Load on l_3. (By symmetry.)

$$M_2 = +\frac{l}{56}\left\{4\ K_2 - K_1\right\} \qquad M_3 = \frac{-l}{14}\left\{4\ K_2 - K_1\right\}$$

$$M_4 = \frac{-l}{14}\left\{\frac{15}{4} K_1 - K_2\right\}$$

Load on l_4.

$$M_2 = -\frac{1}{56}\, l\, K_2. \qquad M_3 = +\frac{1}{14}\, lK_2. \qquad M_4 = \frac{-15}{56}\, lK_2.$$

Influence Line of M_2.

Load on l_1. $M_2 = \frac{-15}{56}\, l\, K_1.$ $\int_{k=0}^{k=1} M_2 = \frac{-15}{224}\, Wl.$

Load on l_2. $M_2 = \frac{-l}{14}\left\{\frac{15}{4} K_2 - K_1\right\}$ $\int_{k=0}^{k=1} M_2 = \frac{-11}{224}\, Wl.$

Load on l_3. $M_2 = \frac{l}{56}\left\{4\ K_2 - K_1\right\}$ $\int_{k=0}^{k=1} M_2 = \frac{3}{224}\, Wl.$

Load on l_4. $M_2 = \frac{-1}{56}\, l\, K_2.$ $\int_{k=0}^{k=1} M_2 = \frac{-1}{224}\, Wl.$

Influence Line of M_3.

Load on l_1. $M_3 = +\frac{1}{14}\, l\, K_1.$ $\int_{k=0}^{k=1} M_3 = \frac{1}{56}\, Wl.$

Load on l_2. $M_3 = \frac{-l}{14}\left\{4\ K_1 - K_2\right\}$ $\int_{k=0}^{k=1} M_3 = \frac{-3}{56}\, Wl.$

Load on l_3. $M_3 = \frac{-l}{14}\left\{4\ K_2 - K_1\right\}$ $\int_{k=0}^{k=1} M_3 = \frac{-3}{56}\, Wl.$

Load on l_4. $M_3 = +\frac{1}{14}\, l\, K_2.$ $\int_{k=0}^{k=1} M_3 = +\frac{1}{56}\, Wl.$

The curves of M_2 and M_3 are shewn in Figs. 9*a* and 9*b*.

Table *9a*.—SEE FIGURES *9a* AND *9b*.

k	$\frac{M_2}{l}$ Load on l_1	$\frac{M_2}{l}$ Load on l_2	$\frac{M_2}{l}$ Load on l_3	$\frac{M_2}{l}$ Load on l_4	$\frac{M_3}{l}$ Load on l_1	$\frac{M_3}{l}$ Load on l_2	$\frac{M_3}{l}$ Load on l_3	$\frac{M_3}{l}$ Load on l_4
	−	−	+	−	+	−	−	+
·05	·0134	·0212	·0057	·0017	·0036	·0076	·0229	·0066
·10	·0265	·0387	·0106	·0031	·0071	·0161	·0418	·0122
·15	·0393	·0527	·0142	·0042	·0105	·0250	·0569	·0169
·20	·0514	·0634	·0171	·0051	·0137	·0343	·0686	·0206
·25	·0628	·0711	·0192	·0059	·0167	·0435	·0770	·0234
·30	·0731	·0761	·0206	·0064	·0195	·0525	·0825	·0255
·35	·0823	·0786	·0213	·0067	·0219	·0608	·0852	·0268
·40	·0900	·0789	·0214	·0069	·0240	·0686	·0857	·0274
·45	·0961	·0771	·0210	·0068	·0256	·0751	·0840	·0274
·50	·1004	·0737	·0201	·0067	·0268	·0804	·0804	·0268
·55	·1027	·0687	·0188	·0064	·0274	·0840	·0751	·0256
·60	·1029	·0626	·0171	·0060	·0274	·0857	·0686	·0240
·65	·1006	·0554	·0152	·0055	·0268	·0852	·0608	·0219
·70	·0956	·0476	·0131	·0049	·0255	·0825	·0525	·0195
·75	·0879	·0394	·0109	·0042	·0234	·0770	·0435	·0167
·80	·0771	·0309	·0086	·0034	·0206	·0686	·0343	·0137
·85	·0632	·0224	·0063	·0026	·0169	·0569	·0250	·0105
·90	·0458	·0143	·0040	·0018	·0122	·0418	·0161	·0071
·95	·0248	·0068	·0020	·0009	·0066	·0229	·0076	·0036
1·00	·0000	·0000	·0000	·0000	·0000	·0000	·0000	·0000

REACTION R_1.

Load on l_1

$$R_1 = \frac{M_2}{l} + (1 - k) = (1 - k) - \frac{15}{56} K_1.$$

$$\int_{k=0}^{k=1} R_1 = + \frac{97}{224} W.$$

Load on l_2

$$R_1 = \frac{M_2}{l} = - \frac{1}{14} \left\{ \frac{15}{4} K_2 - K_1 \right\}$$

$$\int_{k=0}^{k=1} R_1 = - \frac{11}{224} W.$$

Load on l_3

$$R_1 = \frac{M_2}{l} = + \frac{1}{56} \left\{ 4K_2 - K_1. \right\}$$

$$\int_{k=0}^{k=1} R_1 = \frac{3}{224} W_1.$$

Load on l_4. $\quad R_1 = \frac{M_2}{l} = -\frac{1}{56} K_2.$

$$\int_{k=0}^{k=1} R_1 = -\frac{1}{224} W.$$

For Influence Line curve, see Fig. 9*c*.

Reaction R_2.

Load on l_1 $\quad R_2 = (2 - k) - 2R_1 + \frac{M_3}{l} = k + \frac{17}{28} K_1.$

$$\int_{k=0}^{k=1} R_2 = \frac{73}{112} W.$$

Load on l_2 $\quad R_2 = (1-k) - 2R_1 + \frac{M_3}{l} = (1-k) + \frac{1}{28}\left\{17K_2 - 12K_1\right\}$

$$\int_{k=0}^{k=1} R_2 = \frac{61}{112} W.$$

Load on l_3 $\quad R_2 = -2R_1 + \frac{M_3}{l} = -\frac{3}{28}\left\{4K_2 - K_1\right\}$

$$\int_{k=0}^{k=1} R_2 = \frac{-9}{112} W.$$

Load on l_4 $\quad R_2 = -2R_1 + \frac{M_3}{l} = \frac{+3}{28} K_2.$

$$\int_{k=0}^{k=1} R_2 = \frac{+3}{112} W.$$

The Influence Line curve is shewn in Fig. 9*d*.

Reaction R_3.

Load on l_1 $\quad R_3 = (3 - k) - 3R_1 - 2R_2 + \frac{M_4}{l} = \frac{-3}{7} K_1.$

$$\int_{k=0}^{k=1} R_3 = \frac{-3}{28} W.$$

Load on l_2 $$R_3=(2-k)-3R_1-2R_2+\frac{M_4}{l}=k-\frac{3}{7}K_2+\frac{5}{7}K_1.$$

$$\int_{k=0}^{k=1} R_3 = \frac{+4}{7}\,W.$$

Load on l_3 $$R_3=(1-k)-3R_1-2R_2+\frac{M_4}{l}=(1-k)+\frac{5}{7}K_2-\frac{3}{7}K_1.$$

$$\int_{k=0}^{k=1} R_3 = \frac{+4}{7}\,W.$$

Load on l_4 $$R_3 = -3R_1 - 2R_2 + \frac{M_4}{l} = \frac{-3}{7}K_2.$$

$$\int_{k=0}^{k=1} R_3 = \frac{-3}{28}\,W.$$

The last two could be deduced by symmetry.

The Influence Line curve is shewn in Fig. 9*e*.

There is no need to work out R_4 and R_5 on account of symmetry.

Table 9*c*.—SEE FIGURES 9*c*, 9*d* AND 9*e*.

Reactions at 1, 2 and 3.

	R_1				R_2				R_3			
	LOAD ON				LOAD ON				LOAD ON			
k	l_1	l_2	l_3	l_4	l_1	l_2	l_3	l_4	l_1	l_2	l_3	l_4
	+	−	+	−	+	+	−	+	−	+	+	−
·05	·9366	·0212	·0057	·0017	·0803	·9849	·0344	·0099	·0214	·0459	·9948	·0397
·10	·8735	·0387	·0106	·0031	·1601	·9614	·0627	·0183	·0424	·0974	·9797	·0733
·15	·8107	·0527	·0142	·0042	·2390	·9304	·0853	·0253	·0628	·1536	·9557	·1011
·20	·7486	·0634	·0171	·0051	·3166	·8926	·1028	·0309	·0823	·2137	·9234	·1234
·25	·6872	·0711	·0192	·0059	·3923	·8487	·1154	·0352	·1005	·2768	·8839	·1406
·30	·6269	·0761	·0206	·0064	·4658	·7997	·1238	·0382	·1170	·3420	·8380	·1530
·35	·5677	·0786	·0213	·0067	·5365	·7463	·1278	·0402	·1316	·4085	·7865	·1609
·40	·5100	·0789	·0214	·0069	·6040	·6891	·1286	·0411	·1440	·4754	·7303	·1646
·45	·4539	·0771	·0210	·0068	·6679	·6291	·1260	·0411	·1538	·5420	·6702	·1644
·50	·3996	·0737	·0201	·0067	·7277	·5670	·1206	·0402	·1607	·6071	·6071	·1607
·55	·3473	·0687	·0188	·0064	·7829	·5035	·1127	·0385	·1644	·6702	·5420	·1538
·60	·2971	·0626	·0171	·0060	·8331	·4394	·1029	·0360	·1646	·7303	·4754	·1440
·65	·2494	·0554	·0152	·0055	·8779	·3756	·0912	·0329	·1609	·7865	·4085	·1316
·70	·2044	·0476	·0131	·0049	·9167	·3128	·0788	·0292	·1530	·8380	·3420	·1170
·75	·1621	·0394	·0109	·0042	·9492	.2517	·0652	·0251	·1406	·8839	·2768	·1005
·80	·1229	·0309	·0086	·0034	·9749	·1932	·0514	·0206	·1234	·9234	·2137	·0823
·85	·0868	·0224	·0063	·0026	·9932	·1379	·0376	·0157	·1011	·9557	·1536	·0628
·90	·0542	·0143	·0040	·0018	1·0038	·0868	·0241	·0106	·0733	·9797	·0974	·0424
·95	·0252	·0068	·0020	·0009	1·0062	·0406	·0115	·0053	·0397	·9948	·0459	·0214
1·00	·0000	·0000	·0000	·0000	1·0000	·0000	·0000	·0000	·0000	1·0000	·0000	·0000

Case 10.—BEAM OF ONE SPAN FIXED AT ENDS. (Fig. 10.)

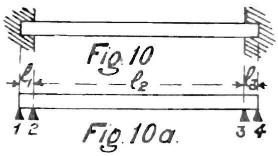


Fig. 10

Fig. 10a.

Bending Moments.

Referring to Fig. 10a.

Load on l_2.

$$M_1\, l_1 + 2M_2\,(l_1 + l_2) + M_3\, l_2 = -\, l_2^2\, K_2.$$

$$M_2\, l_2 + 2M_3\,(l_2 + l_3) + M_4\, l_3 = -\, l_2^2\, K_1.$$

Here $l_1 = l_3 = o$ and $M_1 = M_2$ also $M_3 = M_4$

$$\therefore\ 2M_2 + M_3 = -\, l_2\, K_2$$

$$M_2 + 2M_3 = -\, l_2\, K_1$$

$$\therefore\ M_1 = M_2 = \tfrac{1}{3}\, l\,(K_1 - 2K_2). \quad M_3 = M_4 = \tfrac{1}{3}\, l\,(K_2 - 2K_1).$$

$$\int_{k=0}^{k=1} M_2 = -\, \tfrac{1}{12}\, Wl.$$

Curve is drawn in Fig. 10b.

Table 10b.—Bending Moments at 2. $\dfrac{M_2}{l}$

k	0	1	2	3	4	5	6	7	8	9
	–	–	–	–	–	–	–	–	–	–
0	·0000	·0098	·0192	·0282	·0369	·0451	·0530	·0605	·0577	·0745
1	·0810	·0871	·0929	·0984	·1035	·1084	·1129	·1171	·1210	·1247
2	·1280	·1311	·1338	·1364	·1386	·1406	·1424	·1439	·1452	·1462
3	·1470	·1476	·1480	·1481	·1481	·1479	·1475	·1469	·1461	·1451
4	·1440	·1427	·1413	·1397	·1380	·1361	·1341	·1320	·1298	·1275
5	·1250	·1225	·1198	·1171	·1143	·1114	·1084	·1054	·1023	·0992
6	·0960	·0928	·0895	·0862	·0830	·0796	·0763	·0730	·0696	·0663
7	·0630	·0597	·0564	·0532	·0500	·0469	·0438	·0407	·0378	·0348
8	·0320	·0292	·0266	·0240	·0215	·0191	·0169	·0147	·0127	·0108
9	·0090	·0074	·0059	·0046	·0034	·0024	·0015	·0009	·0004	·0001

Bending Moment at Centre. M_c.

We have $R_2 \times l_2 + M_2 - (1 - k)\, l_2 = M_3$

$$\therefore\ R_2 = (1 - k) + \left\{ M_3 - M_2 \right\} \div l$$

$$= (1 - k) + (K_2 - K_1)$$

and $R_3 = k + (K_1 - K_2)$

Influence Line M_c.

$$M_c = R_2 \times \frac{l}{2} - \left(\frac{l}{2} - k\right) + M_2$$

or $= R_3 \times \dfrac{l}{2} + M_3$ from $k = o$ to $k = \tfrac{1}{2}$

and $= R_2 \times \dfrac{l}{2} + M_2$ from $k = \tfrac{1}{2}$ to $k = 1$

Left half $k = o$ to $k = \frac{1}{2}$

$$M_c = \left\{ k + (K_1 - K_2) \right\} \frac{l}{2} + \tfrac{1}{3} \left\{ K_2 - 2K_1 \right\} l$$

$$= \left\{ \frac{k}{2} - \frac{1}{6} (K_1 + K_2) \right\} l$$

Right half $k = \frac{1}{2}$ to $k = 1$

$$M_c = \left\{ \frac{1 - k}{2} - \frac{1}{6} (K_1 + K_2) \right\} l$$

$$\int_{k=O}^{k=1} M_c = 2 \int_{k=O}^{k=\frac{1}{2}} \frac{k}{2} - \frac{1}{6} \int \left\{ K_1 + K_2 \right\}$$

$$= \left\{ \frac{1}{8} - \frac{1}{12} \right\} Wl = \frac{Wl}{24}$$

The curve is shewn in Fig. 10*c*.

For uniform loading throughout and for beam loaded in the centre, the upward B. Mt. (not the ultimate B. Mt.) is the same as for a beam with free ends, but for any partial loading the upward and also the ultimate B. Mt. are different, and the shears also are different. These points will be referred to again later.

Table 10*c*.—Bending Moment at Centre.

k	0	1	2	3	4	5	6	7	8	9
	+	+	+	+	+	+	+	+	+	+
0	·0000	·0000	·0002	·0004	·0008	·0012	·0018	·0024	·0032	·0040
1	·0050	·0060	·0072	·0084	·0098	·0112	·0128	·0144	·0162	·0180
2	·0200	·0220	·0242	·0264	·0288	·0312	·0338	·0364	·0392	·0420
3	·0450	·0480	·0512	·0544	·0578	·0612	·0648	·0684	·0722	·0760
4	·0800	·0840	·0882	·0924	·0968	·1012	·1058	·1104	·1152	·1200
5	·1250	·1200	·1152	·1104	·1058	·1012	·0968	·0924	·0882	·0840
6	·0800	·0760	·0722	·0684	·0648	·0612	·0578	·0544	·0512	·0480
7	·0450	·0420	·0392	·0364	·0338	·0312	·0288	·0264	·0242	·0220
8	·0200	·0180	·0162	·0144	·0128	·0112	·0098	·0084	·0072	·0060
9	·0050	·0040	·0032	·0024	·0018	·0012	·0008	·0004	·0002	·0090

INFLUENCE LINE OF R_2.

$$R_2 = (1 - k) + (K_2 - K_1) \qquad \int_{k=O}^{k=1} R_2 = \tfrac{1}{2}.$$

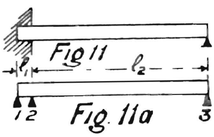
Curve is shewn in Fig. 10*d*.

Table 10*d*.—Reaction at End.

k	0	1	2	3	4	5	6	7	8	9
0	1·000	·9997	·9988	·9973	·9954	·9927	·9896	·9859	·9818	·9771
1	·9720	·9663	·9602	·9537	·9466	·9393	·9314	·9231	·9144	·9055
2	·8960	·8863	·8760	·8657	·8548	·8437	·8324	·8207	·8088	·7965
3	·7840	·7713	·7584	·7451	·7318	·7183	·7044	·6907	·6766	·6623
4	·6480	·6335	·6190	·6043	·5896	·5747	·5598	·5449	·5300	·5149
5	·5000	·4851	·4700	·4551	·4402	·4253	·4104	·3957	·3810	·3665
6	·3520	·3377	·3134	·3093	2956	·2817	·2682	·2549	·2416	·2287
7	·2160	·2035	·1912	·1793	·1676	·1563	·1452	·1343	·1240	·1137
8	·1040	·0945	·0856	·0769	·0686	·0607	·0534	·0463	·0398	·0337
9	·0280	·0229	·0182	·0141	·0104	·0073	·0046	·0027	·0012	·0003

Case 11.—BEAM FIXED AT ONE END AND FREE AT OTHER.

(Fig. 11.)



Bending Moments at end.

Referring to Fig. 11*a*. $M_3 = M_4 = 0$.

$M_1 l_1 + 2 M_2 (l_1 + l_2) + M_3 l_2 = - l_2^2 K_2$.

Here $M_3 = M_4 =$ zero, also $M_1 = M_2$ and $l_1 = l_3 =$ zero

$\therefore\ 2 M_2 l_2 = - l_2^2 K_2$

$$M_2 = - \frac{1}{2} K_2 l. \qquad \int_{k=0}^{k=1} M_2 = - \frac{Wl}{8}$$

Curve is shewn in Fig. 11*b*.

Reactions.

$$R_2 = (1 - k) - \frac{M_2}{l} \text{ since } M_3 = 0$$

$$= (1 - k) + \tfrac{1}{2} K_2. \qquad \int_{k=0}^{k=1} R_2 = \tfrac{5}{8} W.$$

$$R_3 = k + \frac{M_2}{l} = k - \frac{1}{2} K_2.$$

$$\int_{k=0}^{k=1} R_3 = \tfrac{3}{8} W.$$

Curves are shewn in Figs. 11*c*, 11*d*.

BENDING MOMENT at centre.

$$M_c = R_3 \times \frac{l}{2} \text{ from } k = o \text{ to } k = \tfrac{1}{2}.$$

$$\text{and} = R_2 \times \frac{l}{2} + M_2 \text{ from } k = \tfrac{1}{2} \text{ to } k = 1.$$

$$\therefore\ M_c = \left\{k - \tfrac{1}{2}K_2\right\}\frac{l}{2} \text{ from } k = o \text{ to } k = \tfrac{1}{2}$$

$$\text{and} = \left\{(1-k) - \tfrac{1}{2}K_2\right\}\frac{l}{2} \text{ from } k = \tfrac{1}{2} \text{ to } k = 1.$$

$$\int_{k=0}^{k=1} M_c = \frac{1}{16} Wl.$$

Curve is shewn in Fig. 11*e*.

B. Mts. and Shears at any point can be determined from the Reactions and B. Mts. at end in a similar manner to that by which M_c was deduced.

Table II.—Bending Moments and Reactions.

FIGS. 11*b* to 11*e*.

k	$\frac{M_2}{l}$	$\frac{M_c}{l}$	R_2	R_3	k	$\frac{M_2}{l}$	$\frac{M_c}{l}$	R_2	R_3
	−	+	+	+		−	+	+	+
·05	·0463	·0019	·9963	·0037	·55	·1795	·1291	·6295	·3705
·10	·0855	·0073	·9855	·0145	·60	·1680	·1040	·5680	·4320
·15	·1180	·0160	·9680	·0320	·65	·1536	·0812	·5036	·4964
·20	·1440	·0280	·9440	·0560	·70	·1365	·0608	·4365	·5635
·25	·1641	·0430	·9141	·0859	·75	·1172	·0430	·3672	·6328
·30	·1785	·0608	·8785	·1215	·80	·0960	·0280	·2960	·7040
·35	·1877	·0812	·8377	·1623	·85	·0733	·0160	·2233	·7767
·40	·1920	·1040	·7920	·2080	·90	·0495	·0073	·1495	·8505
·45	·1918	·1291	·7418	·2582	·95	·0250	·0019	·0750	·9250
·50	·1875	·1563	·6875	·3125	1·00	·0000	·0000	·0000	1·0000

From the above it will be noticed that for a central load the B. Mt. at centre is reduced in ratio $\frac{156}{250}$ by fixing, *i.e.*, B. Mt. is about ·624 what it would be if the end were not fixed.

For a uniformly distributed load throughout, the B. Mt. at centre is reduced by $\frac{1}{2}$. For partial loading the reduction of B. Mt. is readily deduced from the Reactions and B. Mts. at end.

NOTE.—It may be remarked here that in all cases of summation for partial loading it follows that since k is merely a multiplying factor of l, the area of the curve must be found between the limits specified and multiplied by wl^2 where w is the load per unit length (at first sight it might appear as if the area ought to be multiplied by the load on the portion between the limits). Thus if W_a is the load on the portion between $k = k_1$ and $k = k_2$.

$$\text{B. Mt.} = \frac{W_a}{k_2 - k_1}\, l \int_{k_1}^{k_2} F(k)\, dk.$$

$$\text{Similarly Reactions} = \frac{W_a}{k_2 - k_1} \int_{k_1}^{k_2} F_1(k)\, dk.$$

This follows thus $\int M = \int F(k) \times l \times dW.$

$$\text{and } dW = \frac{W_a\, d(kl)}{k_2 l - k_1 l} = \frac{W_a}{k_2 - k_1}\, dk.$$

$$\therefore \int M = \int \frac{W_a}{k_2 - k_1}\, F(k)\, dk = \frac{W_a}{k_2 - k_1} \int_{k_1}^{k_2} F(k)\, dk.$$

Case 12.—BEAM OF TWO EQUAL SPANS FIXED AT BOTH ENDS.

(Fig. 12.)

Bending Moment at supports.

Load on l_2.

$$2M_2 + M_3 = -K_2 l$$
$$M_2 + 4M_3 + M_4 = -K_1 l$$
$$M_3 + 2M_4 = 0.$$

$$\therefore M_2 = -\frac{1}{12}(7K_2 - 2K_1)\, l \qquad \int_{k=0}^{k=1} M_2 = -\frac{5}{48} Wl$$

$$M_3 = -\frac{1}{6}(2K_1 - K_2)\, l \qquad \int_{k=0}^{k=1} M_3 = -\frac{1}{24} Wl$$

$$M_4 = \frac{1}{12}(2K_1 - K_2)\, l \qquad \int_{k=0}^{k=1} M_4 = \frac{1}{48} Wl.$$

Load on l_3 (by symmetry).

$$M_2 = \frac{1}{12}(2K_2 - K_1)l \qquad \int_{k=0}^{k=1} M_2 = \frac{1}{48} Wl$$

$$M_3 = -\frac{1}{6}(2K_2 - K_1)\,l \qquad \int_{k=0}^{k=1} M_3 = -\frac{1}{24} Wl$$

$$M_4 = \frac{1}{12}(2K_2 - 7K_1)l \qquad \int_{k=0}^{k=1} M_4 = -\frac{5}{48} Wl.$$

The curves for M_2 and M_3 are shewn in Figs. 12*b*, 12*c*.

REACTIONS.

Load on l_2.

$$R_2 = \frac{M_3 - M_2}{l} + (1 - k) = (1 - k) - \frac{1}{4}\left\{2K_1 - 3K_2\right\}$$

$$R_4 = -\frac{M_3 - M_4}{l} = -\frac{1}{4}\left\{2K_1 - K_2\right\} \qquad \int_{k=0}^{k=1} R_4 = -\frac{1}{16} W$$

$$R_3 = \frac{M_2 - M_4}{l} - 2R_4 + k = k + \left\{K_1 - K_2\right\} \qquad \int_{k=1}^{k=0} R_3 = \frac{1}{2} W$$

Load on l_3 (by symmetry).

$$R_4 = k - \tfrac{1}{4}\left\{2K_2 - 3K_1\right\} \qquad \int_{k}^{k} R_4 = \frac{9}{10} W$$

$$R_2 = -\tfrac{1}{4}\left\{2K_2 - K_1\right\} \qquad \int_{k=0}^{k=1} R_2 = \frac{-1}{16} W$$

$$R_3 = (1 - k) + \left\{K_2 - K_1\right\} \qquad \int_{n}^{k=1} R_3 = \frac{1}{2} W.$$

The curves for R_2 and R_3 are shewn Figs. 12*d*, 12*e*.

Table 12.—Bending Moments and Reactions. (Figs. 12—12e).

k	$\frac{M_2}{l}$ Load on l_2	$\frac{M_2}{l}$ Load on l_3	$\frac{M_3}{l}$ Load on l_2	$\frac{M_3}{l}$ Load on l_3	R_2 Load on l_2	R_2 Load on l_3	R_3 Load on l_2	R_3 Load on l_3
	−	+	−	−	+	−	+	+
·05	·0457	·0113	·0012	·0226	·9945	·0338	·0073	·9927
·10	·0863	·0208	·0045	·0415	·9788	·0608	·0280	·9720
·15	·1132	·0271	·0096	0542	·9536	·0813	·0607	·9393
·20	·1360	·0320	·0160	·0640	·9200	·0960	·1040	·8960
·25	·1523	·0352	·0235	·0703	·8789	·1055	·1563	·8437
·30	·1628	·0368	·0315	·0735	·8313	·1103	·2160	·7840
·35	·1678	·0370	·0398	·0739	·7780	·1109	·2817	·7183
·40	·1680	·0360	·0480	·0720	·7200	·1080	·3520	·6480
·45	·1639	·0340	·0557	·0681	·6583	·1021	·4253	·5747
·50	·1563	·0313	·0625	·0625	·5938	·0938	·5000	·5000
·55	·1454	·0279	·0681	·0557	·5274	·0836	·5747	·4253
·60	·1320	·0240	·0720	·0480	·4600	·0720	·6480	·3520
·65	·1166	·0199	·0739	·0398	·3926	·0597	·7183	·2817
·70	·0998	·0158	·0735	·0315	·3263	·0473	·7840	·2160
·75	·0821	·0117	·0703	·0235	·2618	·0352	·8437	·1563
·80	·0640	·0080	·0640	·0160	·2000	·0240	·8960	·1040
·85	·0379	·0048	·0542	·0096	·1420	·0143	·9393	·0607
·90	·0293	·0023	·0415	·0045	·0887	·0068	·9720	·0280
·95	·0137	·0006	·0226	·0012	·0411	·0018	·9927	·0073
1·00	·0000	·0000	·0000	·0000	·0000	·0000	1·0000	·0000

Case 13.—BEAM OF TWO EQUAL SPANS FIXED AT ONE END, FREE AT OTHER. (Fig. 13.)

Bending Moments.

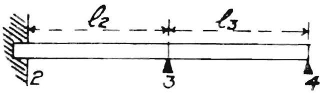


Fig. 13.

Load on l_2.

$$2M_2 + M_3 = -K_2\, l.$$
$$M_2 + 4M_3 = -K_1\, l.$$
$$\therefore\ M_2 = -\tfrac{1}{7}\left\{4K_2 - K_1\right\} l.$$
$$M_3 = -\tfrac{1}{7}\left\{2K_1 - K_2\right\} l.$$

Load on l_3.

$$2M_2 + M_3 = 0.$$
$$M_2 + 4M_3 = -K_2\, l.$$

$$\therefore\ M_2 = \tfrac{1}{7}K_2 l \qquad \int_{k=0}^{k=1} M_2 = \frac{1}{28}Wl. \quad M_3 = -\tfrac{2}{7}K_2 l \qquad \int_{k=0}^{k=1} M_3 = -\frac{\cdot 1}{14}Wl.$$

The curves for these are shewn Figs. 13a, 13b.

Reactions.

Load on l_2.

$$R_2 = \frac{M_3 - M_2}{l} + (1-k) = (1-k) + \tfrac{1}{7}\left\{5K_2 - 3K_1\right\} \qquad \int_{k=0}^{k=1} R_2 = \frac{4}{7}W$$

$$R_4 = \frac{M_3}{l} = -\tfrac{1}{7}\left\{2K_1 - K_2\right\} \qquad \int_{k=0}^{k=1} R_4 = -\frac{1}{28}W$$

$$R_3 = k + \frac{M_2}{l} - 2R_4 = k + \tfrac{1}{7}\left\{5K_1 - 6K_2\right\} \qquad \int_{k=0}^{k=1} R_3 = \frac{13}{28}W$$

Load on l_3.

$$R_4 = \frac{M_3}{l} + k = k - \tfrac{2}{7}K_2. \qquad \int_{k=0}^{k=1} R_4 = \frac{3}{7}W$$

$$R_3 = \frac{M_2}{l} - 2R_4 + (1+k) = (1-k) + \tfrac{5}{7}K_2. \qquad \int_{k=0}^{k=1} R_3 = \frac{19}{28}W$$

$$R_2 = \frac{M_3 - M_2}{l} = -\tfrac{3}{7}K_2. \qquad \int_{k=0}^{k=1} R_2 = \frac{-3}{28}W$$

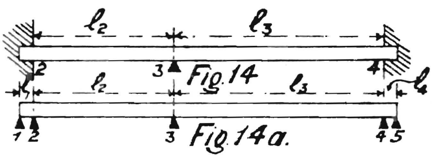
The curves for these are shewn Figs. 13*c*, 13*d*, 13*e*.

Table 13.—Bending Moments and Reactions. (Figs. 13—13*e*.)

k	$\frac{M_2}{l}$		$\frac{M_3}{l}$		R_2		R_3		R_4	
	Load on l_2	Load on l_3	Load on l_2	Load on l_3	Load on l_2	Load on l_3	Load on l_2	Load on l_3	Load on l_2	Load on l_3
	−	+	−	−	+	−	+	+	−	+
·05	·0515	·0132	·0010	·0265	·9948	·0397	·0063	1·0161	·0010	·0235
·10	·0836	·0244	·0039	·0489	·9797	·0733	·0241	1·0221	·0039	·0511
·15	·1139	·0337	·0082	·0674	·9557	·1011	·0525	1·0185	·0082	·0826
·20	·1371	·0411	·0137	·0823	·9234	·1234	·0903	1·0057	·0137	·1177
·25	·1540	·0469	·0201	·0937	·8839	·1406	·1362	·9844	·0201	·1563
·30	·1650	·0510	·0270	·1020	·8380	·1530	·1890	·9550	·0270	·1980
·35	·1706	·0536	·0341	·1073	·7865	·1609	·2476	·9181	·0341	·2427
·40	·1714	·0549	·0411	·1097	·7303	·1646	·3109	·8743	·0411	·2903
·45	·1679	·0548	·0477	·1096	·6702	·1644	·3776	·8240	·0477	·3404
·50	·1607	·0536	·0536	·1071	·6071	·1607	·4464	·7679	·0536	·3929
·55	·1503	·0513	·0583	·1025	·5420	·1538	·5164	·7061	·0583	·4475
·60	·1371	·0480	·0617	·0960	·4754	·1440	·5863	·6400	·0617	·5040
·65	·1219	·0439	·0634	·0877	·4085	·1316	·6549	·5694	·0634	·5623
·70	·1050	·0390	·0630	·0780	·3420	·1170	·7210	·4950	·0630	·6220
·75	·0871	·0335	·0603	·0670	·2768	·1005	·7834	·4174	·0603	·6830
·80	·0686	·0274	·0549	·0549	·2137	·0823	·8411	·3371	·0549	·7451
·85	·0501	·0209	·0465	·0419	·1536	·0628	·8928	·2547	·0465	·8081
·90	·0321	·0141	·0347	·0283	·0974	·0424	·9373	·1707	·0347	·8717
·95	·0153	·0071	·0193	·0143	·0460	·0214	·9734	·0856	·0193	·9357
1·00	·0000	·0000	·0000	·0000	·0000	·0000	1·0000	·0000	·0000	1·0000

Case 14.—BEAM OF TWO UNEQUAL SPANS FIXED AT ENDS.

(FIG. 14.)



BENDING MOMENTS.

Referring to Fig. 14a

Let $\dfrac{l_3}{l_2} = r.$

Load on l_2

$$2M_2+M_3 = -K_2\,l_2$$
$$M_2+2M_3(1+r)M_4r = -K_1\,l_2$$
$$M_3+2M_4 = 0.$$

$$\therefore\ M_2 = \frac{1}{6(1+r)}\left\{2K_1-(4+3r)K_2\right\}l_2.$$

$$M_3 = \frac{-1}{3(1+r)}\left\{2K_1-K_2\right\}l_2.$$

$$M_4 = \frac{+1}{6(1+r)}\left\{2K_1-K_2\right\}l_2.$$

Load on l_3.

Put K_1 for K_2 and *vice versa*, also $r = \dfrac{1}{r}$ and interchange M_4 and M_2 also l_2 and l_3, then

$$M_2 = \frac{+r}{6(1+r)}\left\{2K_2-K_1\right\}l_3$$

$$M_3 = \frac{-r}{3(1+r)}\left\{2K_2-K_1\right\}l_3.$$

$$M_4 = \frac{1}{6(1+r)}\left\{2rK_2-(3+4r)K_1\right\}l_3.$$

The curves and tables are worked out for $r = 1{\cdot}5$. See Figs. 14b, 14c, 14d.

REACTIONS.

Load on l_2.

$$R_2 = \frac{M_3-M_2}{l_2}+(1-k) = (1-k)-\frac{1}{2(1+r)}\left\{2K_1-(r+2)K_2\right\}$$

$$R_4 = \frac{M_3-M_4}{l_3} = -\frac{1}{2r(1+r)}\left\{2K_1-K_2\right\}$$

$$R_3 = \frac{M_2-M_4}{l_2}-R_4(1+r)+k$$

$$= k+\frac{1}{2r}\left\{2K_1-(1+r)K_2\right\}$$

Load on l_3 (by symmetry.)

$$R_2 = \frac{-r^2}{2(1+r)}\left\{2K_2 - K_1\right\}$$

$$R_1 = k - \frac{r}{2(1+r)}\left\{2K_2 - \left(\frac{1}{r}+2\right)K_1\right\}$$

$$= k - \frac{1}{2(1+r)}\left\{2rK_2 - (1+2r)K_1\right\}$$

$$R_3 = 1 - k + \frac{r}{2}\left\{2K_2 - \frac{1+r}{r}K_1\right\}$$

$$= (1-k) + \tfrac{1}{2}\left\{2rK_2 - (1+r)K_1\right\}$$

The curves for these are shewn in Figs. 14*e* - 14*g*. The table is worked out for $r = 1{\cdot}5$.

Table 14.—Bending Moments and Reactions for $r = 1{\cdot}5$. FIGS. 14—14/.

k	$\frac{M_2}{l_2}$	$\frac{M_2}{l_3}$	$\frac{M_3}{l_2}$	$\frac{M_3}{l_3}$	$\frac{M_4}{l_2}$	$\frac{M_4}{l_3}$	R_2		R_3		R_4	
	Load on l_2	Load on l_3	Load on l_2	Load on l_3	Load on l_2	Load on l_3	Load on l_2	Load on l_3	Load on l_2	Load on l_3	Load on l_2	Load on l_3
	−	+	−	−	+	−	+	−	+	+	−	+
·1	·0837	·0243	·0036	·0486	·0018	·0252	·9801	·1094	·0235	1·0328	·0036	·0766
·2	·1376	·0384	·0128	·0768	·0064	·0576	·9248	·1728	·0880	·9920	·0128	·1808
·3	·1659	·0441	·0252	·0882	·0126	·0924	·8407	·1985	·1845	·8943	·0252	·3042
·4	·1728	·0432	·0384	·0864	·0192	·1248	·7344	·1944	·3040	·7560	·0384	·4384
·5	·1625	·0375	·0500	·0750	·0250	·1500	·6125	·1688	·4375	·5938	·0500	·5750
·6	·1392	·0288	·0576	·0576	·0288	·1632	·4816	·1296	·5760	·4240	·0576	·7056
·7	·1071	·0189	·0588	·0378	·0294	·1596	·3483	·0851	·7105	·2633	·0588	·8218
·8	·0704	·0096	·0512	·0192	·0256	·1344	·2192	·0432	·8320	·1280	·0512	·9152
·9	·0333	·0027	·0324	·0054	·0162	·0828	·1009	·0122	·9315	·0347	·0324	·9774
1·0	·0000	·0000	·0000	·0000	·0000	·0000	·0000	·0000	1·0000	·0000	·0000	1·0000

Case 15.—BEAM OF TWO UNEQUAL SPANS, FIXED AT ONE END AND FREE AT OTHER. (FIG. 15.)

BENDING MOMENTS. Let $\frac{l_3}{l_2} = r$

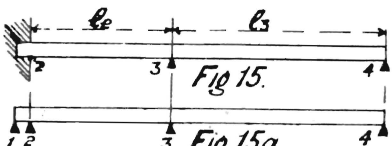


Fig 15.

Fig 15a

Load on l_2.

$$2M_2 + M_3 = -K_2 l_2.$$

$$M_2 + 2M_3(1+r) = -K_1 l_2.$$

$$M_2 = -\frac{1}{4r+3}\left\{2(1+r)K_2 - K_1\right\} l_2.$$

$$M_3 = -\frac{1}{4r+3}\left\{2K_1 - K_2\right\} l_2.$$