THE DETERMINATION OF WORKING STRESSES FOR BRIDGES.

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IN all modern specifications for regulating the intensity of working stresses in the various parts of bridges, some allowance is made for the increased effect of moving live loads. This is more particularly the case in railway bridges, but even in roadway bridges the vibrations set up by moving loads are under certain conditions very perceptible, and can by no means be overlooked.

So far back as 1847 this subject was under consideration, and a Commission, appointed to inquire into the failure of the Dee Bridge under a passing train, made certain experiments on the dynamical effect of moving loads, which resulted in the issue of a regulation by the Board of Trade in 1849, that for cast iron girders (the type then employed for bridges), the factor of safety provided against moving loads should be twice that provided against dead load.

The investigations of the Royal Commission were the means of bringing prominently forward an important aspect of bridge design which had been previously overlooked, and though not embodied in the Board of Trade regulation of 1859, dealing with the use of wrought iron, and of 1877, with the use of steel (both still in force), it has had the effect of causing extra provision to be made against live load stresses in all properly designed structures of recent years. A great number of measurements have been made by different investigators of the actual stresses occurring in the members of railway bridges under the action of moving train loads. A summary of the conclusions deduced from the most extensive of these, viz., those by Professor Turneaure in 1897, is given in Appendix I. It is to be noted that in railway bridges the increased stresses imposed by moving train loads travelling at high speeds are due principally to the action of the unbalanced moving parts of the locomotive, (*i.e.*, the counterweights on the wheels), and the extra load imposed by the vibration of the locomotive on its springs, but also in a less degree, to the lurching action of the train caused by the rails not being exactly level, the shocks due to inequalities in the level of the ends of the rails at joints, flat wheels, and the thrust due to the obliquity of the connecting rod of the locomotive. These causes all tend to set up vibrations in the bridge, and for high speeds, in main girders of 100 feet span and over, these vibrations very often become cumulative, owing to the even spacing of the wheels of the train, and increase the deflection of the structure, especially if the impulses of the loads are timed to agree with the natural rate of vibration of the structure. For short spans, such as the longitudinal stringers of a railway bridge, the vibration is non cumulative and the increase in stress is due to shock or impact solely.

In road bridges, the causes producing impact or vibration are much less marked, and, so far as the authors are aware, have not been investigated, but it is reasonable to suppose that unequal wear in the planking of timber-decked bridges, and inequalities in the surface, and loose metal on the roadway of structures having a metalled deck, must increase the effect of the rolling load due to a traction engine, or other moving load, to some extent, while considerable vibration may be set up by trotting horses or a mob of cattle crossing the bridge.

The Launhardt-Weyrauch Formula.

At this point it may be well to refer to another view of the question of live load stresses, viz., the much debated point as to whether a great number of repetitions of a stress lower than the actual breaking stress of any portion of a structure will cause failure owing to the "fatigue" of the metal. The well-known experiments made by Wöhler, published by him in 1870, and supplemented by those of other investigators, show that a bar of iron or steel will break under a stress considerably less than its static breaking load, if such stress be repeated a great number of times. These experiments caused considerable stir at the time when they were published, and the view taken by many engineers, especially on the Continent, was that the explanation of this phenomenon lay in some molecular alteration inducing "fatigue" in the metal. The result was the enunciation of a formula known as the Launhardt-Weyrauch formula, based on the experimental figures. This formula is of the form, $p = \text{constant}\left(1 + \frac{1}{2}\frac{\text{min.}}{\text{max.}}\right)$, and was very widely used for a time as a "fatigue" formula, even in the proportioning of many road bridges, where, so far from there being any continued repetition of the maximum live load stresses, it is probable that these would recur only at very wide intervals, if they ever actually occurred in the whole life of the structure.*

Later on, it began to be recognised that the "fatigue" theory was hardly justified by the facts, but it was seen that the formula, with perhaps some variation in the constants employed, gave results which were justified by experience, and it continued to have a vogue as a

* Bulletin of the International Railway Congress, August, 1901. Report by Max Edler von Leber "on the question of the Construction and Tests of Metallic Bridges.":—"In conclusion, we would remark that the repetition of strains at very short intervals—a condition which Wöhler took into account—cannot be compared to the way in which any members of structures are strained, for in actual practice the intervals of time elapsing between the successive applications of the live load are long, and allow the metal to recover its original molecular state, as is shown by the experiments made by Mr. Bauschinger."

"fatigue and impact" formula, there being a certain diffidence in abandoning altogether the "fatigue" idea, even while it was thought that it was less necessary to make provision against "fatigue" than against the dynamical effect of moving live loads.

The experiments now being made at the Sydney University on stress repetition, in the machine designed by Professor Warren and Mr. Madsen, B.E., may possibly throw some further light on the imperfectly understood phenomena of Wöhler's experiments. In the meantime, the authors incline to the opinion that the explanation may lie in the extension of micro flaws in the material of the test specimens.

At the present time, the consensus of opinion, especially among American engineers, is in favour of the direct "impact" method of providing against live load stresses; but before leaving the subject of "fatigue," it may be well to refer to the opinion of two eminent English authorities, viz., Professor Unwin and Sir Benjamin Baker, upon the conclusions to be deduced from Wöhler's experiments, as expressed in a recent discussion before the Institution of Civil Engineers.

Professor Unwin, who made the first experiments ever carried out on the reduction of strength due to repetition of stress about 1860, and also many subsequent experiments, was of opinion that the action—"which might for shortness be called the Wöhler action"—was not in any way due to dynamical action, the explanation lying in the extraordinary variability of the real elastic limit, "which might be reduced even to zero stress by loads below what was commonly called the yield point." He considered that "the Wöhler effect could not be properly taken into account by making an empirical allowance for the dynamical action of the train," but, at the same time, thought that the way in which Wöhler's law had been applied to bridges had not been altogether a rational one, and favoured adhering to the rule of dead load – plus – twice – live load as providing margin enough both for repetition of loading and dynamical action.

Sir Benjamin Baker stated, "That it was often said that, in bridges and other structures, the stresses were so far within the limit of elasticity that the experiments of Wöhler and others on iron or steel, strained beyond the elastic limit did not apply." He then proceeded to show that this view of the case was not altogether a correct one, since it did not take into account initial stresses, such as those set up in plates by cold straightening during manufacture; or the increased stresses caused in portions of the flange of a girder by oxidation or unequal temperature, any of which causes might easily raise the local stresses beyond the elastic limit. At the same time, these local variations of stress might go on for billions of times without breaking down the bridge, owing to the fact that a ductile material was employed which elongated regularly beyond the elastic limit, and thus allowed of the most highly strained portions stretching slightly, and equalising the stress throughout the whole section. He considered that "Formulas for the working stresses in bridges founded solely on Wöhler's experiments, and ignoring other practical conditions often of as much importance as the relative proportion of live to dead load, would lead to badly proportioned structures."

So much for the Launhardt-Weyrauch formula, which has been dwelt upon somewhat at length, since it was a formula very widely employed for a number of years, and is still used to some extent—*e.g.*, the French Government formula referred to later on.

The author's opinion is that, whether used as a "fatigue" formula or as a "fatigue and impact" formula, it will, with proper constants, give satisfactory results for ordinary proportions of dead to live load; but, at the same time, that equally good results can be obtained in a simpler and more direct manner by the "impact" method now to be described.

Impact Formulas.

Specifications which make provision against the dynamic stresses set up by live loads in motion are of two kinds:—(*a*) Those which make no direct addition to the live load stresses, but employ certain fixed values for the working stresses to be employed, which are less for live load than for dead load, and are sufficiently low for the former to cover the effect of "impact," or (*b*). Those which add some percentage for impact to the live load stresses, and then employ a constant unit stress, which is the same both for live and dead load.

Theodore Cooper.

The well-known specifications of Theodore Cooper are of the former class. In these, the unit stresses allowed in tension for the deck system (stringers, cross girders, &c.), are the same both for dead and live load, and have a low value, since the dead load stresses in these members are usually small compared with those due to live load; and, at the same time, these members are particularly subject to shock or impact.

For the proportioning of truss members, however, the value of the unit stress specified for live load is only *one half* that for dead load.

For medium steel (Cooper's 1901 specification) having an ultimate strength of from 60,000 to 68,000 lbs. (26·8 to 30·0 tons) per square inch, with an elastic limit of not less than one half the ultimate strength; the factors of safety given by these specifications are as follow, measured on the minimum value of 60,000 lbs. per square inch:—

Railroad Bridges—(1901 specification).

	Factor of safety when stress is entirely due to dead load.	Factor of safety when stress is entirely due to live load.
Stringers and cross girders ...	6·0	6·0
Cords and web members ...	3·0	6·0

*Highway Bridges—(1896 specification).

Stringers and cross girders... ...	4·6	4·6
Chords and web members—eyebars	2·4	4·6
Chords and web members—shapes...	2·75	5·5

These figures are of course valuable only as an indication of what the real factor of safety will be, measured on the sum of the dead load and live load stresses.

While these specifications are based upon a wide experience, and are recognised in American practice as giving satisfactory results; the reason for some of the values specified is not quite clear, *e.g.*, the unit stress for dead load in the chords, etc., for railway bridges is 20,000 lbs.

*Cooper's 1901 Highway Bridges Specification is not yet to hand in New South Wales.

per square inch, and for the same members in highway bridges 22,000 lbs. per square inch, while for live loads it is one half of these values in either case. There is no doubt that an engineer of Mr. Cooper's wide experience, has some good reason for making the factor of safety against *dead* load greater for one type of structure than for another; and also for specifying an almost identical factor of safety (6·0 as agaiust 5·5) for the live load due to a railway train, and for that due to a crowd of people (which latter would certainly appear to require a less rigid provision); but Mr. Cooper's premises are not apparent from the specifications, and it is human nature to wish to know *why* such values are specified.

Waddell and Schneider.

Mr. Waddell is one of the best known of American bridge engineers; while Mr. Schneider's specification is that which has been adopted by the great American Bridge Company in their 1900 specification. Both these gentlemen make direct provision against impact on the (*b*) method, by adding a percentage for impact to the live load stresses, proportioned in the case of the former upon the span of the member, or the length of the live load causing the maximum stress, and in the latter upon the span of the member solely.

Waddell refers to his impact formula as follows: ("De Pontibus," 1899).

"Meanwhile the author has adopted temporarily the formula "$I = \frac{40,000}{L + 500}$ (where I = the percentage to be added to the live "load stress for impact. L = span or length of live load causing "the maximum stress).

"This formula was established to suit the average practice of half "a dozen of the leading bridge engineers of the United States, as given "in their standard specifications, and not because the author considers "that it will give truly correct percentages for impact.

"The assumption made in some specifications, that the live load is "always twice as important and destructive as the dead load is absurd, "and involves far greater errors than those that would be caused by "any incorrectness in the assumed impact formula.

"The author acknowledges that he anticipates finding the values "given by the formula somewhat high, but it must be remembered that "the said formula is intended to cover in a general way also, the effects "of small variations from correctness in shop work, or to provide for "what the noted bridge engineer, the late C. Shaler-Smith, used to "term the factor of ignorance."

Both Waddell's and Schneider's formulas are based solely upon the length of the span or live load for the percentage of "impact" which is to be added, and take no account of the type of structure adopted. For the longer spans, the percentages of impact which they allow appear excessive: viz., Waddell, forty per cent. of the live load stress for a span of 500 feet, and twenty-seven per cent. for 1,000 feet span; and Schneider, thirty-seven and a half per cent. and twenty-three per cent. respectively.

Sydney Harbour Bridge Specification.

In Appendix II. are given the unit stresses prescribed in the specification for the bridge over Sydney Harbour, for which designs and tenders are now being invited, to close on 28th February, 1902. This structure is to have a main span of not less than 1,200 feet in the clear, with a clear headway of 170 feet above high water mark over the 600 feet at centre, and an overall width of about 120 feet, in which are to be embraced a double line of railway, two thirty feet wood-blocked roadways, each having a line of electric tramway, and two twelve feet footpaths. The whole of the material of the trusses and deck system is required to be medium steel, having an ultimate strength of from 60,000 to 70,000 lbs. (26·8 to 31·2 tons) per square inch, with an elastic limit of not less than one-half the ultimate strength.

The designs and tenders are to provide for a total length of structure of 3,000 feet, so that there will, in all probability, be a long and heavy main span, either of the suspension or cantilever type, together with some shorter truss spans on either side of the main span.

The live loads specified are two engines, each weighing 268,000 lbs. (120 tons) followed by a train load of 4,000 lbs. per foot, together with a live load of 100 lbs. per square foot, on the roadways and footways, or else a rolling load of 44,800 lbs. (20 tons) on four wheels spaced ten feet apart longitudinally, and five feet apart transversely.

Here we have a structure of unusual span and width, having a heavy deck and subjected to live load stresses, which will result partly from railway roads, and partly from the loading specified for a city type of road bridge.

Assuming that the "impact" method is, in the present state of our knowledge, the simplest and most rational manner of dealing with live load stresses, the problem was to determine the correct amounts of impact to be added, both to the live load stresses due to train loads, and to those due to a crowd of people, and a rolling load on the roadways.

After consideration and comparison of existing methods, it was determined by the Advisory Board to adopt the method recently formulated by Mr. J. W. Schaub, in October, 1900, and which he states was first proposed by Mr. H. S. Prichard, in 1895, viz. to add a percentage for impact to the static live load stress, proportional upon the ratio which the live load stress bears to the total stress *i.e.* $\frac{L}{L + D}$ in the Diagram. This appears to the authors to be the only rational method of providing for impact, since it takes into account the *inertia* of the structure as resisting the dynamical effect of live load, which is not the case with Waddell's or Schneider's formulas, based as these are solely upon the *length* of the span or member under load, and not in any way upon its *mass*.

For short span girders, such as the stringers of a railway bridge, where the vibrations set up by moving live loads are non-cumulative, this method of allowing for impact follows directly a first law of dynamics, which states that the force required to produce a given change of velocity in a given time is proportional to the number of units of mass of which the body consists (*i.e.*, for a given live load moving at a given speed the deflection varies with the mass to be overcome by the live load during the time in which it is in action). In longer spans, such as main trusses, there will also be cumulative vibration set up, which will depend upon the panel length of truss, spacing of wheels, counterweights, etc., and will be different for every structure. It is impossible to estimate with any accuracy what the amount or effect of this variation will be; and it is equally impossible, except after careful measurements, to arrive at any approximation even of the initial stresses in the material referred to by Sir Benjamin Baker, or the secondary stresses which exist in every structure. It is not, therefore, claimed that the formula now proposed is a perfect one by any means, though, as shown by Diagram I. the factor of safety allowed after providing against impact should, in all cases, be ample to cover vibration and initial and secondary stresses.

Having now determined that impact shall be provided against by adding a percentage to the live load stress, of the Form $I = L \times \frac{L}{L + D}$ [where I = the addition to be made to the live load stress for impact. L = live load stress, and D = the dead load stress in the member], it remains to be settled whether the whole of this amount, or only a proportion thereof, is to be allowed. If we allow the whole, then in members where the live load stress is very large compared with the dead load stress (*e.g.* the stringers of a railway bridge), putting $D = O$, I will be 100% of L.

Unfortunately, we have no experimental results showing what the dynamic effect on railroad stringers actually is, but Turneaure estimates that it will be "something like seventy-five per cent." Adopting this limit as the maximum which should be allowed upon *any* member, the formula then becomes $I = \cdot 75 \times L \times \frac{L}{L + D}$. This is for a railway bridge. The values assumed for road bridge loads, viz.:—

$I = \cdot 30 \times L \times \frac{L}{L + D}$ for rolling load; and $I = \cdot 15 \times L \times \frac{L}{L + D}$ for load due to a crowd of people, are taken as reasonable assumptions in the absence of any experimental data on such structures.

Diagram I.

On Diagram I. are shown three curves, which represent the actual statical stresses in tension on any portion of the structure, due to the various loads prescribed in the Sydney Harbour Bridge specification, after adding a percentage for impact as specified. The first point to settle was, what should be the higher limit to be allowed, *i.e.*, the working stress when the stress was entirely due to dead load, (a hypothetical figure, since in every member there will be some live load stress). This was fixed at 17,000 lbs. per square inch, or a factor of safety of about three and a half on the minimum specified tensile strength of the steel, viz., 60,000 lbs. per square inch. The curves have been plotted for different ratios of $\frac{L}{L + D}$ For example: take the lowest point of the train load curve where $D = O$, and $\frac{L}{L+D} = 1{\cdot}0$ In this case, the amount to be added for impact, $I = {\cdot}75 \times L \times 1{\cdot}0$ and the statical working stress in tension allowed (*i.e.*, the working stress per square inch, if all the load be treated as dead load), will be $17{,}000 \div 1 + {\cdot}75$ or 9,714 lbs. per square inch. Or, again, where $D = L$ and $\frac{L}{L + D} = {\cdot}5$, then $I = {\cdot}75 \times L \times {\cdot}5 = {\cdot}375\ L$, and this working stress becomes $17{,}000 \div D + L\ (1 + {\cdot}375) = 17{,}000 \div {\cdot}5 + {\cdot}5\ (1 + {\cdot}375) = 14{,}316$ lbs. per square inch. Similarly, the remaining points have been fixed for the three classes of live load specified, for all values from $\frac{L}{L + D} = 0$ to $\frac{L}{L + D} = 1{\cdot}0$, and the curves plotted.

Diagram II.

On Diagram II. are shown the curves for working stresses in tension, plotted from the values given by the following, viz. Cooper's railway and highway specifications; the French Government formula; Waddell's railway and highway specifications; and Schneider's specification for railway bridges as adopted by the American Bridge Company. In each case the curves represent the statical working stresses (*i.e.* the working stresses per square inch, if all the load be treated as a dead load), after allowing for impact in the case of the two latter specifications. On Figs. 1, 2 and 3 the working stresses specified for Sydney Harbour Bridge, are plotted on for purposes of comparison, but this was not possible in the case of Figs. 4, 5 and 6, where the impact is dependent upon the length of span and not in any way upon the ratio of the live load to the total load.

Diagram III.

In order to see exactly how the Sydney Harbour Bridge specification would compare with other specifications in actual practice, Diagram III. has been prepared. This embraces the stringers and main girders of a deck span of 150 feet, such as might be included as one of the side spans in the 3,000 feet of bridge for which tenders are being invited. The dead loads have been calculated from experience as what are likely to occur, and the live loads are those prescribed in the specification.

(*a*) As an example of how the values have been arrived at, take the case of the railroad stringer, where $\frac{L}{L + D} = {\cdot}93$. Here $I = {\cdot}75 \times (L \times {\cdot}93) = {\cdot}70\ L$ and the resulting statical working stress $= 17{,}000 \div {\cdot}07 + {\cdot}93\ (1 + {\cdot}70) = 10{,}303$ lbs. per square inch for Sydney Harbour Bridge. This is somewhat higher than the values given by Waddell, Schneider, and Cooper, while it is lower than that obtained from the French Government formula.

(*b*) For railroad main girder, the French Government formula is also the highest, while the Sydney Harbour Bridge specification is nearly identical with the values given by the other authorities.

(*c*) For roadway stringers the Sydney Harbour Bridge specification gives higher working stresses than either Waddell or Cooper, and also than the French Government formula, though it is probable that the latter only refers to railway bridges; the authors are not certain on this point. The extreme value of 16,473 lbs. (7·35 tons) per square inch, given by the Sydney Harbour Bridge specification, for dead load and live load of 100 lbs. per square foot, is not, in the authors' opinion, too high, since it represents a factor of safety of 3 against dead load, and of 4·1 against live load, after allowing for impact; or, putting it another way, the Sydney Harbour Bridge specification allows a working stress in tension of 20,000 lb. (9·0 tons) per square inch for dead load, and of 14,592 lb. (6·5 tons) per square inch for live load, after allowing for impact. Waddell's and Cooper's values appear very low for these members.

(*d*) For roadway main girders, the value given by the Sydney Harbour Bridge specification is somewhat lower than Cooper's and higher than Waddell's.

The foregoing comparison with well known authorities justify the authors in believing that the method now proposed will give satisfactory results in all ordinary cases, and that it has the advantage of simplicity in working, and of keeping always before the mind the relative effects of dead and live load in all portions of the structure under consideration.

Before concluding, it may be of interest to refer to other features in the Sydney Harbour Bridge specification, as given in Appendix II.

Temperature.

The co-efficient for expansion for 1° F. has been taken as ·00000667, or ·0004 for 60° F., as specified. From observations taken at the Forth Bridge, the actual expansion was found to be $\frac{1}{160}$ inch per 1° F. for every 100 feet, equivalent to a co-efficient of expansion of ·0000052, but it is stated that the observations only covered limited ranges of temperature, and the expansion and contraction were considerably less than estimated for.

Wind Pressure.

The wind pressures allowed for, viz.: 50 lbs. per square foot on the unloaded structure, and 30 lbs. per square foot on the loaded structure; are in accord with the most modern practice, as is also the wind pressure of 300 lbs. per lineal foot on the surface of train. Such a wind pressure would overturn any of the rolling stock at present in use in New South Wales, but is by no means too high in view of future requirements, and is in common use in America. In order to avoid making the specification too cumbersome, the co-efficients to be employed, where the exposed surfaces of members of the structure are curved, latticed, etc., have not been included; but it will of course be competent for tenderers to make suitable allowance in such cases.

It is usual to employ higher working stresses for temperature and wind pressure than for dead and live load, and this practice has been followed in the present instance.