STEEL-CONCRETE BRIDGE CONSTRUCTION.

(A paper read before the Sydney University Engineering Society,)

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I. THEORY.

Introductory.

DURING the last few years the subject of Steel-Concrete Construction has received a great deal of attention among engineers, and the theory has been investigated by J. Melan, J. B. Johnson[1], W. Beer[2], Considère[3], and many other writers. Various theories for the moment of resistance of a steel-concrete beam have been proposed, in some of which, including the earlier ones, the tensile strength of the concrete is neglected. It has lately been suggested that the ordinary beam theory should be modified so as to take into account the imperfection of the elasticity of concrete[4].

It seems evident that considerable errors may be introduced into the calculation of the strength of steel-concrete structures by the assumption that the properties of concrete when employed in large masses and in conjunction with metal, are the same as those deduced from small size tests of concrete or mortar alone. Thus, until recently, it has been assumed that cracks were formed on the under surface of a steel-concrete beam when the proportionate distortion exceeded that of ordinary concrete, and that these cracks were too fine and too greatly

1. *Engineering News*, January 3rd, 1895; "Materials of Construction," 1st Ed., p. 72.
2. "The Monier System of Construction," Proc. Inst. C.E., vol. cxxxiii., p. 376.
3. For an account of M. Considères work, see Proc. Am. Soc. C.E., August, 1901, also *Engineering News*, February 27th, 1902.
4. "Theory of the Strength of Beams with Reinforced Concrete," W. Kendrick Hatt, *Engineering News*, February 27th, 1902; also *Engineering News*, July 17th, 1902.

distributed to be visible. The careful experiments undertaken by the French engineers, M. Considère and Harel de la Nöe, as well as those described by Mr. A. L. Johnson[1], have proved beyond any reasonable doubt that these cracks do not exist, and that the proportionate elongation may reach a value of from ten to twenty times that at which an unarmed concrete test piece would fail in tension. M. Considère's experiments are particularly conclusive. In one case he first subjected an armed concrete beam to twenty times the bending which would have been sufficient to break it had it not been reinforced with metal, and then to 139 repetitions of loading, the deformations ranging from four and a half to thirteen times the maximum value for plain concrete. A cylindrical bar of concrete surrounding one of the metal rods was then carefully removed, and on being tested was found to develop nearly the maximum tensile strength of the material—a result which would manifestly have been impossible had any cracks been formed at the tension surface of the beam.

This property of steel-concrete is clearly illustrated in a series of tests by Messrs. Fowler and Baker[2], included in which were eight concrete slabs 6 ft. 6 in. span, 2 ft. wide, and 3 in. thick; four slabs were fortified with expanded metal, the remaining four being composed of ordinary concrete and cement mortar. The deflection of the former averaged $\frac{7}{16}$ inch at *one-half* the ultimate load, while the latter broke without appreciable deflection. It is difficult to suggest any adequate reason for this remarkable decrease in the value of the modulus of elasticity of concrete, when used in combination with metal bars or sheets, but it seems probable however, that the effect is confined to the concrete in the vicinity of the metal, and is not distributed over the whole section.

Moments of Inertia of Different Sections.

It is usually considered that the discrepancy between the tensile strength and modulus of rupture of a material is caused by the permanent set altering the distribution of stress in such a manner as to increase the moment of resistance of the section.

In a very complete and valuable series of tests on concrete beams and bars, with and without metal, which were carried out by Prof. Kendrick Hatt,[3] at Purdue University, U.S.A., the load-deflection diagrams for the steel-concrete beams are curved until a point is reached at which the load has a value of about $\frac{2}{3}$ that causing the first crack at the tension surface of the concrete, after which the diagrams are represented very nearly by straight lines, until the load causing total failure is approached. The permanent set for each test is shown in a separate curve; it commences in the *initial stages* of the tests and increases at a fairly uniform rate throughout, the maximum set having an average value of about $\frac{1}{5}$ of the total deflection.[4] No change in

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1. Proc. Am. Soc. C.E., August, 1901.
2. For an account of these, see paper by Mr. A. T. Walmisley, M. Inst. C.E., on "The Use of Expanded Metal in Concrete," *The Builder*, Sept. 15th, 1900.
3. "Tests of Reinforced Concrete Beams," W. Kendrick Hatt, *Eng. News*, July 17th, 1902. A paper read at the Annual Meeting of the American Society for Testing Materials.
4. The test beams here referred to were composed of Portland cement concrete of the composition of 1 p.c., 2 sand, and 4 broken limestone, 1 in. guage and under. The span was 80 in. and the section 8 in. x 8 in. A summary is given in Table I.

the direction of the load-deflection diagram is noticeable at the first crack in the concrete.

Considering the present imperfect state of our knowledge of the subject of steel-concrete construction it would perhaps be sufficiently accurate to adopt the usual conditions of the "beam theory" and assume that the modulus of section is multiplied by a certain factor "F" which increases with the permanent set. The ratio between the modulus of rupture and tensile strength in a number of tests of various kinds of stone recorded by Prof. J. B. Johnson[1] is 2 : 1. In Prof. Hatt's experiments on 1-2-4 concrete this ratio is 1·9 : 1. For a steel-concrete beam it would therefore seem reasonable to assume a value of 2 for F for the point at which the concrete commences to crack.

It has been established by the Austrian experiments on steel-concrete arches at Puckersdorf, that the stresses taken by the two materials are directly proportional to their respective moduli of elasticity. As far as the author is aware, the only theories of steel-concrete construction founded on a correct distribution of stress between the two materials, are those brought forwarded by Prof. J. B. Johnson and Mr. Walter Beer. In an interesting and comprehensive paper published in the Proceedings of the Inst. C.E., Mr. Beer deduces general equations for determining the stresses in a body consisting of two or more materials of different moduli of elasticity under the action of direct forces and bending moments, and gives expressions for the moment of inertia of a Monier beam, both before and after the concrete fails in tension.

The moments of inertia and positions of the neutral axes of various sections of steel-concrete will now be determined by first principles from the "transformed section" in accordance with Prof. Johnson's method. Since the stresses are distributed in direct proportion to the moduli of elasticity, it may be assumed that the metal is virtually replaced by an amount of concrete of equal depth, and of width equal to the width of metal in the section, multiplied by the ratio of the moduli of elasticity of the two materials.

For the case where the steel or iron is in the form of bars or sheets whose thickness is small compared to the depth of the beam, which is a condition often fulfilled in practice, the expressions for neutral axis and moment of inertia may be deduced as follows:—

Adhering to Mr. Beer's Notation,

Let E_2 = modulus of elasticity of steel.

E_1 = modulus of elasticity of concrete.

$$\mu = \frac{E_2}{E_1}.$$

D = depth of a beam of unit width.

d = thickness of metal layer per unit width.

a = distance of metal layer from surface of beam.

Z = distance of neutral axis from under surface of beam.

I = moment of inertia of a beam of unit width about its neutral axis.

1. "Materials of Construction," 1st Ed., Table XLI., p. 643.

Then, for the case of a beam, as shown in Fig. 1, with one layer of metal near the under surface, and taking moments about that surface,

$$Z = \frac{D^2 + 2(\mu - 1)\,da}{2\{D + (\mu - 1)\,d\}} \quad \ldots\ldots\ldots\ldots(1),$$

the neutral axis passing through the centre of gravity of the transformed section. The moment of inertia about the neutral axis can be found by first principles, thus,

$$I = \frac{Z^3}{3} + \frac{(D - Z)^3}{3} + (\mu - 1)\,d\,(Z - a)^2 \quad \ldots\ldots\ldots\ldots(2).$$

If μ be substituted for $(\mu - 1)$ these expressions will be found to correspond with Mr. Beer's values.

If there are two layers of metal, each being at a distance "a" from the upper and lower surfaces of the concrete respectively,

$$Z = \frac{D}{2}$$

and $$I = \frac{D^3}{12} + 2(\mu - 1)\,d\left(\frac{D}{2} - a\right)^2 \quad \ldots\ldots\ldots\ldots(3).$$

By making μ equal to unity, the well-known value for the moment of inertia of a rectangular beam of unit width may be obtained.

If the metal consists of rolled girders, or other shapes, embedded in the concrete at intervals, then for the case where the neutral axes of the metal girders, or other shapes, coincide with the neutral axis of the concrete, as in Fig. 2.,

$$I = \frac{D^3}{12} + (\mu - 1)I^1 \quad \ldots\ldots\ldots\ldots(4),$$

where I is the moment of inertia of the cross section per unit width, and I^1 is the moment of inertia of one of the metal girders about its neutral axis, divided by the distance between them.

If the girders, or other shapes, are placed unsymmetrically, as in Fig. 3, then by taking moments,

$$Z = \frac{D^2 + 2(\mu - 1)Aa}{2\{D + (\mu - 1)A\}} \quad \ldots\ldots\ldots\ldots(5),$$

and $$I = \frac{Z^3}{3} + \frac{(D - Z)^3}{3} + \{\mu - 1\}\{I^1 + A(Z - a)^2\} \ldots(6),$$

"a" being, in this case, the distance from the centre of gravity of the girders from the under surface of the concrete beam, A the area of one of the girders divided by the distance between them, and the other symbols having the same meaning as before.

In practice μ may be substituted for $(\mu - 1)$ in all the foregoing expressions without introducing appreciable errors.

Having found expressions for the position of the neutral axes and moments of inertia of different forms of steel-concrete beams, the maximum stresses in the concrete and metal can easily be determined as follows :—

$$\text{Tensile stress in concrete} = \frac{MZ}{FI}.$$

M being the maximum bending moment per unit width, and F being a constant depending on the amount of permanent set.

The maximum tensile stress in the metal would probably be μ times this value.

It must be noted that these stress formulæ are correct only up to the point at which the concrete commences to crack.

Variation of the tensile stress in the concrete with the value of μ.

The strength of a steel-concrete beam or arch is practically governed by the maximum tensile stress (f) developed in the concrete. As the tensile stress depends in part on the value of μ assumed, and as this value cannot be found with any great degree of accuracy, it becomes a a matter of some importance to determine the relation between f and μ, and this will therefore be done for the three classes of steel-concrete sections most constantly occurring in practice.

For the purpose of the following investigation the equation

$$f = \frac{MZ}{I}, \text{ will be assumed as correct.}$$

Therefore for the case of one layer of metal in the concrete,

$$f = \frac{MZ}{\frac{Z^3}{3} + \frac{(D - Z)^3}{3} + (\mu - 1)\, d\, (Z - a)^2}.$$

Substituting for Z its value found in (1) and simplifying,

$$f = \frac{6MD^2 + 12Mda\,(\mu - 1)}{D^4 + 4Dd(D^2 - 3aD + 3a^2)\,(\mu - 1)} \dots\dots\dots\dots (7).$$

This equation can be put in the form,

$$\alpha f + \beta\mu f - \gamma\mu - \epsilon = 0,$$

which represents a rectangular hyperbola referred to axes parallel to the asymtotes, these being situated at distances of $-\frac{\alpha}{\beta}$ and $\frac{\gamma}{\beta}$ from the axis of f and μ respectively.

In a similar manner, when there are two layers of metal embedded in the concrete,

$$f = \frac{6\,MD}{D^3 + 24d\left(\frac{D}{2} - a\right)^2 (\mu - 1)} \dots\dots\dots\dots\dots\dots (8).$$

$$\text{or } \alpha f + \beta\,\mu f - \epsilon = 0.$$

This also is the equation of a rectangular hyperbola, one asymtote coinciding with the axis of μ, and the other one being parallel to, and at a distance $-\frac{\alpha}{\beta}$ from, the axis of f.

When the metal is in the form of rolled girders, or other shapes, so placed that the neutral axis of the concrete section passes through their centres of gravity, it can be shown that

$$f = \frac{6MD}{D^3 + 12I^1(\mu - 1)} \dots\dots\dots\dots (9).$$

This equation represents the same curve as for the last case, the value of the co-efficient β being $12I^1$ instead of $24d\left(\frac{D}{2} - a\right)^2$.

These three cases are illustrated respectively by curves 1, 2, and 3, in Fig. 4, the data assumed in each case being as follows:—

Bending moment (M)	= 4,000 inch lbs.
Thickness of concrete (D)	= 15 inches.
Distance from surface of steel to under surface of concrete	= 2·5 inches.
Area of steel per lineal foot of width of section	= 3 sq. inches.

The equivalent of this area of steel is :—For curve No. 1 : One layer of 6in. x $\frac{1}{2}$in. bars spaced 12 inches apart near under surface of concrete. For curve No. 2 : Two layers of 3in. x $\frac{1}{2}$in. bars spaced 12 inches apart near upper and under surfaces of concrete respectively. For curve No. 3 : 10in. x 5in. rolled girders placed symmetrically in the concrete 2 feet 10 inches apart, the area of each girder being 8·53 square inches and its moment of inertia in $(\text{inch})^4$ units, 141·67.

II. Estimation of the Value of the Modulus Ratio (μ).

Although the modulus of elasticity of iron and steel varies only within narrow limits, and for steel is almost identical for hard and soft material[1], the modulus of elasticity of concrete and cement mortar is a very uncertain quantity. It differs in tension and compression, and varies with the age and proportions of the mixture, nature of aggregate, and manner of mixing, and also decreases as the load is increased.

The values of μ assumed by different writers differ widely among themselves. This fact is not surprising when it is considered that most of them are based on the modulus of elasticity of concrete and mortar of various proportions, the tests being made on plain concrete beams or bars without metal. For the reasons mentioned in the opening section of this paper, such a course is liable to lead to serious errors in the determination of μ.

In his paper on "Monier Construction," Mr. Beer takes the modulus of elasticity of concrete at 2,800,000lbs. per sq. in., basing this value on Hartig's experiments, and allowing for the fact that these were conducted with small specimens. This value is, presumably, intended to apply for loads near the breaking point, and for the usual Monier proportion of 3 to 1 mortar or fine concrete; the age is not stated. The modulus for steel is taken at 35,000,000lbs. per sq. in., and μ is given a value of 12.

Mr. Edwin Thacher, of the firm of Keepers and Thacher, U.S.A., assumes for the modulus of elasticity of steel 28,000,000lbs. per sq. in., and for concrete 1,400,000lbs. per sq. in., thus giving μ a value of 20. The modulus of elasticity for concrete is that deduced from the test of the 75·4ft. span concrete arch at Puckersdorf, the main portion of the arch being constructed of concrete of the proportions : 1 P.C., 2 broken stone, 3 gravel, and 3 sand, and the test taking place after two months and three weeks. This value is probably the average taken during the application of the load, but the author was unable to obtain definite information on this point.

1. J. A. Ewing, "The Strength of Materials," p. 93-94.

Professor J. B. Johnson, in his discussion of this question[1], draws attention to the uncertainty of the value of the modulus of elasticity of concrete, and assumes, as a general average, 1,000,000lbs. per square inch, giving μ a value of 28.

In an interesting paper[2] on Monier construction, which was con-communicated to this Society in the year 1900, Mr. J. J. C. Bradfield states that the value of μ usually assumed in the calculation of Monier structures is 40, the modulus for the metal rods being taken at 30,000,000lbs. per square inch, and that for the cement mortar being the average deduced during the testing of the Monier arch at Puckersdorf. This arch was 75·4ft. span, and was constructed of 1 P.C. to 3 river sand, and was tested when two months old.

The experiments of M. Considère and Mr. A. L. Johnson on the deflection of steel-concrete beams have already been referred to. The results of Professor Hatt's tests confirm M. Considère's conclusions, the average deflection of the steel-concrete beams for the load producing first crack, being sixteen times that of the beams not being reinforced (see Table I). The elongation of the steel-concrete bars tested in direct tension was six times as great as that of the bars without steel. The section of these test bars was 4 in. × 4 in., the composition of the concrete 1-2-4, and the age of the tests twenty-six to thirty-five days.

Judging from these results it appears that μ may have a value of at least 100 for beams in which there is one layer of iron or steel bars near the tension surface.

Experimental data on the behaviour of concrete beams containing *two* layers of steel bars, or rolled girders, are urgently required.

III. Properties of Portland Cement Concrete, and Mortar.

The most important properties of concrete and mortar, in relation to steel-concrete construction are—

(1) Co-efficient of expansion.
(2) Modulus of elasticity.
(3) Tensile strength.
(4) Adhesion to iron or steel.
(5) Imperviousness to moisture.

Of these the modulus of elasticity has already been considered; the other properties will be dealt with in order.

Co-efficient of Expansion.

This is a most important quality, as the practicability of steel-concrete construction depends upon a close correspondence between the expansion of the metal and that of the concrete. The following average values per deg. F. have been compiled from the somewhat limited data available :—

Material.	Co-efficient of Expansion per deg. F.	Authority.
Iron (rolled)	·0000067	Trautwine
1 to 3 Cement Mortar	·0000079	J. J. C. Bradfield[3]
1-2-4 ,, Concrete	·0000055	W. D. Pence[4]

1. "The Materials of Construction," 1st Ed., p. 72.
2. "Some Notes on Monier Construction," John J. C. Bradfield, M.E., Assoc. M. Inst. C.E., Jour. Sydney Uni. Eng. Soc., Vol. V., 1900.
3. "Some Notes on Monier Construction," John J. C. Bradfield, M.E., Assoc. M. Inst. C.E., Journal of the Sydney University Eng. Soc., Vol. V., 1900.
4. "The Co-efficient of Expansion of Concrete," W. D. Pence, M.W.S.E., *Eng. News*, Nov. 21st, 1901.

The bars of concrete which were tested by Mr. Pence were composed of 1 P.C., 2 sand, and 4 broken limestone. A bar of the limestone, tested at the same time, gave practically the same value for the co-efficient of expansion as the concrete, and it was therefore concluded that this property depends chiefly on the nature of the stone of which the concrete is composed.

Tensile Strength.

The tensile strength of the mortar and concrete employed in steel-concrete construction is of great importance, as the strength of a beam or arch is practically governed by it. Although when the concrete cracks at the tension surface the structure may still be capable of a high degree of resistance, the section must be so proportioned as to have a considerable factor of safety against cracking, as the cracks, besides being unsightly, would allow moisture to find its way to and corrode the metal, and this would, sooner or later, cause a general failure.

It is very difficult to assign a common average value for the tensile strength of concrete and mortar, as it varies with every brand of cement and variety of sand and stone, and the proportions in which they are used. It is also influenced by the method of mixing and other factors. Although no attempt will be made to deal with the matter very fully in this paper, it is hoped that the figures in Table II. will act as a guide in determining what value should be allowed for the tensile strength. The figures for concrete have mostly been selected from tests on large size specimens, as it is considered that in the case of a non-homogeneous material such as concrete very little reliance can be placed on the result of experiments made on a small scale. Mr. Henby's tests on small bars, however, are of great value in showing the *relative* strength of different mixtures, and two of his average results are included in the table for the purpose of comparison.

With regard to the ratio between the tensile strength and the modulus of rupture of concrete, perhaps this may be taken at one-half, this being the average ratio deduced from a number of tests of granites, sandstones, and limestones which are given in Table XLI., page 643, of J. B. Johnson's "Materials of Construction" (1st ed.).

Assuming this value for the ratio of tensile strength to modulus of rupture, and making due allowance for differences in age and composition, &c., of the results recorded, the author is of the opinion that the following figures may be adopted as representing the tensile strength at an age of one month of two of the principal varieties of concrete and mortar employed in steel-concrete construction:—

For 1-2-4 concrete	300 to 350lbs. per sq. in.
For 1-3 mortar	200 to 250lbs. per sq. in.

The compressive strength is comparatively unimportant in steel-concrete construction, as from bending stresses it is never likely to reach a very high value. From a careful comparison of various authorities, about 2,500lbs. per sq. in. would seem to be a fair average value for 1-2-4 concrete, and 2,000lbs. per sq. in. for 1-3 mortar, at an age of one month.

Imperviousness and Adhesive Qualities.

In his paper on "Monier Construction," Mr. Beer attaches great importance to the necessity for using concrete with a fine aggregate, both to ensure imperviousness, and to cause greater adhesion between the metal and concrete.

In the author's opinion, both these conditions can be satisfied by using a concrete with moderately large aggregate, as long as it is properly proportioned so as to have the voids entirely filled, which can only be the case when an amount of mortar in excess of the actual voids in the broken stone or gravel is provided, the additional amount being required in order to surround each stone with a layer of mortar. There should be small danger of water percolating through such a concrete in quantities sufficient to damage the metal, and enough mortar being provided to entirely surround the iron or steel bars, their adhesion should not be diminished by stones bearing directly upon them.

One of the most usual proportions for concrete in combination with steel is 1 P.C., 2 sand. and 4 broken stone or gravel, and the extent to which the voids are filled in this mixture will now be estimated.

The proportions of voids for broken stone and gravel are approximately, as follows :—

Screened broken stone (harder qualities) ..	50 per cent.
,, ,, ,, (softer qualities).. ..	45 ,, ,,
Unscreened broken stone (harder qualities) ..	40 ,, ,,
Mixture, 2 broken stone and 1 gravel	40 ,, ,,
Unscreened broken stone (softer qualities) ..	35 ,, ,,
Gravel	35 ,, ,,

From various authorities[1] the volume of mortar derived from mixing 1 P.C. with 2 sand, is about 2·5. In proportioning a concrete, an allowance of 10 per cent. of mortar over the volume of the voids in the broken stone is sometimes allowed. According to the experiments of Professor Baker[2], however, an excess of mortar of 40 per cent.[3] is required to entirely fill the voids, each stone being surrounded with a layer of mortar.

Taking the highest percentage of voids, and allowing, say, 10 per cent. for shrinkage under ramming.

Volume of voids in stone, for 1-2-4 concrete,

$$= 4 \times 0{\cdot}9 \times 0{\cdot}5 = 1{\cdot}8,$$

and amount of mortar required,

$$= 1{\cdot}8 \times 1{\cdot}4 = 2{\cdot}52.$$

It will be noticed that even with the highest percentage of voids the amount of mortar for the proportion 1 P.C., 2 sand, and 4 stone, will be sufficient to ensure a solid and impervious concrete.

With regard to adhesive resistance, Prof. Bauschinger found that the adhesion of iron to concrete was about 600lbs. per sq. inch after the latter was thoroughly set. In Prof. J. B. Johnson's

1. Ira O. Baker, "A Treatise on Masonry Construction," 9th Ed., Table 11, p. 88.
G. W. Rafter, "On the Theory of Concrete," Table No. 3, Proc. Am. Soc. C.E., April, 1899.
L. K. Sherman, *Eng. News*, Jan. 9th, 1902.
2. "A Treatise on Masonry Construction," 9th Ed., p. 112b.
3. This result has been confirmed by the experiments of Messrs. Hawley and Krahl, *Eng. News*, June 7th, 1900.