Load on right span.

$$R_3 = (1 - k) - R_2\left(1 + \frac{1}{r}\right)$$

$$= (1 - k) + \frac{r}{2} K_2$$

$$\int_{k=0}^{k=1} R_3 = \left\{ \frac{1}{2} + \frac{r}{8} \right\} W_3$$

In each case $R_2 + R_3 + R_4 =$ unity.

Table 5.—Bending Moments and Reactions.

FIGS. 5*a*—5*d*.

$l_3 = 15. \quad l_2 = 12. \quad r = 5/4.$

	Load on l_2.				Load on l_3.			
k	M_3	R_2	R_3	R_4	M_3	R_2	R_3	R_4
·1	− ·264	+ ·878	+ ·139	− ·018	− ·712	− ·059	+1·007	+ ·053
·2	− ·512	+ ·757	+ ·277	− ·034	−1·200	− ·100	+ ·980	+ ·120
·3	− ·728	+ ·639	+ ·409	− ·049	−1·487	− ·124	+ ·923	+ ·201
·4	− ·896	+ ·525	+ ·534	− ·060	−1·600	− ·134	+ ·840	+ ·293
·5	−1·000	+ ·417	+ ·650	− ·067	−1·562	− ·130	+ ·734	+ ·396
·6	−1·024	+ ·315	+ ·754	− ·068	−1·400	− ·117	+ ·610	+ ·507
·7	− ·952	+ ·221	+ ·843	− ·063	−1·137	− ·095	+ ·471	+ ·634
·8	− ·768	+ ·136	+ ·915	− ·051	− ·800	− ·067	+ ·320	+ ·747
·9	− ·456	+ ·062	+ ·968	− ·030	− ·412	− ·034	+ ·162	+ ·873
1·0	− ·000	+ ·000	+1·000	− ·000	− ·000	− ·000	+ ·000	+1·000

Some applications will be given of the above.

B. Mt. at 3 due to left span fully loaded with uniform load w per unit of length, Fig. 5*e*.

$$= \frac{-l_2}{2(1+r)} \int_0^1 (k - k^3)dk \times wl_2$$

$$= \frac{-wl_2{}^2}{8(1+r)} = \frac{-W_2 l_2}{8(1+r)} \text{ where } W_2 \text{ is total load on } l_2$$

B. Mt. at 3, right span fully loaded. (Fig. 5*f*.)

$$= \frac{-l_3 r}{2(1+r)} \int_0^1 \left\{ (1-k) - (1-k)^3 \right\} d(1-k)\, l_3 w$$

$$= \frac{-wl_3^2r}{8(1+r)} = \frac{-W_3 l_3 r}{8(1+r)} \text{ where } W_3 \text{ is total load on } l_3$$

The latter result may be obtained from the former by putting $r = \frac{1}{r}$, W_3 for W_2 and l_3 for l_2.

These examples will serve to illustrate the meaning of the integral sign and the method of deducing the results given above.

$R_1 + R_2 + R_3 = W_2$ or W_3 according to whether load is as in Fig. 5*e* or Fig. 5*f*.

The diagram for M_3 Fig. 5*a*, shews graphically how Clapeyron's equation is connected with the general equation for a concentrated load, and the last results all check with results worked directly from Clapeyron's equation.

By means of these diagrams the points of contraflexure (where B. Mt. is zero), and also shears may be readily deduced.

Case 6.—THREE SPANS ON FOUR SUPPORTS, THE SPANS BEING EQUAL IN LENGTH. (Fig. 6.)

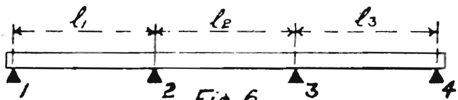


Fig. 6

Bending moment at 2 (Fig. 6*a*.)

Load on span, l_1.

$$M_1 l_1 + 2M_2(l_1 + l_2) + M_3 l_2 = -l_1^2(K_1).$$

here $M_1 = O$ and $l_1 = l_2 = l$ say.

$\therefore\ 4M_2 + M_3 = -lK_1$ considering 1, 2, 3 supports

$M_2 + 4M_3 = O$ „ 2, 3, 4 „

$$\therefore\ M_2 = -\frac{4l}{15}K_1 \text{ and } M_3 = \frac{l}{15}K_1.$$

Load on span l_2

$4M_2 + M_3 = -lK_2$ considering 1, 2, 3 supports

$M_2 + 4M_3 = -lK_1$ „ 2, 3, 4 „

$$\therefore\ M_2 = -\frac{l}{15}\left\{4K_2 - K_1\right\} \quad M_3 = -\frac{l}{15}\left\{4K_1 - K_2\right\}$$

Load on span l_3.

By symmetry M_2 for load on l_3 is same as M_3 for load on l_1 with $(1-k)$ put for k.

$$M_2 = \frac{l}{15}K_2.$$

The influence line of B. Mt. at 3 is deduced by symmetry from M_2.

Reaction R_1.

By similar reasoning to Case 4.

Load on end span l_1 $\quad R_1 = \frac{M_2}{l} + (1-k) = (1-k) - \frac{4}{15}(K_1)$

$$\int_{k=0}^{k=1} R_1 = \frac{13}{30} W$$

Load on mid. span l_2 $\quad R_1 = \frac{M_2}{l} = -\frac{1}{15}\left\{ 4K_2 - K_1 \right\}$

$$\int_{k=0}^{k=1} R_1 = \frac{-1}{20} W$$

Load on end span l_3 $\quad R_1 = \frac{M_2}{l} = \frac{1}{15} K_1.$

$$\int_{k=0}^{k=1} R_1 = \frac{+1}{60} W.$$

Reaction R_2.

Load on l_1 $\quad R_2 = \frac{M_3}{l} + (2-k) - 2R_1 = \frac{9}{15} K_1 + k.$

$$\int_{k=0}^{k=1} R_2 = \frac{13}{20} W.$$

Load on l_2 $\quad R_2 = \frac{M_3}{l} + (1-k) - 2R_1 = (1-k) + \frac{1}{5}\left\{ 3K_2 - 2K_1 \right\}$

$$\int_{k=0}^{k=1} R_2 = \frac{+11}{20} W.$$

Load on l_3 $\quad R_2 = \left\{ (1-k) - 3R_1 - R_3 \right\} \frac{1}{2} = \frac{-6}{15} K_2.$

$$\int_{k=0}^{k=1} R_2 = \frac{-1}{10} W.$$

R_3 is deduced from R_2 by symmetry, and R_4 is deduced from R_1 in same way.

Fig 5.

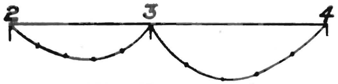


Fig. 5a. B.M.t at 3

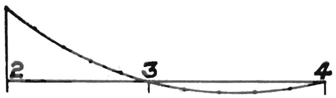


Fig. 5b. Reaction at 2

Fig. 5c. Reaction at 4

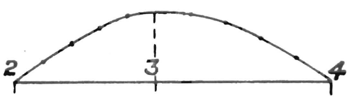


Fig. 5d. Reaction at 3.

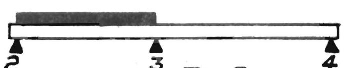


Fig. 5e

Fig 5f

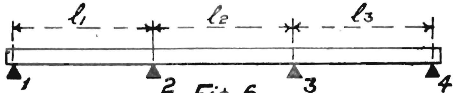


Fig. 6

Fig. 6a. B.M.t at 2

Fig. 6b. Reaction at 1

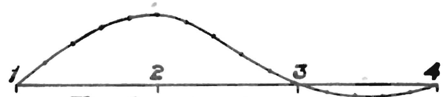


Fig. 6c. Reaction at 2

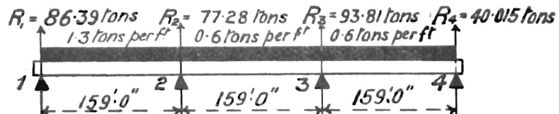


Fig. 6d

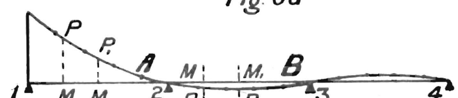


Fig. 6e.

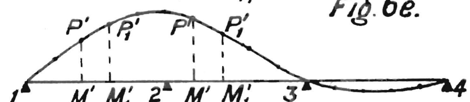


Fig. 6f.

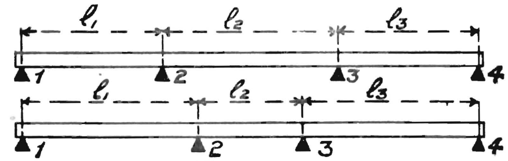
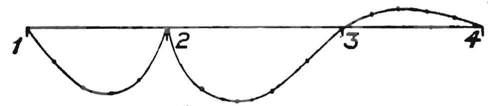


Fig. 7.



B.M.t at 2 Fig. 7a

Reaction at 1 Fig. 7b

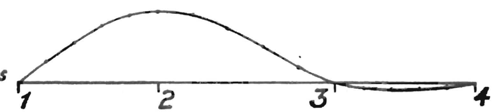


Fig. 7c. Reaction at 2

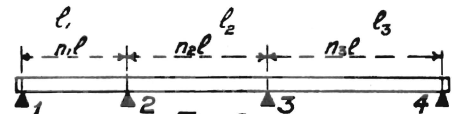


Fig. 8

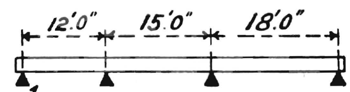


Fig. 8a

Table 6.—Shewing Bending Moments and Reactions for Three Spans on Four Supports.

	Load on l_1.			Load on l_2.			Load on l_3.		
k	$\frac{M_2}{l}$	R_1	R_2	$\frac{M_2}{l}$	R_1	R_2	$\frac{M_2}{l}$	R_1	R_2
	−	+	+	−	−	+	+	+	−
·05	·0133	·9368	·0799	·0214	·0214	·9856	·0062	·0033	·0370
·10	·0264	·8736	·1594	·0390	·0390	·9630	·0114	·0066	·0684
·15	·0391	·8109	·2380	·0531	·0531	·9329	·0156	·0098	·0944
·20	·0512	·7488	·3152	·0640	·0640	·8960	·0192	·0128	·1152
·25	·0626	·6875	·3906	·0719	·0799	·8531	·0219	·0156	·1312
·30	·0728	·6272	·4638	·0770	·0770	·8050	·0238	·0182	·1428
·35	·0819	·5681	·5343	·0796	·0796	·7524	·0250	·0205	·1502
·40	·0896	·5104	·6016	·0800	0800	·6960	·0256	·0224	·1536
·45	·0957	·4543	·6653	·0784	·0784	·6366	·0255	·0239	·1534
·50	·1000	·4000	·7250	·0750	·0750	·5750	·0250	·0250	·1500
·55	·1023	·3477	·7802	·0701	·0701	·5119	·0239	·0256	·1436
·60	·1024	·2976	·8304	·0640	·0640	·4480	·0224	·0256	·1344
·65	·1001	·2499	·8752	·0569	·0569	·3841	·0205	·0250	·1228
·70	·0952	·2048	·9142	·0490	·0490	·3210	·0182	·0238	·1092
·75	·0875	·1625	·9469	·0406	·0406	·2594	·0156	·0219	·0938
·80	·0768	·1232	·9728	·0320	·0320	·2000	·0128	·0192	·0768
·85	·0629	·0871	·9915	·0234	·0234	·1436	·0098	·0157	·0586
·90	·0456	·0544	1·0026	·0150	·0150	·0910	·0066	·0114	·0396
·95	·0247	·0253	1·0056	·0070	·0070	·0429	·0033	·0062	·0200
1.00	·0000	·0000	1·0000	0000	·0000	·0000	·0000	·0000	·0000

It will be seen from the above that the design of a continuous girder, so far as stresses are concerned, is as direct as that of a plain girder. The equations and theory used depend on the supports moving uniformly (it is beyond the limit of this paper to investigate the effects of supports not moving uniformly), and have all the liability to error due to imperfect elasticity and such other factors as operate against a definite knowledge of the stresses in any structure which is not of the "simple" kind; but for the finding of stresses according to the ordinary continuous beam theory, the writer thinks that the method is clearer and neater than other methods.

The curves for reactions need only be plotted for the spans under consideration, and the reactions for any loading are readily deduced from the tables (or the value of the ordinates might be written on them if there were room). From these the bending moments and shears are readily deduced.

For instance, in the working out of a continuous girder, Warren's Engineering Construction, pp. 168-175, in getting the maxima bending moments, five cases are taken and the moments worked out for each; in the method used there the calculations could be simplified by the use of influence lines to give the reactions, and the bending moments and shears worked directly.

In short, the Influence Lines work out the required function once for all for the structure; while without them much of the same ground is covered several times.

Take Case 1, in example mentioned above (Fig. 6*d*).

l_1 loaded with live and dead loads $= 1{\cdot}3$ tons per ft.

l_2, l_3 „ „ dead load only $= 0{\cdot}6$ „ „

$l_1 = l_2 = l_3 = 159$ ft.

$$\text{Thus } M_1 = -\frac{1}{15}\, lW_\alpha - \frac{1}{20}\, lW_\beta + \frac{1}{60}\, lW_\gamma.$$

Where W_α = load on span 12 = 159 ft. × 1·3 tons per ft.

W_β = „ „ 23 = 159 ft. × ·6 „ „

W_γ = „ „ 34 = 159 ft. × ·6 „ „

M_1 = B. Mt. at first pier from left.

$$\text{Thus } M_1 = 159^2 \left\{ -\frac{1}{15}(1{\cdot}3) - \frac{1}{20}({\cdot}6) + \frac{1}{60}({\cdot}6) \right\}$$

$= 2696{\cdot}6$ as in text.

This saves the labour and liability to error in deducing and solving the simultaneous equations.

For same case.

$$R_1 = \frac{13}{30} W_\alpha - \frac{1}{20} W_\beta + \frac{1}{60} W_\gamma.$$

$$= 159 \text{ ft.} \left\{ \frac{13}{30} \times 1{\cdot}3 \text{ tons} - \frac{1}{20} \times {\cdot}6 \text{ tons} + \frac{1}{60} \times {\cdot}6 \text{ tons} \right\}$$

$= +\ 86{\cdot}39$ tons.

$$R_2 = \frac{13}{20} W_\alpha + \frac{11}{20} W_\beta - \frac{1}{10} W_\gamma.$$

$= +\ 177{\cdot}285$ tons.

$$R_3 = \frac{13}{20} W_\gamma + \frac{11}{20} W_\beta - \frac{1}{10} W_\alpha$$

$= +\ 93{\cdot}81$ tons.

$$R_4 = \frac{13}{30} W_\gamma - \frac{1}{20} W_\beta + \frac{1}{60} W_\alpha$$

$= 40{\cdot}015$ tons.

We have now the loads and reactions as shewn in Fig. 6*d*. From this can be deduced the B. Mt. or shear at any point and the full effect of the loading known in detail as with a simple beam.

The other cases could be similarly treated, and, if required, influence lines of B. Mt. or Shear at any point could be constructed.

It is stated on p. 175, Warren's Engineering Construction. "In consequence of the change of position of the points of contraflexure during the passage of a rolling load, the shearing stresses cannot be accurately determined without considerable labour," and the text shews approximate methods of dealing with moving loads, distributed and concentrated.

With diagrams such as Figs. 6*a*, 6*b*, 6*c* drawn to scale, the shears for any position of a rolling load offer no difficulty.

An example will be given. (Figs. 6*e* and 6*f*.)

Take a rolling load, $W_1\ W_2$ on span 12.

Shear at any point A on span 12.

$$= \{W_1 \times PM\} + \{W_2 \times P_1 M_1\} - (W_1 + W_2) \text{ Fig. } 6e.$$

Again, shear at any point B on span 23. (Figs. 6*e* and 6*f*.)

$$= W_1 \times \{PM + P^1 M^1\} + W_2 (P_1 M_1 + P_1^1 M_1^1) - (W_1 + W_2)$$

Take the loads to have rolled on span 23.

Shear at A $= - \{W_1 (PM) + W_2 (P_1 M_1)\}$ Fig. 6*e*. Span 23.

Shear at B $= - \{W_1 (PM) + W_2 (P_1 M_1)\} +$
$\{W_1 (P^1 M^1) + W_2 (P_1^1 M_1^1)\}$
$- (W_1 + W_2)$* Figs. 6*e* and 6*f*. Span 23.

This example will suffice to shew the method of working. Results could be similarly deduced for concentrated or distributed loads in any position.

Case 7.—THREE SPANS, THE TWO END SPANS BEING EQUAL IN LENGTH. (Fig. 7.)

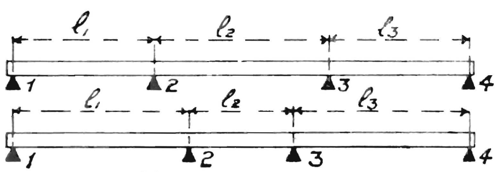


Fig. 7.

Let $l_1 = l_3 = l$ say.
and $l_2 = nl$, where n may be $\lesseqgtr$ unity.

Let m represent the quantity $4 (1 + n)^2 - n^2$ which will be often used.

As before $K_1 = k - k^3$ and $K_2 = (1 - k) - (1 - k)^3$

Bending Moments.

From general equation $M_2 l_2 + 2M_3 (l_2 + l_3) + M_4 l_3 = - l_2^2 (k - k^3)$

---

* The last term non-existent when B is on left of M.

Load on l_1. Since $M_1 = M_4 =$ zero.

$$2M_2 (l_1 + l_2) + M_3 l_2 = - l_1^2 K_1$$
$$\text{also } M_2 l_2 + 2M_3 (l_2 + l_3) = O$$
$$\therefore M_2 = \frac{2(1+n)}{m} K_1 l \text{ and } M_3 = \frac{nK_1}{m} l$$

Load on l_2.

$$2M_2 (l_1 + l_2) + M_3 l_2 = - l_2^2 K_2$$
$$M_2 l_2 + 2M_3 (l_2 + l_3) = - l_2 K_1$$
$$\therefore M_2 = \frac{-ln^2}{m} \left[2(1+n) K_2 - n K_1\right]$$
$$\text{and } M_3 = \frac{-ln^2}{m} \left[2(1+n) K_1 - n K_2\right]$$

Load on l_3. By symmetry.

$$M_2 = + \frac{n}{m} K_2 l \text{ and } M_3 = - \frac{2(1+n)}{m} K_2 l$$

Thus for Influence Lines of M_2

Load on l_1. $M_2 = \frac{-2(1+n)}{m} K_1 l \quad \int_{k=O}^{k=1} M_2 = \frac{-(1+n)}{2m} Wl.$

Load on l_2. $M_2 = \frac{-n^2}{m} \left\{2(1+n) K_2 - n K_1\right\} l$

$$\int_{k=O}^{k=1} M_2 = \frac{-n^2}{m} \cdot \frac{2+n}{4} Wl.$$

Load on l_3. $M_2 = \frac{n}{m} K_2 l.$

$$\int_{k=O}^{k=1} M_2 = \frac{n}{4m} Wl.$$

The curve is shewn in Fig. 7*a*, see also Table 7.

Reaction R_1.

Load on l_1. $R_1 = \frac{M_2}{l} + (1-k) = (1-k) - \frac{2(1+n)}{m} K_1.$

$$\int_{k=O}^{k=1} R_1 = \left\{\frac{1}{2} - \frac{1+n}{2m}\right\} W.$$

Load on l_2. $R_1 = \frac{M_2}{l} = \frac{-n^2}{m} \left\{2(1+n) K_2 - n K_1\right\}$

$$\int_{k=O}^{k=1} R_1 = \frac{-n_2}{m} \cdot \frac{2+n}{4} W.$$

Load on l_3. $R_1 = \frac{M_2}{l} = \frac{n}{m} K_2.$

$$\int_{k=0}^{k=1} R_1 = \frac{n}{4m} W.$$

The curve is shewn in Fig. 7*b*, see also Table 7.

Reaction R_2.

Load on l_1. $R_2 = \left\{ \frac{M_3}{l} + (1 + n - k) - R_1 (1 + n) \right\} \frac{1}{n}.$

$$= k + (k - k^3) \left\{ \frac{2 (1 + n)^2 + n}{m\ n} \right\}$$

$$\int_{k=o}^{k=1} R_2 = \left[\frac{1}{2} + \frac{1}{4} \left\{ \frac{2 (1 + n)^2 + n}{m\ n} \right\} \right] W.$$

Load on l_2. $R_2 = \left\{ \frac{M_3}{l} + (1 - k)\ n - R_1 (1 + n) \right\} \frac{1}{n}.$

Where $\frac{M_2}{l} = R_1.$

$$= \frac{M_3 - M_2}{ln} - \frac{M_2}{l} + (1 - k)\ n.$$

$$= (1 - k) + \frac{n}{m} \left[\left\{ 2\ (1 + n)^2 + n \right\} K_2 - (1 + n)\ (2 + n)\ K_1 \right]$$

$$\int_{k=O}^{k=1} R_2 = \left\{ \frac{1}{2} + \frac{n\ (2n + n^2)}{4m} \right\} W.$$

Load on l_3. $R_2 = \frac{M_3}{ln} - \frac{R_1 (1 + n)}{n}$

$$= \frac{- (1 + n)\ (2 + n)}{m\ n} K_2.$$

$$\int_{k=O}^{k=1} R_2 = \frac{- (1 + n)\ (2 + n)}{4\ m\ n} W.$$

On account of symmetry, R_3 and R_4 will be same as R_2 and R_1 respectively, taking k from right end instead of from left.

Table 7.—Shewing Bending Moments and Reactions for Case 7.

Figures are worked out for $n = 5/4$.

k	Load on l_1. $\frac{M_2}{l}$	Load on l_1. R_1	Load on l_1. R_2	Load on l_2. $\frac{M_2}{l}$	Load on l_2. R_1	Load on l_2. R_2	Load on l_3. $\frac{M_2}{l}$	Load on l_3. R_1	Load on l_3. R_2
	−	+	+	−	−	+	+	+	−
.05	·0120	·9380	·0743	·0296	·0296	·9960	·0062	·0619	·0290
·10	·0238	·8762	·1482	·0540	·0539	·9816	·0114	·0114	·0535
·15	·0353	·8147	·2214	·0734	·0733	·9574	·0158	·0157	·0738
·20	·0462	·7538	·2935	·0883	·0881	·9250	·0193	·0192	·0902
·25	·0564	·6936	·3641	·0989	·0989	·8850	·0219	·0219	·1027
·30	·0657	·6343	·4329	·1058	·1057	·8414	·0239	·0238	·1118
·35	·0739	·5761	·4995	·1091	·1091	·7855	·0251	·0251	·1175
·40	·0809	·5191	·5636	·1094	·1092	·7278	·0257	·0257	·1202
·45	·0864	·4636	·6248	·1068	·1066	·6662	·0257	·0256	·1201
·50	·0903	·4097	·6826	·1019	·1018	·6017	·0251	·0250	·1174
·55	·0924	·3576	·7367	·0949	·0950	·5355	·0240	·0240	·1123
·60	·0925	·3075	·7870	·0863	·0862	.4677	·0225	·0224	·1052
·65	·0904	·2596	·8328	·0763	·0762	·3996	·0205	·0205	·0961
·70	·0860	·2140	·8738	·0654	·0655	·3332	·0183	·0182	·0855
·75	·0790	·1710	·9097	·0539	·0540	·2678	·0157	·0157	·0734
·80	·0694	·1306	·9401	·0421	·0421	·2051	·0128	·0128	·0601
·85	·0568	0932	·9649	·0305	·0305	·1464	·0098	·0098	·0459
·90	·0412	·0588	·9832	·0194	·0193	·0917	·0066	.0066	·0310
·95	·0223	·0277	·9951	·0091	·0091	·0426	·0033	·0033	·0156
1·00	·0000	·0000	1·0000	·0000	·0000	·0000	·0000	·0000	·0000

Case 8.—THREE UNEQUAL SPANS. (Fig 8).

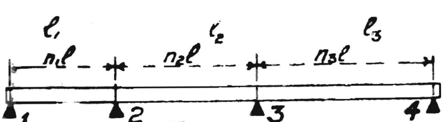


Fig. 8

Let $l_1 = n_1\ l.$

$l_2 = n_2\ l.$

$l_3 = n_3\ l.$

Let $4\ (n_1 + n_2)\ (n_2 + n_3) - n_2^2 \equiv m_2.$

Bending Moments.

Load on l_1. $M_1\ l_1 + 2M_2\ (l_1 + l_2) + M_3\ l_2 = -\ l_1^2\ (K_1).$

Here $M_1 = o \qquad l_1 = n_1 l \qquad l_2 = n_2 l.$

$\therefore\ 2M_2\ (n_1 + n_2) + M_3\ n_2 = -\ n_1^2 l\ K_1$..............(i.)

also $M_2\ l_2 + 2M_3\ (l_2 + l_3) + M_4\ l_3 = o.$

$\therefore\ M_2\ n_2 + 2M_3\ (n_2 + n_3) = o$..........................(ii.)

$$\therefore\ M_2 = \frac{-\ 2n_1^2\ (n_2 + n_3)}{m_2}\ K_1\ l \text{ and } M_3 = \frac{n_1^2\ n_2}{m_2}\ K_1\ l.$$

Load on l_2. $2M_2(n_1 + n_2) + M_3 n_2 + n_2^2 l K_2 = o.$

$$M_2 n_2 + 2 M_3 (n_2 + n_3) + n_2^2 l K_1 = o.$$

$$\therefore M_2 = \frac{-n_2^2 l}{m_2}\left[2(n_2 + n_3) K_2 - n_2 K_1\right]$$

$$\text{and } M_3 = \frac{-n_2^2 l}{m_2}\left[2(n_2 + n_1) K_1 - n_2 K_2\right]$$

Load on l_3. M_2 similar to M_3 in 1st Case with K_2 for K_1.
M_3 ,, M_2 ,, ,, K_1 for K_2.
In both cases n_3 and n_1 are interchanged.

$$\therefore M_2 = \frac{+ n_3^2 n_2}{m_2} K_2 l \text{ and } M_3 = \frac{-2 n_3^2 (n_2 + n_1)}{m_2} K_2 l.$$

Reaction R_1.

Load on l_1. $R_1 = \frac{M_2}{l_1} + (1 - k) = (1 - k) - \frac{2 n_1 (n_2 + n_3)}{m_2} K_1.$

Load on l_2. $R_1 = \frac{M_2}{l_1} = \frac{-n_2^2}{n_1 m_2}\left[2(n_2 + n_3) K_2 - n_2 K_1\right]$

Load on l_3. $R_1 = \frac{M_2}{l} = \frac{n_3^2 n_2}{n_1 m_2} K_2.$

Reaction R_2.

Load on l_1. $R_2 = \frac{M_3}{l_2} + \frac{l_1 + l_2 - kl_1}{l_2} - R_1 \frac{l_1 + l_2}{l_2}$

$$= k + K_1\left[\frac{n_1^2 n_2 + 2 n_1 (n_1 + n_2)(n_2 + n_3)}{m_2 n_2}\right]$$

Load on l_2. $R_2 = \frac{M_3}{l_2} + (1 - k) - R_1 \frac{l_1 + l_2}{l_2}$

$$= (1 - k) + \frac{n_2}{m_2}\left[\left\{n_1 n_2 + 2(n_1 + n_2)(n_2 + n_3)\right\} K_2. - (n_1 + n_2)\left\{\frac{n_2}{n_1} + 2\right\} K_1\right]$$

Load on l_3. $R_2 = \frac{M_3}{l_2} - R_1 \frac{l_1 + l_2}{l_2}$

$$= \frac{-n_3^2 (n_1 + n_2)(2 n_1 + n_2)}{m_2 n_2} K_2.$$

Reaction R_3.

Similar to R_2 with 1 put for 3 throughout.
k put for $(1 - k)$.
K_1 put for K_2.

Load on l_1. $R_3 = \frac{-n_1^2 (n_3 + n_2)(2 n_3 + n_2)}{m_2 n_2} K_1.$

Load on l_2. $R_3 = k + \frac{n_2}{m_2}\left[\left\{n_3 n_2 + 2(n_3 + n_2)(n_1 + n_2)\right\} K_1. - (n_3 + n_2)\left(\frac{n_2}{n_3} + 2\right)\right\} K_2\Big]$

Load on l_3.

$$R_3 = (1 - k) + K_2 \left\{ \frac{n_3^2 \, n_2 + 2n_3 \, (n_1 + n_2) \, (n_2 + n_3)}{m_2 \, n_2} \right\}$$

Reaction R_4.

Similar to R_1 with n_3 for n_1 and K_2 for K_1 and *vice-versa*.

Load on l_1. $R_4 = \dfrac{+ \, n_1^2 \, n_2}{n_3 \, m_2} K_1.$

Load on l_2. $R_4 = \dfrac{- \, n_2^2}{n_3 \, m_2} \left[2 \, (n_2 + n_1) \, K_1 - n_2 \, K_2 \right]$

Load on l_3. $R_4 = k - \dfrac{2 \, (n_3) \, (n_2 + n_1)}{m_2} K_2.$

The curves are similar in shape to Case 7, in fact Case 7 may be deduced from Case 8, by putting $n_1 = n_3 = 1$ and $n_2 = n$.

As a practical example of Case 8, take a beam supporting a building with supports 12 feet, 15 feet and 18 feet apart. Fig. 8*a*.

Here $n_1 = 4$, $n_2 = 5$, $n_3 = 6$, and $l = 3$.

Then $m_2 = 4 \, (n_1 + n_2) \, (n_2 + n_3) - n_2^2 = 371.$

Reaction R_1.

For load on l_1. $R_1 = (1 - k) - \dfrac{88}{371} K_1.$

" l_2. $= \dfrac{- \, 25}{1484} \left\{ 22 \, K_2 - 5 \, K_1 \right\}$

" l_3. $= \dfrac{+ \, 45}{371} \left\{ K_2 \right\}$

Reaction R_2.

For load on l_1. $R_2 = k + \dfrac{872}{1855} K_1.$

" l_2. $= (1 - k) + \dfrac{5}{1484} \left\{ 3248 \, K_2 - 117 \, K_1 \right\}$

" l_3. $= - \dfrac{585}{371} K_2.$

Reaction R_3.

For load on l_1. $R_3 = \dfrac{- \, 2992}{1855} K_1.$

" l_2. $= k + \dfrac{5}{2226} \left\{ 1308 \, K_1 - 187 \, K_2 \right\}$

" l_3. $= (1 - k) + \dfrac{1368}{1855} K_2.$

Reaction R_4.

For load on l_1. $R_4 = \dfrac{+ \, 40}{1113} K_1.$

" l_2. $= \dfrac{- \, 25}{2226} \left\{ 18 \, K_1 - 5 \, K_2 \right\}$

" l_3. $= k - \dfrac{108}{371} K_2.$

The fractions above can be put into decimals and the curves traced.

Table 8.—Reactions for Beams, 12′, 15′, 18′ Spans Continuous.

k	R1 LOAD ON			R2 LOAD ON			R3 LOAD ON			R4 LOAD ON		
	l_1	l_2	l_3	l_1	l_2	l_3	l_1	l_2	l_3	l_1	l_2	l_3
	+	−	+	−	+	−	−	−	+	+	−	+
·1	·8765	·0550	·0209	·1465	2·732	·2696	·1597	·319	1·0261	·0036	·0104	·0502
·2	·7545	·0906	·0351	·2906	3·884	·4541	·3097	·643	1·0124	·0069	·0226	·1162
·3	·6352	·1093	·0434	·4283	4·511	·5629	·4403	·952	·9633	·0098	·0352	·1961
·4	·5203	·1140	·0467	·5579	4·675	·6055	·5419	1·226	·8832	·0121	·0474	·2882
·5	·4111	·1074	·0456	·6763	4·464	·5913	·6048	1·444	·7765	·0135	·0548	·3908
·6	·3089	·0922	·0409	·7805	3·930	·5298	·6194	1·587	·6478	·0138	·0587	·5022
·7	·2153	·0711	·0332	·8678	3·157	·4305	·5758	1·634	·5013	·0128	·0568	·6205
·8	·1317	·0468	·0234	·9354	2·194	·3027	·4645	1·565	·3416	·0103	·0474	·7441
·9	·0594	·0223	·0121	·9804	1·121	·1561	·2758	1·361	·1730	·0061	·0290	·8712
1·0	·0000	·0000	·0000	1·0000	1·000	·0000	·0000	1·000	·0000	·0000	·0000	1·0000

Case 9.—FOUR EQUAL SPANS ON FIVE SUPPORTS. (Fig 9).

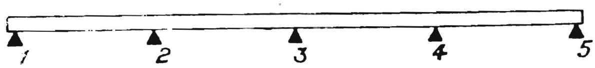


Fig. 9.

These might be the stringers of a bridge, or a large beam in a building.

From general equation.

$$M_2\, l_2 + 2M_3\,(l_2 + l_3) + M_4\, l_3 = -\, l_2^2\, K_1.$$

since $l_1 = l_2 = l_3 = l_4 = l$ say.

$M_2 + 2M_3 + M_4 = -\, l\, K$ where K may be K_1 or K_2 according to which span the load is on.

Bending mements.

Load on l_1.

$$\left.\begin{aligned} M_1 + 4\,M_2 + M_3 &= l\,K_1 \\ M_2 + 4\,M_3 + M_4 &= o \\ M_3 + 4\,M_4 + M_5 &= o \end{aligned}\right\} \text{where } M_1 = M_5 = o.$$

$$\therefore\ M_2 = \frac{-15}{56}\, lK_1 \qquad M_3 = +\frac{1}{14}\, lK_1 \qquad M_4 = -\frac{1}{56}\, lK_1.$$

Load on l_2.

$$4M_2 + M_3 = -\, lK_2.$$
$$M_2 + 4M_3 + M_4 = -\, lK_1.$$
$$M_3 + 4M_4 = o.$$

$$\therefore\ M_2 = \frac{-\,l}{14}\left\{\frac{15}{4}\, K_2 - K_1\right\} \qquad M_3 = \frac{-\,l}{14}\left\{4K_1 - K_2\right\}$$

$$M_4 = \frac{l}{56}\left\{4K_1 - K_2\right\}$$

Fig. 9

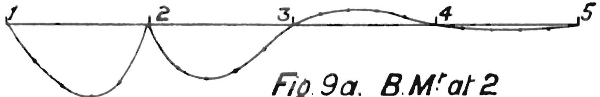


Fig. 9a. B.Mt at 2

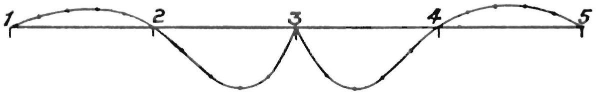


Fig. 9b. B.Mt at 3.

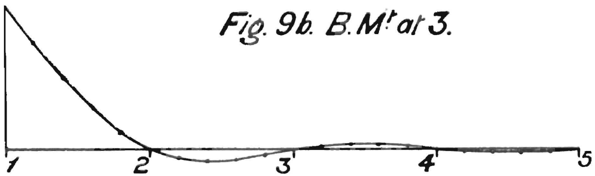


Fig. 9c. Reaction at 1

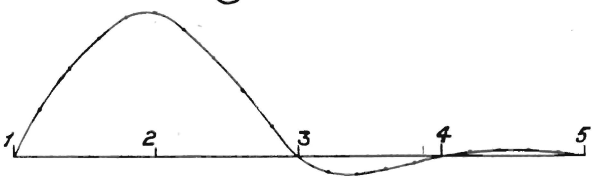


Fig. 9d. Reaction at 2.

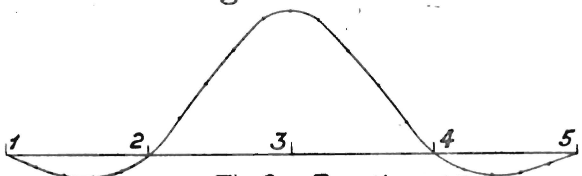


Fig. 9e. Reaction at 3.

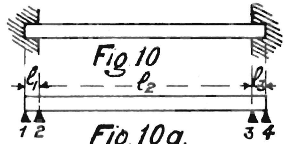


Fig 10

Fig. 10a.

Fig. 10b. B.Mt at 2

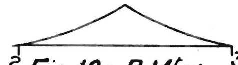


Fig. 10c B.Mt at centre

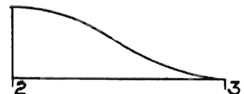


Fig. 10d. Reaction at 2

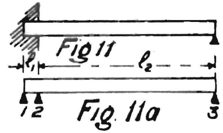


Fig 11

Fig. 11a

Fig. 11b. B.Mt at 2

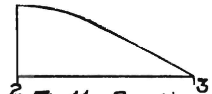


Fig. 11c. Reaction at 2

Fig. 11d. Reaction at 3

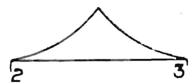


Fig. 11e B.Mt at centre