THE DESIGN OF RIVETED JOINTS IN GIRDERS.

BY

W. H. WARREN, M. INST., C.E., WH. SC., M. AM. SOC. C.E.,
CHALLIS PROFESSOR OF ENGINEERING.

SECTION I.

DESCRIPTION AND RESULTS OF TESTS.

ALTHOUGH considerable attention has been devoted to riveted joints as applied to the construction of steam boilers, comparatively little has been done experimentally in regard to the group joints necessary in girders.

The following experiments have been made in order to ascertain the correct proportion of the rivet area to the plate area in group joints, and to demonstrate the behaviour of certain types of joint when subjected to tensile stress. The joints in compression will be abundantly safe if they are constructed precisely the same as the joints in tension. Figs. 1 to 6, Plate I. and Photos 1 to 6, Plate II., show the joints which have been tested; the former give the arrangements of plates, covers and rivets before testing, and the photographs show the various joints after the tests were made. The physical properties of the steel used in the plates and covers are shown by the table in Appendix I.

The rivets were of ordinary rivet steel and their shearing strength is shown in the various tests made, Nos. 1 to 6.

The holes were drilled in the plates and the riveting done by hydraulic pressure; the plates were then planed on their edges to the dimensions shewn in Figs. 1 to 6, Plate I.

The specimens prepared in the manner above described were tested in tension in the ordinary way.

RESULTS OF TESTING RIVETED JOINTS.

NO. 1.—FIG. 1.—PHOTO. 1.

Rivet area	= $4 \times 0{\cdot}4418$	=	1·77 square inches
Plate area, net	= $3\,(2{\cdot}25 - 0{\cdot}75)\,\frac{3}{8}$	=	1·70 ,, ,,
Plate area, gross	= $3 \times 2{\cdot}25 \times \frac{3}{8}$	=	2.53 ,, ,,
Bearing area	= $4 \times \frac{3}{4} \times \frac{3}{8}$	=	1·125 ,, ,,

The joint failed by the tearing of the three plates across the rivet holes in one end group. The stress on the plates was

$$\frac{39{\cdot}8}{1{\cdot}7} = 23{\cdot}4 \text{ tons per square inch.}$$

The stress on the rivets $= \frac{39 \cdot 8}{1 \cdot 77} = 22 \cdot 4$ tons per square inch

The pressure on the bearing area

$$= \frac{39 \cdot 8}{1 \cdot 125} = 35 \cdot 3 \text{ tons per square inch}$$

The rivets cut into the plates elongating the rivet holes, but the rivets did not shear.

At 20 tons the joint yielded slightly and the stresses were as follows :—

On the plates $= \frac{20}{1 \cdot 7} = 11 \cdot 7$ tons per square inch

On the rivets $= \frac{20}{1.77} = 11 \cdot 2$ tons per square inch

The pressure on the bearing area

$$= \frac{20}{1 \cdot 125} = 17 \cdot 8 \text{ tons per square inch}$$

No. 2. Fig. 2. Photo 2.

Rivet area	=	$4 \times 0 \cdot 4418$	=	1·77 square inches
Plate area, net	=	$3(2 \cdot 25 - 0 \cdot 75)\frac{3}{8}$	=	1·70 " "
Plate area, gross	=	$3 \times 2 \cdot 25 \times \frac{3}{8}$	=	2·53 " "
Bearing area	=	$4 \times \frac{3}{4} \times \frac{3}{8}$	=	1·125 " "

The joint failed by the tearing of the three plates across the rivet holes in one end group.

The stress on the plates was :—

$$\frac{39 \cdot 5}{1 \cdot 7} = 23 \cdot 2 \text{ tons per square inch.}$$

The stress on the rivets was :—

$$\frac{39 \cdot 5}{1 \cdot 77} = 22 \cdot 3 \text{ tons per square inch.}$$

The pressure on the bearing area was :—

$$\frac{39 \cdot 5}{1 \cdot 125} = 35 \cdot 1 \text{ tons per square inch.}$$

The rivets cut into the plates elongating the rivet holes, but the rivets did not shear.

At 20 tons the joint yielded slightly and the stresses were as follows :—

On the plates $= \frac{20}{1 \cdot 7} = 11 \cdot 7$ tons per square inch.

On the rivets $= \frac{20}{1 \cdot 77} = 11 \cdot 2$ tons per square inch.

The pressure on the bearing area

$$= \frac{20}{1 \cdot 125} = 17 \cdot 8 \text{ tons per square inch.}$$

No. 3. Fig. 3. Photos 3 and 3a.

Rivet area	=	$6 \times 0{\cdot}4418$	=	2.65 square inches.
Plate area net	=	$4(2{\cdot}25 - 0{\cdot}75)\frac{3}{8}$	=	2.25 „ „
Plate area gross	=	$4 \times 2{\cdot}25 \times \frac{3}{8}$	=	3·375 „ „
Bearing area	=	$6 \times \frac{3}{4} \times \frac{3}{8}$	=	1·7 „ „

The plates failed outside the joints by tearing across a countersunk rivet hole at the ends, but the two cover plates were considerably necked, showing that they were on the point of failure.

Half the load was therefore concentrated on the two rivets forming the central group, the rivets in each of the two end groups being stressed only to half this amount.

The total load applied was 58·2 tons, and the stresses were as follows :

On the four plates $= \dfrac{58{\cdot}2}{2.25} =$ 25·8 tons per square inch.

On the two rivets in the central group, in triple shear, equivalent to twice single shear, $= \dfrac{29{\cdot}1}{0{\cdot}8836} =$ 32.9 tons per square inch.

The pressure on the bearing area, where the central plate has twice the pressure on it as the outer plate, (*See photos* 3 *and* 3*a*.)

$$= \frac{29{\cdot}1}{0{\cdot}85} = 34{\cdot}1 \text{ tons per square inch.}$$

At 34·8 tons the joint yielded slightly and the stresses were as follows :—

On plates $= \dfrac{34{\cdot}8}{2{\cdot}25} =$ 15·4 tons per square inch.

On rivets $= \dfrac{17{\cdot}4}{0{\cdot}8836} =$ 18·5 „ „ „

Pressure on bearing area $= \dfrac{17{\cdot}4}{0{\cdot}85} =$ 20·5 „ „ „

The joint was, after testing, cut along its length through the rivet holes. (*See photo* 3*a*.)

No. 4.—Fig. 4.—Photo. 4.

Rivet area	=	$7 \times 0{\cdot}4418$	=	3·10 square inches
Plate area, net	=	$4(2{\cdot}25 - 0{\cdot}75)\frac{3}{8}$	=	2·25 „ „
Plate area, gross	=	$4 \times 2{\cdot}25 \times \frac{3}{8}$	=	3·375 „ „
Bearing area	=	$7 \times \frac{3}{4} \times \frac{3}{8}$	=	1·97 „ „

The joint failed by the tearing of one cover plate across a rivet hole, and also of two of the plates in a similar manner. The rivets in the end group were sheared, and the other cover plate was on the point of failing. In this case the three rivets in the central group must have resisted about one-half of the total load applied or

$$\frac{59{\cdot}7}{2} = 29{\cdot}85 \text{ tons}$$

The stress on the rivets in the central group in triple shear which was actually equivalent to twice the single shear was :—

$$\frac{29{\cdot}85}{1{\cdot}3254} = 22{\cdot}5 \text{ tons per square inch}$$

The pressure on the bearing area was :—

$$\frac{29{\cdot}85}{1{\cdot}26} = 23{\cdot}6 \text{ tons per square inch}$$

The stress on the plates was :—

$$\frac{59{\cdot}7}{2{\cdot}25} = 26{\cdot}5 \text{ tons per square inch}$$

No. 5. Figs. 6 and 6a. Photo. 5.

Rivet area	=	$9 \times 0{\cdot}4418$	=	3·97 square inches
Plate area, net	=	$4(3 - 0{\cdot}75)\frac{3}{8}$	=	3·37 ,, ,,
Plate area, gross	=	$4 \times 3 \times \frac{3}{8}$	=	4·50 ,, ,,
Bearing area	=	$9 \times \frac{3}{4} \times \frac{3}{8}$	=	2·53 ,, ,,

The joint failed by the tearing of the two cover plates across a rivet hole at the same time as the the three rivets sheared in the central group. The total pull causing fracture was 59.8 tons or

$$\frac{59{\cdot}8}{3{\cdot}37} = 18 \text{ tons per square inch in the effective plate area.}$$

The two cover plates would resist :—

$$1{\cdot}685 \times 18 = 29{\cdot}9 \text{ tons per square inch.}$$

Leaving 29·9 tons to be resisted by the three rivets, or :

$$\frac{29{\cdot}9}{1{\cdot}325} = 22{\cdot}5 \text{ tons per square inch}$$

The pressure on the bearing area is

$$\frac{29{\cdot}9}{1{\cdot}843} = 35{\cdot}4 \text{ tons per square inch}$$

The diagram A, Fig. 6a, Plate III, shows that there was a slight yield at 28 tons and a decided yield at 44 tons.

At the former pressure the stresses are as follows :—

On the plates net section $= \frac{28}{3{\cdot}37} = 8{\cdot}3$ tons per square inch

On the two covers net section $= 1{\cdot}685 \times 8{\cdot}3 = 14$ tons

On the three rivets in central group

$$\frac{14}{1{\cdot}325} = 10{\cdot}4 \text{ tons per square inch}$$

Pressure on the bearing area is

$$\frac{14}{0{\cdot}843} = 16{\cdot}6 \text{ tons per square inch}$$

No. 6., Figs. 5, and 5a., Photo. 6.

Rivet area	=	$8 \times 0{\cdot}4418$	=	3·53 square inches
Plate area, net	=	$4(3 - 0{\cdot}75)\frac{3}{8}$	=	3·37 ,, ,,
Plate area, gross	=	$4 \times 3 \times \frac{3}{8}$	=	4·50 ,, ,,
Bearing area	=	$8 \times \frac{3}{4} \times \frac{3}{8}$	=	2·25 ,, ,,

The joint failed by the cover plates tearing across a rivet hole, and at the same time the rivets in the central group cutting into the plates.

The total pull causing fracture was 81 tons.

The stress on the net section of the plates was :—

$$\frac{81}{3{\cdot}37} = 24 \text{ tons per square inch}$$

The resistance of the two covers was, therefore, 40·5 tons, leaving 40·5 tons to be resisted by the two rivets in the central group, and 20·25 tons in each of the two end groups.

The stress on the rivets in the central group was :—

$$\frac{40{\cdot}5}{0{\cdot}8836} = 45{\cdot}8 \text{ tons per square inch}$$

or twice as much as in the rivets in joint No. 5.

The rivets in the central group were, therefore, equal to twice the value of the same number of rivets subjected to single shear. Actually these rivets are in triple shear and the intensity of shearing stress is :—

$$\frac{40{\cdot}5}{2 \times 3 \times 0{\cdot}4418} = 15{\cdot}3 \text{ tons per square inch}$$

In this case one rivet in triple shear equals 68 per cent. of three rivets in single shear.

The pressure on the bearing area is found by observing that the intensity of pressure on each of the two middle plates is twice that on each of the two outer plates, (*See Photos.* 3 *and* 3*a*) ; hence

$$\frac{40{\cdot}5}{0{\cdot}85} = 47{\cdot}6 \text{ tons per square inch}$$

The excessive pressure on the bearing area caused the rivets to cut through the plates instead of shearing at three sections.

The stress (Diagram B, Fig. 6a, Plate III.) shows a yield point at 36 tons, and another yield at 55 tons.

The stress on the four plates net section :—

$$= \frac{36}{3{\cdot}37} = 10{\cdot}7 \text{ tons per square inch}$$

On the two rivets in the central group

$$= \frac{18}{0{\cdot}8836} = 23{\cdot}7 \text{ tons per square inch}$$

The pressure on the bearing area

$$= \frac{18}{0{\cdot}85} = 21{\cdot}1 \text{ tons per square inch}$$

SECTION II.

Design of Riveted Joints in Girders.

Figs. 7 to 11, Plates I. and IV., represent various group joints suitable for the tension flanges of girders.

The arrangement of plates, angles and rivets, shown in Figs. 7 to 9, illustrate the method of designing a group joint in the tension flange of a plate web girder bridge. The thickness of the cover plates on the top and bottom must be made slightly thicker than that of the plates covered, as two joints in the plates come in the same vertical plane. In Fig. 10 the thickness of each of the two cover plates should be equal, as the lever arm with reference to the joint in the upper plate on the right and the top cover is the same as the lever arm with reference to the joint in the lower plate on the left and the bottom cover. If the plates are of equal thickness there will be a slight excess in strength if the covers are also made the same thickness as the plates. If the plates vary in thickness it will depend upon their position in so far as they effect the length of the lever arms of the covers in regard to the joint. If each of the four plates shown in Fig. 10 is half-inch thick the cover plate should also be made half-inch thick, and the excess in area may be neglected.

With reference to the group of joints shown in Figs. 7, 8, 9 and 11, it follows from the foregoing reasoning that the thickness of the cover plates should be about one-fifth greater than one of the four plates, so that it will be sufficiently correct to make these covers five-eighths inch thick if the plates are half-inch thick.

In Figs. 7, 8 and 9 the angle bars uniting the web to the flange plates are stopped against the inner cover plate, and a recess is cut out of the web plate the length and thickness of the inner cover plate. The angle covers or wrappers rest upon the top of the inner cover plate and are prolonged beyond it sufficiently to provide the necessary rivet area. The rivet area in the portion of the angle covers extending beyond the inner cover plate, must be 1·4 times the effective area of the two main angle bars.

The effective area of the two main angle bars, five inches x five inches x five-eighths of an inch in tension is 10·75 square inches, so that the rivet area necessary is fifteen square inches and the number of rivets seven-eighths of an inch in diameter

$$\frac{15}{0{\cdot}6} = 25$$

The joints, Figs. 7, 8 and 9, show seven rivets on each side in double shear, and fourteen rivets in single shear, which is at least equivalent to twenty-five rivets in single shear.

The special arrangement of rivets, angles, plates and covers illustrated in Figs. 7, 8 and 9 was suggested to me by Mr. J. H. Frazer, of the Railway Department, Melbourne.

In the joints illustrated in Figs. 7 to 11 failure may occur in three ways—

1. By the shearing of all the rivets in the joints between 1 and 6.

2. By the shearing of the rivets in the central group or groups between 2 and 5, and the tearing of both cover plates at the same time.

3. By the rivets cutting into the plates, or the plates into the rivets from insufficiency of bearing area or from excessive pressure upon the bearing area.

In regard to the first mode of failure :—

Let n = the total number of rivets in the joint.
p = the number of plates.
b = the effective width of a plate after deducting the diameters of the rivet holes at a section.
t = the thickness of a plate.
a = the area of a rivet.
d = the diameter of a rivet.

Then we require the number of rivets to resist shearing in Fig. 10:—

$$n = \frac{1{\cdot}4\ p\ b\ t}{a}$$

If m = the number of rivets in the end groups, Figs. 1 and 2.
r = the number of rivets in each of the central groups, Fig. 10, and in the central group, Fig. 11.

Then for the first mode of failure, Fig. 10 :—

$$n = \frac{1{\cdot}4\ p\ b\ t}{a} = 2\ m + (p - 1)\ r \ldots\ldots\ldots(1)$$

For the second mode of failure, if the thickness of each cover plate be made equal to t then :—

The cover plate area	=	2 b t
The number of rivets in the central groups, Fig. 10	=	(p – 1) r
and in Fig. 11	=	r
The rivet area in central group, Fig. 10	=	(p – 1) r a
and in Fig. 11	=	r a

$$\text{Equivalent plate area, Fig. 10} = \frac{(p - 1)\ r\ a}{1{\cdot}4}$$

$$\therefore\ 2\ b\ t + \frac{(p - 1)\ r\ a}{1{\cdot}4} = p\ b\ t \ldots\ldots\ldots(2)$$

The rivet area in the central group, Fig. 11, is equivalent to at least twice that of the rivets in Fig. 10, since they would be sheared at three sections, but the results of experiments made on joints, Figs. 1 to 6, show that it will be better to take the effective area of these rivets as 2 r a

$$\therefore\ 2\ b\ t^1 + \frac{2\ r\ a}{1{\cdot}4} = p\ b\ t \ldots\ldots\ldots(3)$$

where t^1 is the thickness of the cover plates. Equations (2) and (3) may be solved to find r, which may be substituted in equation (1) to find m.

In regard to the safe pressure on the bearing area: It may happen that the number of rivets resisting for shearing may be insufficient to provide for the necessary bearing area. If we allow ten tons per square inch as the greatest permissible intensity of pressure on structural steel, such as that used in bridge work, the ratio of the intensity of pressure on the bearing area to the tensile resistance of the plates should not exceed 1·6, *i.e.* the net sectional area of the plates in tension should not be more than 1·6 times the bearing area.

The pressure on the bearing area is most conveniently examined after the joint has been designed to resist the first and second mode of failure, and if found to be excessive, additional rivets may be added to reduce the pressure to the assigned limit.

SECTION III.

Numerical Example.

To design a group joint similar to Figs. 10 and 11, Plate iv., having given that the four plates are twenty-four inches wide and a half inch thick, riveted with four rows of steel rivets seven-eighths of an inch in diameter and three inches pitch.

If the plates are arranged as in Fig. 10, the total number of rivets required, since they are all in single shear, is :—

$$n = \frac{1{\cdot}4 \times 4 \times 10{\cdot}25}{0{\cdot}6} = 96$$

The cover plate area $= 2 \times 10{\cdot}25 = 20{\cdot}5$ square inches.
The rivet area in each of the central groups $= r \times 0{\cdot}6$
And in the three central groups $= 3\,r \times 0{\cdot}6 = 1{\cdot}8\,r$

The equivalent plate area $= \frac{1{\cdot}8\,r}{1{\cdot}4} = 1{\cdot}28\,r$

For the second mode of failure we have :—

$$20{\cdot}5 + 1{\cdot}28\,r = 10{\cdot}25 \times 4 = 41 \therefore r = 16 \text{ about}$$

For the first mode of failure we have :—

$$2\,m + 3 \times 16 = 96 \therefore m = 24$$

The number of rivets must be a multiple of four since there are four rows.

The bearing area of the rivets in the central groups is —

$$48 \times \tfrac{7}{8} \times \tfrac{1}{2} = 21 \text{ square inches.}$$

and the stress in the rivets according to the second mode of failure must be one half the total stress in the four plates, hence the ratio of the tearing to the bearing area in the central groups :—

$$\frac{20{\cdot}5}{21} = 0{\cdot}98 \text{ which is well within the limit } 1{\cdot}6.$$

The total bearing area in the joint is :—

$$96 \times \tfrac{7}{8} \times \tfrac{1}{2} = 42 \text{ square inches.}$$

The total tearing area is 41, so that there is no danger of failure from excessive pressure on the bearing area.

The joint may be designed as in Figure 10.

If the plates are arranged as in Fig. 11, the resistance of a rivet in the central group will be equivalent to twice the resistance of a rivet in the end groups for reasons already stated.

$$\text{The equivalent plate area} = \frac{2\ \mathrm{r} \times 0{\cdot}6}{1{\cdot}4} = 0{\cdot}85\ \mathrm{r}$$

The resistance of the two cover plates $\frac{5}{8}$ of an inch thick

$$= 20{\cdot}5 \times 2 \times \tfrac{5}{8} = 25{\cdot}6 \text{ square inches}$$

For the second mode of failure we have :—

$$25{\cdot}6 + 0{\cdot}85\ \mathrm{r} = 41 \quad \therefore \quad \mathrm{r} = \text{over } 18$$

If we make r = 24 we shall have a slight excess of rivet area in the central group. For the first mode of failure we require 96 rivets in single shear, or 72 rivets if the 24 rivets in the central group are considered as equivalent to 48 rivets in single shear. In this case it follows that m = r = 24.

The maximum pressure on the bearing area is found as follows :— In the central group when the cover plates fail, the two remaining plates provide a bearing area of :—

$$2 \times 24 \times \tfrac{7}{8} \times \tfrac{1}{2} = 20{\cdot}1 \text{ square inches}$$

But since the intensity of pressure is twice as great on the inner plate as on the plate nearest the cover, the equivalent bearing area is only 15·75 square inches.

The effective area of the two cover plates is :—

$$2(24 - 4 \times \tfrac{7}{8})\tfrac{5}{8} = 25{\cdot}625 \text{ square inches}$$

The effective area of the four plates is :—

$$4(24 - 4 \times \tfrac{7}{8})\tfrac{1}{2} = 41 \text{ square inches.}$$

The stress on the rivets in the central group is equal to the resistance of a plate area represented by :—

$$41 - 25{\cdot}625 = 15{\cdot}375 \text{ square inches.}$$

Therefore the ratio of the tearing to the bearing area is :—

$$\frac{15.375}{15.75} = \text{less than } 1$$

which is well within the limit of 1·6.
Hence the joint may be designed as in Fig. 11.

APPENDIX I.

TENSILE TESTS OF STEEL CUT FROM GIRDER PLATES.

Test Number.	Description.	Original Dimensions.			Stress in Pounds.		Stress in Tons. Per Sq. Inch.	Apparent Limit of Elasticity from Auto-diagrams.	Ratio of Limit to Break.	Contracted Dimensions.			Contraction of Area, per cent.	Elongations Measured after Fract'r.		Local Elongation.	General Elongation. Per cent.	Total Elongat'n per cent. on 8 in.	Co-efficient of Quality.	Remarks.
		Breadth	Thick-ness.	Area.	Total.	Per Sq. Inch.			%	Breadth	Thick-ness.	Area.		on 8 in.	on 4 in.					
A	Flat Bar	1·644	·38	·62472	34,500	55,220	24·6	22,500	65	1·31	·235	·30785	51	2·3	1·33	·36	24·2	29	7·1	Sound Clean Fracture.
B	Flat Bar.	1·644	·38	·62472	35,400	56,660	25·2	23,000	65	1·30	·22	·286	54	2·3	1·33	·36	24·2	29	7·3	do.
C	Flat Bar.	1·64	·38	·6232	35,900	57,600	25·7	23,100	64	1·30	·24	·3180	49	2·3	1·4	·50	22·5	29	7·4	do.
D	Flat Bar.	1·64	·38	6232	35,450	56,880	25·4	20,000	56	1·29	·223	·28769	53	2·4	1·5	·60	22·5	30	7·6	do.

Modulus of elasticity, calculated from elongations recorded by Marten's Mirror Apparatus, 24,990,000 lbs. per square inch.