b_i + S_i\}\right] + \mathsf{E}\left[(\alpha - S_i) \cdot I\{z > b_i + S_i\}\right].$$

Now using the definition of $\phi(b,S)$ in (5) and $\Phi(b,s) = \mathsf{E}_z\phi(b,S)$

$$\mathsf{E}^W C|i = \mathsf{E}z + \mathsf{E}\phi(b_i, S_i) + \alpha\mathsf{E}\left[\Pr\left[z \geq (b_i + S_i)\right]\right]. \tag{8}$$

Thus, in our formulation the expected cost of the buyer and the continuation payoff of the winner (6) are related in a particular way. Importantly, the relationship between them resembles that in the classic auction for the good (in i.p.v. setting with no ex-post shocks) with $\phi(b,S)$ playing the role of the payment that the winning bidder makes to the seller. In comparison to such classic auctions the above expected cost has an extra term that captures the extra costs that the buyer faces *after* the tender if the cost overruns prompt the winner to default on the project. We already know from Proposition 1 that due to that term the tender format matters for the expected cost even in a symmetric i.p.v. setting. It is important, however, that $\mathsf{E}^W C|i$ has an additive separable structure, which allows to tackle its terms in isolation.

As with classic auctions the need to respect the contractor's incentive compatibility and participation constraints has major implications on the optimal tender format. Consider an incentive compatible tender A. Consider the truth-telling equilibrium in the direct mechanism that corresponds to A. Denote with $q_i^A(c_i, \mathbf{c}_{-i})$ for all i the allocation rule in such tender and with $b_i^A(c_i, \mathbf{c}_{-i})$ and $S_i^A(c_i, \mathbf{c}_{-i})$

the transfers defined as above, that is $b_i^A(c_i, \mathbf{c}_{-i})$ is the transfer that i receives after tender A if the realized profile of the costs is $(c_i, \mathbf{c}_{-i})$ and i successfully finishes the project and $S_i^A(c_i, \mathbf{c}_{-i})$ is what he pays if he defaults. Suppose contractor i with cost c_i instead reports x. Then her expected payoff is $\Pi_i^A(c_i, x) = \int_{\mathbf{C}_{-i}} \mathsf{E}_z\left[\left(\phi\left(b_i^A(x, \mathbf{c}_{-i}), S_i^A(x, \mathbf{c}_{-i})\right) - c_i\right) \cdot q_i^A(x, \mathbf{c}_{-i})\right] dF(\mathbf{c}_{-i})$. Since this expected payoff is quasi-linear in *true type* c_i by the standard argument incentive compatibility alone implies that the expected equilibrium payoff

$$\Pi_i^A(c_i, c_i) = \max_{x \in \mathcal{C}_i} \int_{\mathbf{C}_{-i}} \mathsf{E}_z\left[\left(\phi\left(b_i^A(x, \mathbf{c}_{-i}), S_i^A(x, \mathbf{c}_{-i})\right) - c_i\right) \cdot q_i^A(x, \mathbf{c}_{-i})\right] dF(\mathbf{c}_{-i}) \tag{9}$$

is decreasing and concave in c_i. Incentive compatibility also implies that

$$\Pi_i^A(c_i, c_i) = \Pi_i^A(C) + \int_{c_i}^{C} Q_i^A(x)\, dx, \tag{10}$$

where C is the worst possible type and $Q_i^A(c_i) = \int_{\mathbf{C}_{-i}} q_i^A(c_i, \mathbf{c}_{-i})\, dF(\mathbf{c}_{-i})$. The above result is familiar interim payoff equivalence, see i.e. Krishna (2002). Since $\Pi_i^A(c_i, c_i)$ is concave and $Q_i^A(c_i) = -\partial \Pi_i^A(c_i, c_i) / \partial c_i$ at every point where the derivative exists the allocation $Q_i^A(c_i)$ is decreasing in c_i, i.e. the contractor with the lowest cost wins in any incentive compatible tender, which results in efficient allocation in our model. If hypothetically, $S = 0$ then incentive compatibility also implies that $b_i^A(c_i, \mathbf{c}_{-i})$ is increasing in c_i, hence the contractor with the lowest cost also has the highest probability of default. The expected cost considerations then require a distortion of the allocation rule away from efficiency as in Burguet et.al. (2012). Accounting for optimal S, however, as we further show alleviates the tension between the expected cost (revenue) and efficiency considerations.

Consider all the tenders in the above class that share the same allocation rule $q_i^A(\mathbf{c})$ for all i and that have the same $\Pi_i^A(C)$, where C is the highest possible cost. In any such tender in equilibrium $\Pi_i^A(c_i, c_i)$ is the same. The expected cost of the buyer in a mechanism with allocation rule $q_i^A(\mathbf{c})$ for all i satisfies

$$\mathsf{E}^A C = \mathsf{E}z + \int_{\mathbf{C}} \sum_i q_i^A(\mathbf{c})\, \mathsf{E}_z \phi\left(b_i^A(\mathbf{c}), S_i^A(\mathbf{c})\right) dF(\mathbf{c}) + \tag{11}$$

$$\alpha \int_{\mathbf{C}} \sum_i q_i^A(\mathbf{c}) \left[1 - F_z\left(\left(b_i^A(\mathbf{c}) + S_i^A(\mathbf{c})\right) \wedge Z\right)\right] dF(\mathbf{c}) \tag{12}$$

In (12) $F_z\left(\left(b_i^A(\mathbf{c}) + S_i^A(\mathbf{c})\right) \wedge Z\right)$ is (capped from above by 1) probability of contractor i *not defaulting* given the realized $b_i^A(\mathbf{c}) + S_i^A(\mathbf{c})$. The three terms in $\mathsf{E}^A C$ mimic those in $\mathsf{E}^W C$ given by (8). The first term on (11) is the expected value of the cost overrun, it is invariant to the choice of the tender and the buyer expects to pay it eventually. The second term is familiar from Myerson (1981) optimal auction. It represents the expected cost of the first stage of the project together with the informational rents left to the contractors at the tender stage. With full commitment

(11) would be the entire expected cost of the buyer. Starting from the interim payoff equivalence, using (10) together with (9) and performing the usual change in the variables of integration (11) net of $\mathsf{E}z$ transforms into

$$\int_{\mathbf{C}} \sum_i q_i^A(\mathbf{c})\, \mathsf{E}_z \phi\left(b_i^A(\mathbf{c}), S_i^A(\mathbf{c})\right) dF(\mathbf{c}) = N \cdot \Pi_i^A(C) + \int_{\mathbf{C}} \sum_i q_i^A(\mathbf{c}) \left(c_i + \frac{F(c_i)}{f(c_i)}\right) dF(\mathbf{c})$$

which implies that for (11) the choice of $b^A(\mathbf{c})$ and $S^A(\mathbf{c})$ is inconsequential as long as incentive compatibility is respected and the resulting $\Pi_i^A(C)$ do not violate the participation constraints. Put differently, treating S as exogenous, the choice of S has no effect on (11) inasmuch the allocation rule $q_i^A(\mathbf{c})$ is unaffected since in any incentive compatible mechanism $\int_{\mathbf{C}_{-i}} \left[q_i^A(\mathbf{c})\, \mathsf{E}_z \phi\left(b_i^A(\mathbf{c}), S_i^A(\mathbf{c})\right)\right] dF(\mathbf{c}_{-i})$ is constant by (10). Therefore in our formulation S has a potential to decrease (12) without affecting (11) if it leaves the allocation rule unchanged. A recent paper by Burguet et.al. (2012) approaches the optimal tender problem in the model where the contractors' budgets are their private information. In such model introducing S leads to additional complications since it is in the same "dimension" as the bidders' private information. In addition, in the expected cost in Burguet et.al. (2012) the informational rents enter multiplicatively with the probability of default. As in this paper, the probability of default decreases with the contractor's type, however, the informational rents increase. As a result the characterization of the optimal mechanism is complicated.

From the perspective of (11), just like in Myerson (1981) the optimal tender is deterministic in the allocation rule and assigns $q_i^A(\mathbf{c}) = 1$ to the contractor whose $c_i + \frac{F(c_i)}{f(c_i)}$ is the lowest at $\mathbf{c}$. Given the regularity assumption incentive compatibility by itself implies that the contractor with the lowest cost c_i wins the tender, hence $Q_i^*(c_i) = (1 - F(c_i))^{N-1}$ for all i. In our formulation V is high enough so that the project is always procured, however, this does not deemphasize the expected cost minimization.

The term on (12) reflects the impact of the expected probability of default on the expected cost and captures the effects of the higher moments of the revenue distribution as was illustrated by Proposition 1. The logic of Proposition 1 is that adding randomness (from the winner's perspective) to the tender outcome, that is considering b that depend on the costs other than the winner's, increases the probability of the winner's default. Proposition 2 below generalizes Proposition 1 and shows that the tender where b only depends on the winner's private information minimizes the probability of the winner's default.

Proposition 2 *Consider incentive compatible tenders with the same allocation rule and same payoff to the lowest possible type. Suppose the default payment S is given exogenously in each tender. To achieve the same probability of default the tender where b is entirely determined by the winner's c_i requires the lowest S.*

The proof is in Appendix. In our model the contractors are assumed to have unlimited budgets. This may not hold in reality and requiring high default payment may decrease the participation in the tender which in itself will drive the expected cost up. The fact that the tender with "simpler" payment rule requires lower default payment is therefore remarkable. Proposition 2 and (11) both call for the use of the lowest bid tender with appropriately chosen S. As long as S is such that the winner's probability of default is zero, the optimal tender simply minimizes (11). It is well known that the lowest bid tender achieves this minimum. By Proposition 2 the lowest bid tender also requires the lowest S to ensure against the winner's default, so that (11) and (12) are alighned in our model. Importantly, the instruments that we show to be optimal are already used in procurement. The following Section briefly describes the concept of the surety bonds and analyses their interplay with the lowest bid tender.

6 Lowest Bid Tender with Surety Bond

Surety bonds were introduced in the USA by the Heard Act of 1894 and are also quite common in Canada and Japan. The Heard Act was replaced in 1935 by the Miller Act. The Miller Act requires the contractor to provide surety bond for any Federal construction contract over $100'000 in value. All the U.S states have since adopted similar legislation, through the acts known as "Little Miller Acts". The American Institute of Architects also recommends the use of the surety bond in its standard building contract.[22] A surety bond guarantees the buyer that the contractor will complete the contract according to the terms including the price and time frame. The surety company evaluates the contractor's capacity to perform the project and also his financial capacity to pay the bond in case of default. In some cases the surety also requires the contractor, or the contractor indemnitors, to pledge enough assets to serve as a collateral on the bond. Our model is already well geared to incorporate surety bonds, in what follows surety bond S is an exogenously given default payment that applies uniformly to any contractor who wins the tender. The buyer chooses S ex-ante and informs the bidders about the chosen value. The value of S influences the bids in the lowest bid tender and the subsequent expected cost but that influence is already fully captured by (8).

We further discuss the optimal size of the surety bond. Such question is addressed with numerical simulations in Calveras et.al. (2004) for the second lowest bid tender in Zheng's model. The rest of this section revisits the lowest bid tender with surety bond S as discussed above. We have already noted that with $S \geq Z$ the winner will never default. In fact, such high S is excessive. As before in the symmetric

[22] Article 11.4 of General Conditions of the Contract for Construction states: "The Buyer shall have the right to require the Contractor to furnish bonds covering the faithful preformance of the Contract and payments of obligations arising thereunder as stipulated in bidding requirements or specifically required in the Contract Documents on the date of the execution of the contract."

equilibrium of the lowest bid tender the equilibrium bid solves

$$\Phi\left(b\left(c,S\right),S\right) = \mathsf{E}\left[Y^1|Y^1 \geq c\right], \tag{13}$$

with $\Phi\left(b,S\right)$ given by (6).

Upon winning the tender the contractor with cost c expects positive continuation payoff from the investment of c, hence she makes such investment in equilibrium. Note that $b\left(c,S\right)$, the solution to (13) for given S is unique and increasing in c, so that the contractor with the lowest cost wins in equilibrium. Also from (7) note that $b\left(c,S\right)$ is increasing in S, with

$$\frac{db\left(c,S\right)}{dS} = \frac{1}{F_z\left(b+S\right)} - 1 > 0. \tag{14}$$

In addition from (14) $d\left(b\left(c,S\right)+S\right)/dS = 1/F_z\left(b+S\right) > 0$ for any c, hence when S increases the probability of the winner's default and the buyer's expected cost decrease. Thus in our setting higher S gets reflected in higher bids in the tender, however, that by itself does not change the expected cost, (11) stays constant since (13) holds for any $S \leq Z$. Surety bond is therefore a "cheap" and effective instrument that discourages default. Not only the winner loses her bond if she is not willing to cover the cost overrun, she is also forfeiting the higher award from the tender.

Remark 1 *Waehrer (1995) examines the effect of the surety bonds in the setting where the buyer may default upon learning the value of the item after the auction. Under "strong seller" assumption where the seller captures the buyer's value after the default the expected revenue increases in default probability hence the buyer prefers not to use surety bonds.*

The optimal S^* ensures that the probability of default is zero, that is $b\left(c,S^*\right) + S^* = Z$ for any c. Since $b\left(c,S\right)$ is increasing in c, optimal S^* discourages the winner with own cost of 0 from defaulting. Such winner bids according to $\Phi\left(b,S\right) = \mathsf{E}\left[Y^1|Y^1 \geq 0\right] = \mathsf{E}c_2$. Therefore optimal S^* is given by

$$S^* = \int_0^Z F_z\left(x\right)dx - \mathsf{E}\left[c_2\right] \tag{15}$$

Clearly $S^* < Z$. Proposition 2 also shows that for any other tender format optimal surety bond would be higher than S^* given by (15). Since the lowest bid tender with surety bond is efficient it implements the optimal tender, that is minimizes (11), if the surety bond satisfies (15) the tender also minimizes (12).

Proposition 3 *In the lowest bid tender the probability of default and the buyer's expected cost decreases with the surety bond level. The lowest bid tender with optimally chosen surety bond, given by (15), implements the optimal tender and minimizes the buyer's expected cost. Such combination of the procurement policies ensures zero probability of default after the tender.*

Appendix I: Proof of Proposition 2

Proposition 4 *Consider incentive compatible tenders with the same allocation rule and same payoff to the lowest possible type. If F_z is log-concave then the winner's probability of default is minimized by the tender where the winner's report determines b and S.*

Proof. Consider the probabilities of the winner's default in the mechanism WA where $b = b(c_i)$, that is the winner conditional on the winning faces no further uncertainty in the mechanism, with any other incentive compatible A with the same allocation rule $q_i(\mathbf{c})$ for all i and the same exogenously given S. When $b(c_i) \geq Z - S$ the winner does not default after WA. The rest of the proof concentrates on the realizations c_i where $b(c_i) < Z - S$. Fix any such c_i and for the ease of notations introduce $b = b(c_i)$. From the interim payoff equivalence

$$\int_{\mathbf{C}_{-i}} q_i(\mathbf{c}) \left[\Phi(b, S) - c_i\right] dF(\mathbf{c}_{-i}) = \int_{\mathbf{C}_{-i}} q_i(\mathbf{c}) \left[\Phi\left(b_i^A(\mathbf{c}), S_i^A(\mathbf{c})\right) - c_i\right] dF(\mathbf{c}_{-i})$$

For $b < Z - S$ the left hand side of the above is

$$\int_{\mathbf{C}_{-i}} q_i(\mathbf{c}) \left[\int_0^{b+S} F_z(x)\, dx - S - c_i\right] dF(\mathbf{c}_{-i}). \tag{16}$$

The right hand side may involve the realizations of $\mathbf{c}_{-i}$ where $b_i^A(\mathbf{c}) + S_i^A(\mathbf{c}) \geq Z$ and those where $b_i^A(\mathbf{c}) + S_i^A(\mathbf{c}) < Z$. Since for every c_i the interim expected payoff is non-negative removing the realizations where $b_i^A(\mathbf{c}) + S_i^A(\mathbf{c}) \geq Z$ from consideration reduces the right-hand side so that

$$\int_{\mathbf{C}_{-i}} q_i(\mathbf{c}) \left[\int_0^{b+S} F_z(x)\, dx - S - c_i\right] dF(\mathbf{c}_{-i}) \geq$$

$$\geq \int_{\mathbf{C}_{-i}} q_i(\mathbf{c}) \left[\int_0^{\left(b_i^A(\mathbf{c})+S_i^A(\mathbf{c})\right)\wedge Z} F_z(x)\, dx - S - c_i\right] dF(\mathbf{c}_{-i})$$

In the above $\wedge$ denotes min operator. For given c_i and S

$$\int_{\mathbf{C}_{-i}} q_i(\mathbf{c}) \left[\int_0^{b+S} F_z(x)\, dx - \int_0^{\left(b_i^A(\mathbf{c})+S\right)\wedge Z} F_z(x)\, dx\right] dF(\mathbf{c}_{-i}) \geq 0.$$

Next we concentrate on the inner integrals. Partition the set of $\mathbf{c}_{-i}$ into set Γ, where $b \geq b_i^A(c_i, \mathbf{c}_{-i})$ and the complementary set Γ^C. Then the above integral can be rewritten as

$$\int_{\mathbf{\Gamma}} q_i(\mathbf{c}) \int_{b_i^A(\mathbf{c})+S}^{b+S} F_z(x)\, dx dF(\mathbf{c}_{-i}) - \int_{\mathbf{\Gamma}^C} q_i(\mathbf{c}) \int_{b+S}^{\left(b_i^A(\mathbf{c})+S\right)\wedge Z} F_z(x)\, dx dF(\mathbf{c}_{-i}).$$

Dividing and multiplying both integrals by $f_z(x)$

$$\int_{\mathbf{\Gamma}} q_i(\mathbf{c}) \int_{b_i^A(\mathbf{c})+S}^{b+S} \tfrac{F_z(x)}{f_z(x)} f_z(x)\, dx dF(\mathbf{c}_{-i}) - \int_{\mathbf{\Gamma}^C} q_i(\mathbf{c}) \int_{b+S}^{\left(b_i^A(\mathbf{c})+S\right)\wedge Z} \tfrac{F_z(x)}{f_z(x)} f_z(x)\, dx dF(\mathbf{c}_{-i}) \leq$$
$$\frac{F_z(b+S)}{f_z(b+S)} \left[\int_{\mathbf{\Gamma}} q_i(\mathbf{c}) \int_{b_i^A(\mathbf{c})+S}^{b+S} f_z(x)\, dx dF(\mathbf{c}_{-i}) - \int_{\mathbf{\Gamma}^C} q_i(\mathbf{c}) \int_{b+S}^{\left(b_i^A(\mathbf{c})+S\right)\wedge Z} f_z(x)\, dx dF(\mathbf{c}_{-i}) \right].$$

The inequality follows if F_z is log-concave, so that F_z/f_z is increasing. Recall that the argument applies to $b+S<Z$. We have therefore argued that if the distribution F_z is log-concave then

$$\int_{\mathbf{\Gamma}} q_i(\mathbf{c}) \int_{b_i^A(\mathbf{c})+S}^{b+S} f_z(x)\, dx dF(\mathbf{c}_{-i}) - \int_{\mathbf{\Gamma}^C} q_i(\mathbf{c}) \int_{b+S}^{\left(b_i^A(\mathbf{c})+S\right)\wedge Z} f_z(x)\, dx dF(\mathbf{c}_{-i}) \geq 0.$$

Recalling the definitions of sets Γ and Γ^C we have established that

$$\int_{\mathbf{C}_{-i}} q_i^A(\mathbf{c}) \left[\int_0^{b+S} f_z(x)\, dx - \int_0^{\left(b_i^A(\mathbf{c})+S\right)\wedge Z} f_z(x)\, dx \right] dF(\mathbf{c}_{-i}) \geq 0,$$

so that

$$\int_{\mathbf{C}} \sum_i q_i^A(\mathbf{c}) F_z(b(c_i)+S)\, dF(\mathbf{c}) \geq \int_{\mathbf{C}} \sum_i q_i^A(\mathbf{c}) F_z\left(\left(b_i^A(\mathbf{c})+S\right)\wedge Z\right) dF(\mathbf{c}).$$

The left hand side is the aggregate probability of *solvency* after the tender where the own bid of the winner $b(c_i)$ determines her award after completion. The right hand side is the aggregate probability of no default after the tender with the same allocation rule where the award is determined by $b_i^A(c_i, \mathbf{c}_{-i})$. ■

References

[1] Arozamena, L. and E. Cantillon, (2004) "Investment Incentives in Procurement Auctions," *Review of Economic Studies*, 71, 1-18.

[2] Asker, J. and E. Cantillon (2008) "Properties of Scoring Auctions," *RAND Journal of Economics*, 39, 1, pp. 69-85.

[3] Asker, J. and E. Cantillon (2010) "Procurement when Price and Quality Matters," *RAND Journal of Economics*, 41, 1, pp. 1-34.

[4] Ashley, D. and Workman A. "Incentives in Construction Contracts." Document SD-8, Austin, Tex.: The Construction Industry Institute, 1986.

[5] Bajari, P., R. McMillan and S. Tadelis (2008) "Auctions versus Negotiations in Procurement: an Empirical Analysis," *Journal of Law, Economics & Organization*, 25, 2, pp. 373-399.

[6] Board, S. (2007), "Bidding into the Red: A Model of Post-Auction Bankruptcy," *Journal of Finance*, 62, 6, 2695-2723.

[7] Burguet R., J,-J. Ganuza and E, Hauk (2012): "Limited Liability and Mechanism Design in Procurement," *Games and Economic Behavior*, 76, pp 15-25.

[8] Calveras, A., J.-J. Ganuza and E. Hauk (2004): "Wild Bids: Gambling for Resurrection in Procurement Contracts," *Journal of Regulatory Economics*, 26,1 pp. 41-68.

[9] Carpineti, L., G. Piga and M. Zanza, "The Variety of Procurement Practice: Evidence from Public Procurementm," Handbook of Procurement, Cambridge University Press (2006).

[10] Chang W-S, B. Chen and T.C. Salmon (2013), "An Investigation of the Average Bid Mechanism for Procurement Auctions," *mimeo.*

[11] Chen, J., L. Xu and A. Whinston (2010), "Managing Project Failure Risk through Contingent Contracts in Procurement Auctions," *Decision Analysis*, 7, 1, pp. 23-39.

[12] Che, Y.K. (1993) "Design Competition through Multidimensional Auctions," *RAND Journal of Economics*, 24, 4, pp. 668-680.

[13] Chillemi and C. Mezzetti (2013), "

[14] Decarolis, F (2009) "When the Highest Bidder Loses the Auction: Theory and Evidence from Public Procurement," *mimeo.*

[15] DeMarzo, P.M, I. Kremer and A Skrzypach (2005), "Bidding with Securities: Auctions and Security Design," *American Economic Review*, 95, pp. 936-959.

[16] Eso, P. and L. White (2004), "Precautionary Bidding in Auctions," *Econometrica*, 72, 1, pp. 77-92.

[17] Eso, P. and G. Futo (1999), "Auction Design with a Risk Averse Seller," *Economics Letters*, 65, pp. 71-74.

[18] European Commission, (2002). "Prevention, Detection and Elimination of Abnormally Low Tenders in the European Construction Industry", http://ec.europa.eu/enterprise/construction/alo/altfin.htm

[19] Ewerhart, C. and K. Fieseler (2003) "Procurement Auctions and Unit Price Contracts," *RAND Journal of Economics*, 34, 3, pp. 569-581.

[20] Fox, J.R. Arming America: How the US Buys Arms. Cambridge: Harvard University Press, 1974.

[21] Gil, R. and J-M Oudot, (2009) "Award Mechanism, Relational Contracting and Ex-Post Renegotiation: Evidence from French Defence Procurement and Spanish Movie Exhibition," *mimeo.*

[22] Kirkegaard, R. (2013) Online Supplement to "Mechanism Design Approach to Ranking Asymmetric Auctions," *Econometrica*, 80, 5, pp. 2349-2364.

[23] Krishna, V. "Auction Theory," San-Diego, Academic Press, 2002.

[24] Laffont, J.J. and J. Tirole, (1986) "Using Cost Observation to Regulate Firms," *Journal of Political Economy*, 94, 3, pp. 614-641.

[25] ——— AND ——— (1987) "Auctioning Incentive Contracts," *Journal of Political Economy*, 95, 5 , 921-937.

[26] ——— AND ———, A Theory of incentives in Procurement and Regulation. Cambridge. MIT Press, 1993.

[27] Lewis, G. and P. Bajari, (2011a) "Procurement Contracting with Time Incentives: Theory and Evidence," *Quarterly Journal of Economics*, 126, pp. 1173-1211.

[28] Lewis, G. and P. Bajari, (2011b) "Incentives and Adaptation: Evidence from Highway Procurement in Minnesota," *mimeo.*

[29] McAfee, P.R.. and J. McMillan, (1986). "Bidding for Contracts: A Principal-Agent Analysis," *RAND Journal of Economics*, Vol. 17, No. 3, pp. 326-338.

[30] ——— AND ———. Incentives in Government Contracting. Toronto: University of Toronto Press, 1988

[31] Parlane, S. (2003) "Procurement Contracts under Limited Liability," *Economic and Social Review* 34(1): 1—21.

[32] Riordan, M.H. and D. Sappington (1986) "Awarding Monopoly Franchises," *American Economic Review*, Vol. 77, 375-387.

[33] Rhodes-Kropf, M. and S. Viswanathan (2000): "Corporate Reorganizations and Non-Cash Auctions," *Journal of Finance*, 55, 4, 1807-1849.

[34] Rhodes-Kropf, M. and S. Viswanathan (2005): "Financing Auction Bids," *RAND Journal of Economics,* Winter, 36, 4, 789-815.

[35] Tirole, J. (1986), "Procurement and Renegotiation," *Journal of Political Economy*, Vol. 94, No. 2, pp. 235-259.

[36] Waehrer, K. (1995), "A Model of Auction Contracts with Liquidated Damages," *Journal of Economic Theory*, 67, 2, 531-555.

[37] Zheng, C., (2001), "High Bids and Broke Winners," *Journal of Economic Theory*, 100, pp. 129-171.