Load on l_3.

$$2M_2 + M_3 = 0.$$

$$\frac{1}{r} M_2 + 2M_3 (1 + r) = - K_2 l_3.$$

$$M_2 = \frac{r}{4r(1 + r) - 1} K_2 l_3.$$

$$M_3 = \frac{-2r}{4r(1 + r) - 1} K_2 l_3.$$

The curves for these are shewn Figs. 15b, 15c, for case $r = 1{\cdot}5$.

REACTIONS.

Load on l_2.

$$R_2 = \frac{M_3 - M_2}{l_2} + (1-k) = (1-k) + \frac{1}{4r+3}\left\{(3+2r)K_2 - 3K_1\right\}$$

$$R_4 = \frac{M_3}{l_3} = -\frac{1}{r(4r+3)}\left\{2K_1 - K_2\right\}$$

$$R_3 = k + \frac{M_2}{l_2} - (1+r)R_4 = k - \frac{1}{r(4r+3)}\left\{(1+r)(1+2r)K_2 - (2+3r)K_1\right\}$$

Load on l_3.

$$R_4 = \frac{M_3}{l_3} + k = k - \frac{2r}{4r(1+r) - 1} K_2.$$

$$R_3 = \frac{M_2}{l_2} - (1+r)R_4 + (1+kr) = (1-k) + \frac{3r^2+2r}{4r(1+r)-1} K_2$$

$$R_2 = \frac{M_3 - M_2}{l_2} = -\frac{3r^2}{4r(1+r)-1} K_2.$$

The curves for Reactions are shewn, Figs. 15d-15f, for $r = 1{\cdot}5$.

Table 15.—Bending Moments and Reactions.

k	$\frac{M_2}{l_2}$	$\frac{M_2}{l_3}$	$\frac{M_3}{l_2}$	$\frac{M_3}{l_3}$	R_2		R_3		R_4	
	Load on l_2	Load on l_3	Load on l_2	Load on l_3	Load on l_2	Load on l_3	Load on l_2	Load on l_3	Load on l_2	Load on l_3
	−	+	−	−	+	−	+	+	−	+
·10	·0840	·0183	·0030	·0366	·9810	·0824	·0217	1·0191	·0020	·0634
·20	·1387	·0308	·0107	·0617	·9280	·1389	·0791	1·0006	·0071	·1383
·30	·1680	·0382	·0210	·0765	·8470	·1721	·1670	·9486	·0140	·2235
·40	·1760	·0411	·0320	·0823	·7440	·1851	·2773	·8674	·0213	·3177
·50	·1667	·0402	·0417	·0804	·6250	·1808	·4028	·7611	·0277	·4196
·60	·1440	·0360	·0480	·0720	·4960	·1620	·5360	·6340	·0320	·5280
·70	·1120	·0292	·0490	·0585	·3630	·1316	·6697	·4901	·0326	·6415
·80	·0747	·0206	·0427	·0411	·2320	·0926	·7964	·3337	·0284	·7589
·90	·0360	·0106	·0270	·0212	·1090	·0477	·9090	·1689	·0180	·8788
1·00	·0000	·0000	·0000	·0000	·0000	·0000	1·0000	·0000	·0000	1·0000

Case 16.—ARCH WITH THREE HINGES. (Fig. 16.)

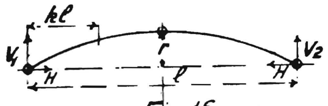


Fig 16.

Let r = rise.
l = span.
c = half span $= \frac{l}{2}$

Vertical Reactions and Horizontal Thrust.

Load on left half.

$$V_1 + V_2 = 1.$$

Moments about Abut. 2.

$$V_1 \times l = (1 - k)\, l.$$

Moments about crown.

$$V_1 \times \frac{l}{2} - H \times r - (\tfrac{1}{2} - k)\, l = 0.$$

$$\therefore\ V_1 = (1 - k) \text{ and } H = \frac{kl}{2r} = k\,\frac{c}{r}.$$

The curves for these are shewn in Figs. 16a, 16b.
The reasoning is the same for loads on right half.

Case 17.—ARCH WITH TWO HINGES. (Fig. 17.)

Fig 17.

Let r = rise of arch if parabolic or rise of equivalent parabola if circular (*i.e.* = rise of parabola of equal area to the circular segment.)

l = span.

c = half span $= \frac{l}{2}$.

b = distance from centre.

y_0 = height of load boundary.

$S = \frac{\text{half span}}{\text{rise}} = \frac{c}{r}$.

kl = distance from left abutment.

Vertical Reactions.

Since there is no Bending Moment at abutment,

$$V = (1 - k).$$

From the principles of elasticity,*

$$H = \frac{c^2 - b^2}{2c\,y_0} = \frac{c^2 - (c - kl)^2}{2c\left\{\dfrac{32\,c^2 r}{25\,c^2 - 5\,(c - kl)^2}\right\}}$$

$$= \frac{S\left\{1 - (1 - 2k)^2\right\}\left\{5 - (1 - 2k)^2\right\}}{12{\cdot}8}$$

since $y_0 = \dfrac{32\,c^2 r}{25c^2 - 5b^2}$.

The curves of vertical reaction and horizontal thrust are shewn in Figs. 17*a* and 17*b*.

Tables of Horizontal Thrust.

$$H = \frac{S}{12{\cdot}8}\left[\left\{1 - (1 - 2k)^2\right\}^2 + 4\left\{1 - (1 - 2k)^2\right\}\right]$$

Table is worked out for S = unity. Figures are to be multiplied by S *i.e.*, $\dfrac{\text{half span}}{\text{rise}}$.

(*a*) *k* in decimals.

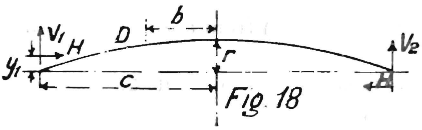
k	0	1	2	3	4	5	6	7	8	9
0	·0000	·0125	·0250	·0370	·0498	·0622	·0745	·0866	·0988	·1107
1	·1226	·1345	·1461	·1565	·1688	·1797	·1907	·2015	·2119	·2222
2	·2320	·2418	·2514	·2608	·2697	·2785	·2869	·2949	·3029	·3102
3	·3175	·3245	·3312	·3375	·3434	·3490	·3542	·3593	·3540	·3680
4	·3720	·3754	·3787	·3818	·3838	·3857	·3874	·3890	·3899	·3903
5	·3906									

* See Johnson's "Framed Structures," or Merriman & Jacoby's "Higher Structures."

(*β*) *k* in fractions. Numerator in column. Denominator in row.

k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	·0000	·3906	·3395	·2785	·2320	·1979	·1718	·1517	·1357	·1225	·1118	·1028	·0980	·0889	·0803	·0775	·0730	·0690	·0654	·0622
2		·0000	...	·3906	·3721	·3395	·3072	·2785	·2534	·2320	·2136	·1979	·1840	·1718	·1601	·1517	·1432	·1357	·1288	·1226
3			·0000	...	...	·3906	·3811	·3616	·3395	·3175	·2971	·2785	·2813	·2459	·2320	·2199	·2081	·1979	·1883	·1797
4				·0000	...	...	...	·3906	·3848	·3720	·3572	·3395	·3230	·3062	.2922	·2785	·2654	·2534	·2423	·2320
5					·0000	...	...	...	...	·3906	·3868	·3777	·3659	·3529	·3395	·3262	·3134	·8011	·2894	·2785
6						·0000	...	...	...	...	...	·3906	·3879	·3811	·3721	·3616	·3507	·3395	·3284	·3175
7							·0000	...	...	...	...	...	...	·3906	·3885	·3814	·3761	·3677	·3584	·3490
8								·0000	...	...	...	...	...	...	...	·3906	·3890	·3848	·3789	·3720
9									·0000	...	...	...	...	...	...	...	...	·3906	·3893	·3857
10										·0000	...	...	...	...	...	...	...	...	...	·3906

Case 18.—ARCH FIXED AT ENDS. (Fig. 18.)



For this case the method is particularly suited, as here we have a few crucial points by which the maximum and minimum sections are determined. The maximum B. Mt. is at the abutment and the minimum at the crown. Of intermediate points the ¼ point is worked out. For a short span the B. Mts. as found from these points fixes the shape and size of the section to be allowed.

From the principles of elasticity the following results are deduced.*

Let y_1 = imaginary equivalent height *above* horizontal at which horizontal thrust acts.
b = distance from centre.
c = half span.
r = rise of arch.
$\frac{b}{c} = x.$ and $S = \frac{c}{r}.$

Moments are reckoned positive when in same direction as B. Mt. due to vertical reaction.

Influence Line y_1.

$$y_1 = \frac{2}{15}\frac{c-5b}{c-b}r = \left\{\frac{2}{15}\cdot\frac{1-5x}{1-x}\right\}r.$$ See Fig. 18*c*.

Influence Line Vertical Reaction.

$$V = \frac{1}{4}(2-x)(1+x)^2.$$ See Fig. 18*a* and Table 18*a*.

Influence Line Horizontal Thrust.

$$H = \frac{15}{32}(1-x^2)^2 \times S.$$ See Fig. 18*b* and Table 18*b*.

Bending Moment at Abutment.

$$M_1 = Hy_1 = \frac{c}{16}(1-x^2)(1+x)(1-5x).$$ See {Fig. 18*d*. / Table 18*d*.}

Bending Moment at Crown.

$$M_c = V_1 \times c - H(r-y_1) - b$$
$$= \frac{c}{32}(3-16x+18x^2-5x^4).$$ See {Fig. 18*e*. / Table 18*e*.}

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* See Footnote to Case 17, Page 93.

Bending Moment at $\frac{1}{4}$ Point (*i.e.*, Point D., Fig. 18).

Let r_1 = rise at $\frac{1}{4}$ point = ar where r is rise of arch.

$$M_d = V_1 \times \frac{c}{2} - H(r - y_1) - \left(b - \frac{c}{2}\right) \text{ from abut. 1 to } \tfrac{1}{4} \text{ point.}$$

$$M_d = V_1 \times \frac{c}{2} - H(r - y_1) \text{ from } \tfrac{1}{4} \text{ point to abut. 2.}$$

These become

$$M_d \text{ from 1 to D} = \tfrac{1}{4}(2 - x)(1 + x)^2 \times \frac{c}{2}$$

$$- \frac{15}{32}(1 - x^2)^2 c \left(a - \frac{2}{15}\cdot\frac{1 - 5x}{1 - x}\right) - c\left(x - \frac{1}{2}\right)$$

$$\text{,, ,, D to 2} = \tfrac{1}{4}(2 - x)(1 + x)^2 \times \frac{c}{2}$$

$$- \frac{15}{32}(1 - x^2)^2 c \left(a - \frac{2}{15}\cdot\frac{1 - 5x}{1 - x}\right)$$

These might also be deduced from fact that:

$$\text{B. Mt. at } \tfrac{1}{4} \text{ point} = \text{B. Mt. at } 2 + V \times \frac{c}{2} - H \times ar - \left(b - \frac{c}{2}\right)$$

Last term omitted when $b < \frac{c}{2}$.

Figs. 18*d*, 18*e*, 18*f* are drawn to same scale.

Table 18a.—Vertical Reactions.

$$V = \tfrac{1}{4}(2 - x)(1 + x)^2.$$

x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9
0	·5000	·5075	·5150	·5225	·5300	·5375	·5450	·5524	·5599	·5673
1	·5748	·5822	·5896	·5970	·6043	·6117	·6190	·6263	·6336	·6408
2	·6480	·6552	·6624	·6695	·6766	·6836	·6906	·6976	·7045	·7114
3	·7183	·7251	·7318	·7385	·7452	·7518	·7583	·7648	·7713	·7777
4	·7840	·7903	·7965	·8026	·8087	·8147	·8207	·8265	·8323	·8381
5	·8438	·8493	·8548	·8603	·8656	·8709	·8761	·8812	·8862	·8911
6	·8960	·9008	·9054	·9100	·9145	·9188	·9231	·9273	·9314	·9354
7	·9392	·9430	·9467	·9502	·9537	·9570	·9602	·9634	·9664	·9692
8	·9720	·9746	·9772	·9796	·9818	·9840	·9860	·9879	·9896	·9913
9	·9928	·9941	·9953	·9964	·9974	·9982	·9988	·9993	·9997	·9999

Table 18b.—Horizontal Thrust.

$H = S \times \frac{15}{32}(1 - x^2)^2$. Figures are for S = unity.

x	0	1	2	3	4	5	6	7	8	9
0	·4687	·4685	·4684	·4680	·4670	·4662	·4652	·4640	·4626	·4610
1	·4593	·4573	·4551	·4529	·4504	·4478	·4450	·4420	·4388	·4353
2	·4319	·4281	·4245	·4205	·4163	·4120	·4075	·4027	·3980	·3932
3	·3882	·3830	·3777	·3721	·3668	·3609	·3550	·3491	·3431	·3370
4	·3307	·3243	·3179	·3114	·3048	·2981	·2914	·2846	·2777	·2707
5	·2636	·2566	·2495	·2424	·2352	·2281	·2209	·2136	·2065	·1992
6	·1920	·1847	·1776	·1704	·1634	·1563	·1493	·1423	·1354	·1287
7	·1219	·1152	·1092	·1022	·0959	·0897	·0836	·0776	·0718	·0662
8	0607	·0554	·0503	·0453	·0406	·0361	·0318	·0277	·0239	·0202
9	·0169	·0138	·0111	·0085	·0063	·0045	·0029	·0016	·0007	·0001

Table 18*d*.—Bending Moment at Abutment.

Figures are for c = unity.

$$M_1 = \frac{c}{16}(1 - x^2)(1 + x)(1 - 5x).$$

x	0	1	2	3	4	5	6	7	8	9
0	+ ·0625	+ ·0599	+ ·0573	+ ·0547	+ ·0519	+ ·0491	+ ·0463	+ ·0433	+ ·0402	+ ·0371
1	+ ·0340	+ ·0309	+ ·0276	+ ·0243	+ ·0210	+ ·0176	+ ·0142	+ ·0107	+ ·0071	+ ·0036
2	·0000	– ·0036	– ·0073	– ·0109	– ·0146	– ·0183	– ·0220	– ·0258	– ·0295	– ·0333
3	– ·0370	– ·0407	– ·0444	– ·0481	– ·0518	– ·0555	– ·0592	– 0629	– ·0666	– ·0702
4	– ·0735	– ·0770	– ·0806	– ·0838	– ·0875	– ·0903	– ·0935	– ·0965	– ·0995	– ·1025
5	– ·1055	– ·1083	– ·1110	– ·1136	– ·1160	– ·1182	– ·1204	– ·1226	– ·1245	– ·1263
6	– ·1280	– ·1295	– ·1308	– ·1320	– ·1332	– ·1340	– ·1347	– ·1351	– ·1353	– ·1355
7	– ·1355	– ·1351	– ·1346	– ·1339	– ·1331	– ·1316	– ·1301	– ·1284	– .1263	– ·1239
8	– ·1214	– ·1188	– ·1154	– ·1120	– ·1084	– ·1042	– ·1000	– ·0951	– ·0900	– ·0848
9	– ·0790	– ·0728	– ·0663	–.0595	– ·0522	– ·0445	– ·0365	– ·0288	– ·0191	– ·0098

Table 18e.—Bending Moment at Crown.

Figures are for c = unity.

$$M_c = \frac{c}{32}(3 - 16x + 18x^2 - 5x^4).$$

x	0	1	2	3	4	5	6	7	8	9
0	+·0938	+·0905	+·0874	+·0842	+·0809	+·0776	+·0743	+·0708	+·0673	+·0638
1	+·0603	+·0567	+·0532	+·0496	+·0460	+·0424	+·0387	+·0350	+.0312	+·0277
2	+·0240	+·0203	+·0166	+·0129	+·0093	+.0055	+·0018	—·0019	—·0055	—·0092
3	—·0128	—·0164	—·0200	—·0235	—·0270	—·0305	—·0339	—·0373	—·0407	—·0440
4	—·0472	—·0504	—·0536	—.0567	—·0597	—.0626	—·0654	—·0682	—·0709	—·0736
5	—·0762	—·0787	—·0810	—·0833	—·0854	—·0875	—·0894	—·0912	—·0929	—·0945
6	—·0960	—·0974	—·0986	—·0996	—·1005	—·1013	—·1019	—·1023	—·1026	—·1028
7	—·1028	—·1026	—·1021	—·1016	—·1009	—·1000	—·0989	—·0974	—·0959	—·0942
8	—·0923	—.0901	—·0876	—·0849	—·0822	—·0791	—·0756	—·0720	—·0681	—·0640
9	—·0596	—·0550	—·0500	—·0448	—·0394	—·0336	—·0275	—·0210	—·0144	—·0059

The ordinary text books* have not applied the method in the way illustrated above (except part of Case 2) and the curves and tables have been prepared for this paper, so that any corrections or suggestions as to the reasoning or figures would be gladly received, as the author hopes to extend this contribution.

The writer desires to thank Mr. H. H. Dare, M.E., Assoc. M. Inst. C.E., for the use of some figures applying to Cases 17 and 18; also Mr. A. J. Gibson, Assoc. M. Inst. C.E., who prepared the tracings of diagrams and Mr. H. S. Mort, B.Sc., for great assistance by working out numbers for the tables.

* *e.g.*, Johnson's "Braced Structures," Merriman and Jacoby's "Higher Structures," Warren's "Engineering Construction," Fidler's "Bridge Construction," Lanza's "Applied Mechanics," have no reference to Influence Lines.

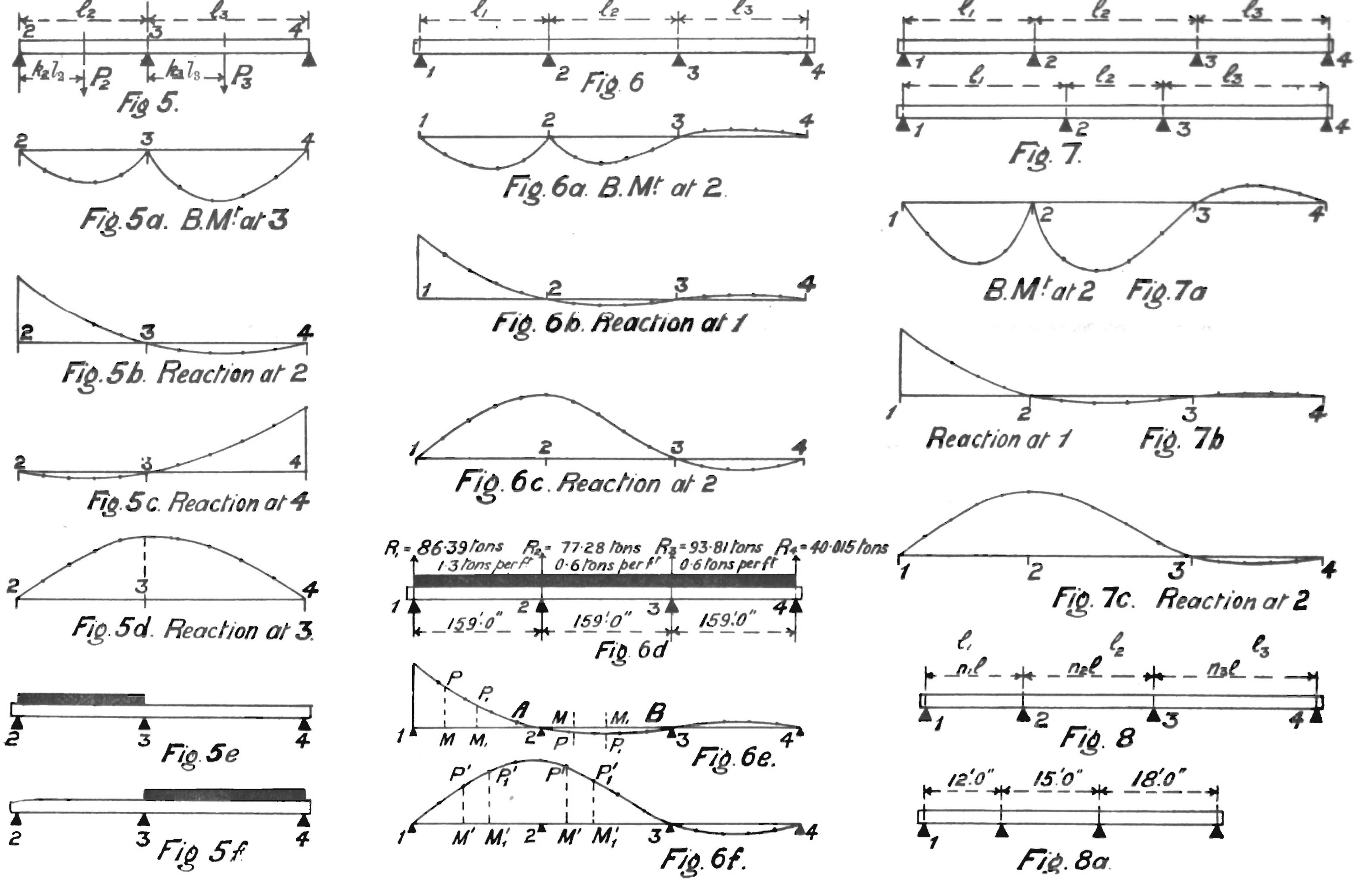
Fig 5.
Fig 5a. B.Mt at 3
Fig. 5b. Reaction at 2
Fig. 5c. Reaction at 4
Fig. 5d. Reaction at 3.
Fig. 5e
Fig 5f.
Fig 6
Fig. 6a. B.Mt at 2.
Fig. 6b. Reaction at 1
Fig. 6c. Reaction at 2
R1 = 86·39 Tons R2 = 77·28 Tons R3 = 93·81 Tons R4 = 40·015 Tons
1·3 Tons per Ft 0·6 Tons per Ft 0·6 Tons per Ft
159'·0" 159'·0" 159'·0"
Fig 6d
Fig. 6e.
Fig. 6f.
Fig. 7.
B.Mt at 2 Fig. 7a
Reaction at 1 Fig. 7b
Fig. 7c. Reaction at 2
Fig. 8
12'·0" 15'·0" 18'·0"
Fig. 8a

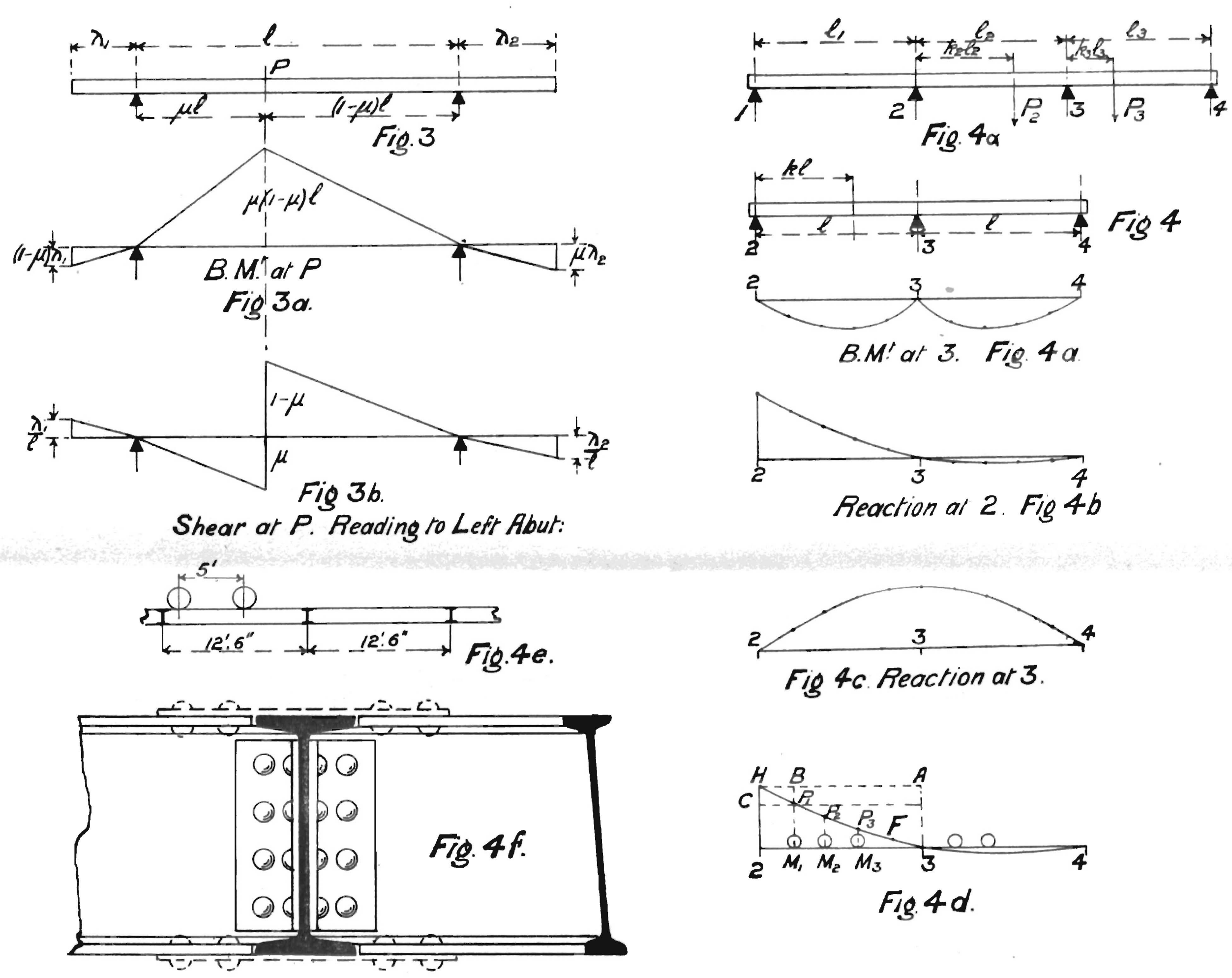
Fig. 3
B.Mt at P
Fig 3a.
Fig 3b.
Shear at P. Reading to Left Abut:
Fig. 4e.
Fig. 4f.
Fig. 4a
Fig 4
B.Mt at 3. Fig. 4a
Reaction at 2. Fig 4b
Fig 4c. Reaction at 3.
Fig. 4d.

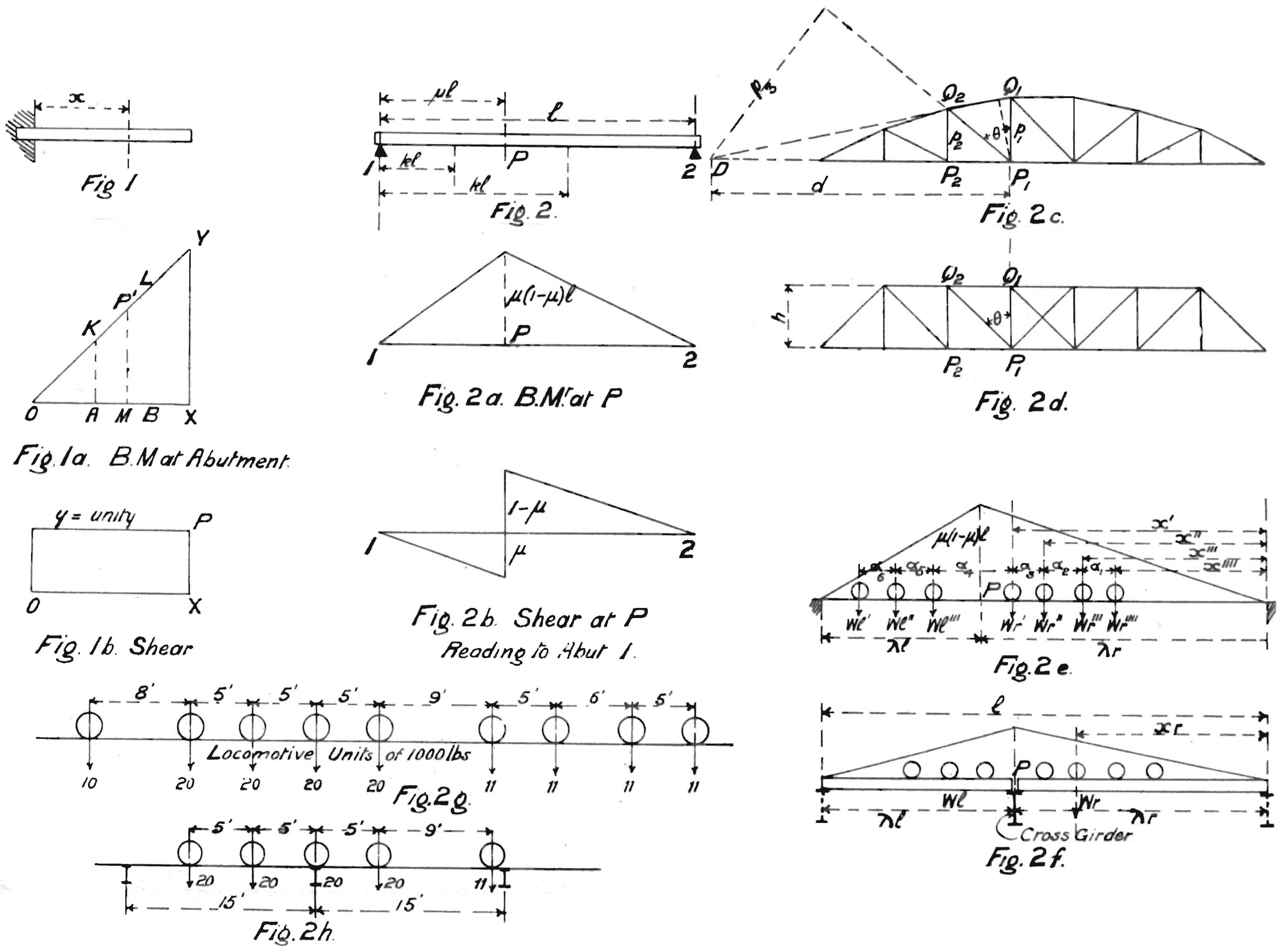
Fig 1
Fig. 1a. B.M at Abutment
y = unity
Fig. 1b. Shear
Fig. 2.
Fig. 2a. B.Mt at P
Fig. 2b. Shear at P
Reading to Abut 1.
Locomotive Units of 1000 lbs
Fig. 2g.
Fig. 2h.
Fig. 2c.
Fig. 2d.
Fig. 2e.
Cross Girder
Fig. 2f.

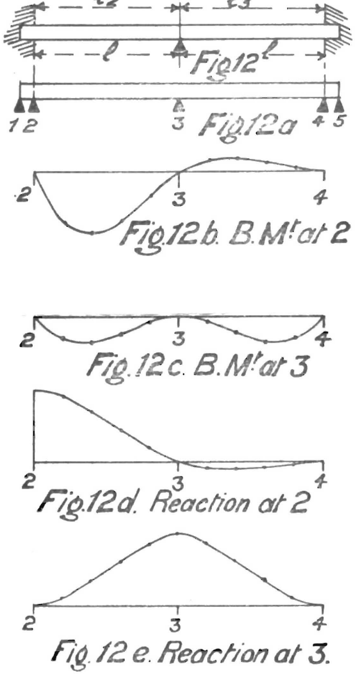

Fig. 12. Fig. 12a. Fig. 12b. B.M.t at 2. Fig. 12c. B.M.t at 3. Fig. 12d. Reaction at 2. Fig. 12e. Reaction at 3.

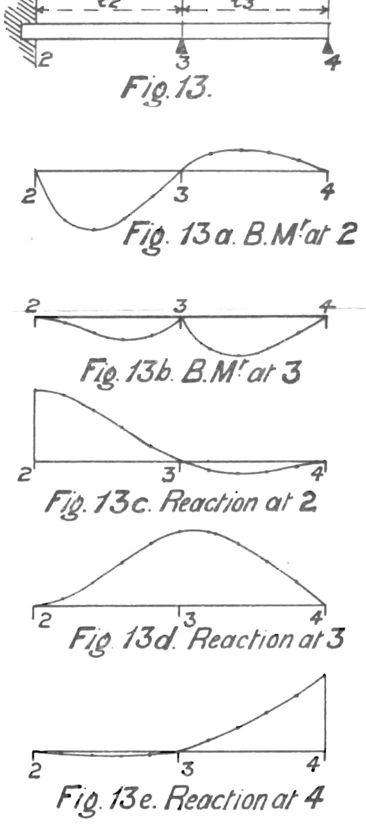

Fig. 13. Fig. 13a. B.M.t at 2. Fig. 13b. B.M.t at 3. Fig. 13c. Reaction at 2. Fig. 13d. Reaction at 3. Fig. 13e. Reaction at 4.

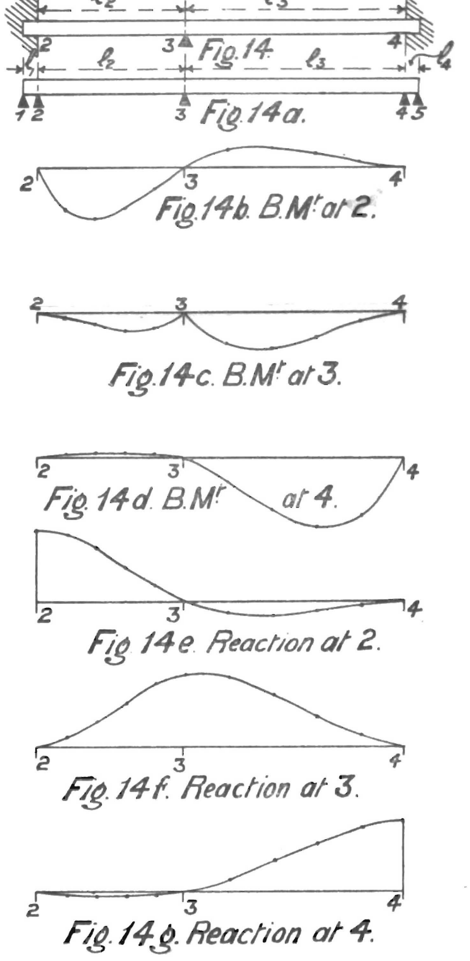

Fig. 14. Fig. 14a. Fig. 14b. B.M.t at 2. Fig. 14c. B.M.t at 3. Fig. 14d. B.M.t at 4. Fig. 14e. Reaction at 2. Fig. 14f. Reaction at 3. Fig. 14g. Reaction at 4.

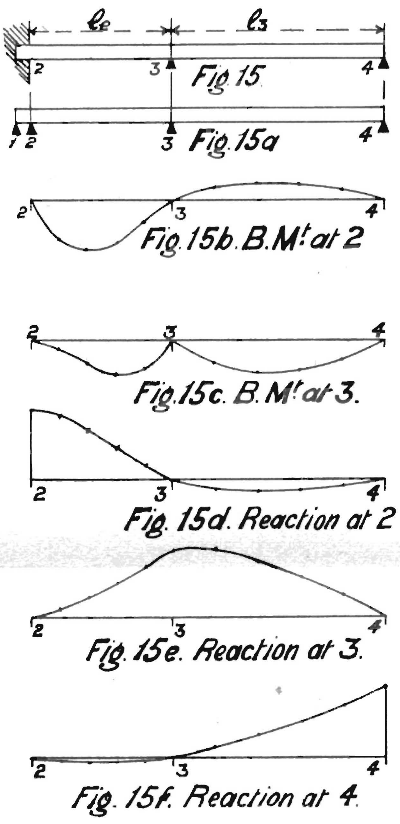

Fig. 15. Fig. 15a. Fig. 15b. B.M.t at 2. Fig. 15c. B.M.t at 3. Fig. 15d. Reaction at 2. Fig. 15e. Reaction at 3. Fig. 15f. Reaction at 4.

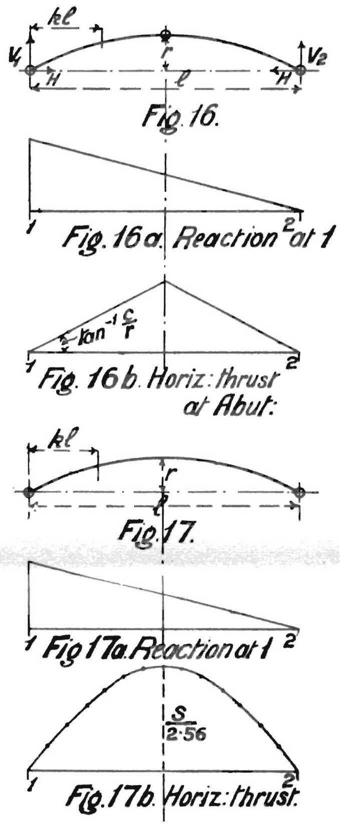

Fig. 16. Fig. 16a. Reaction at 1. Fig. 16b. Horiz: thrust at Abut: Fig. 17. Fig. 17a. Reaction at 1. Fig. 17b. Horiz: thrust.

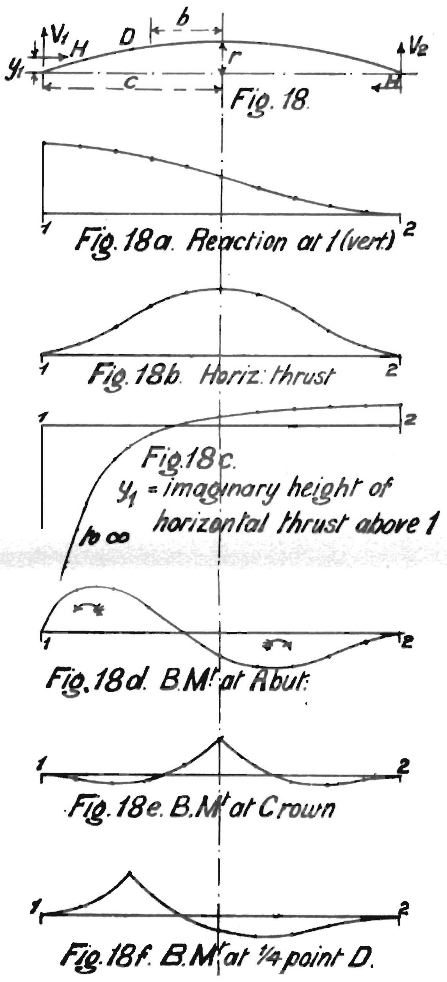

Fig. 18. Fig. 18a. Reaction at 1 (vert). Fig. 18b. Horiz: thrust. Fig. 18c. y_1 = imaginary height of horizontal thrust above 1. Fig. 18d. B.M.t at Abut. Fig. 18e. B.M.t at Crown. Fig. 18f. B.M.t at ¼ point D.

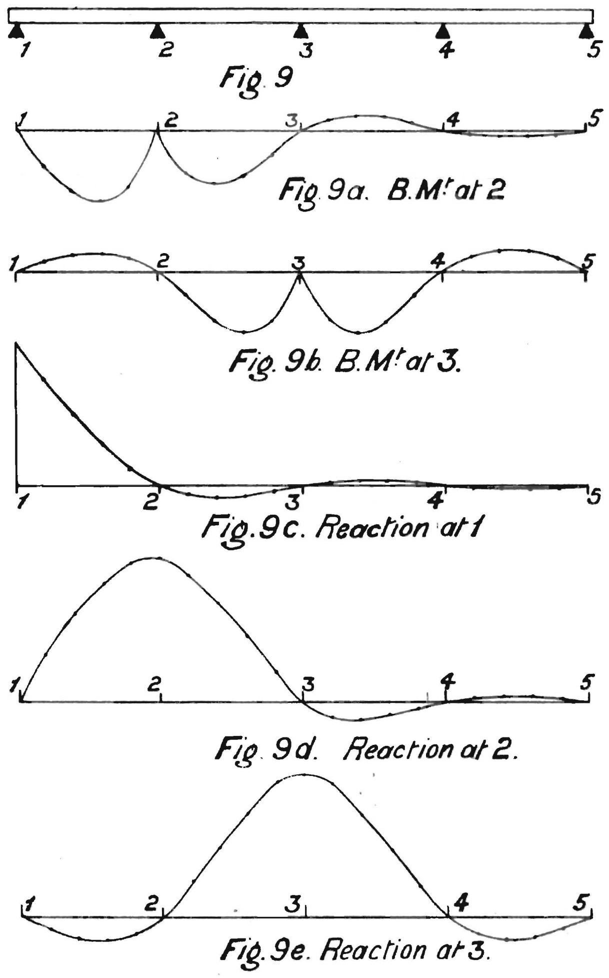

Fig. 9. Fig. 9a. B.M.t at 2. Fig. 9b. B.M.t at 3. Fig. 9c. Reaction at 1. Fig. 9d. Reaction at 2. Fig. 9e. Reaction at 3.

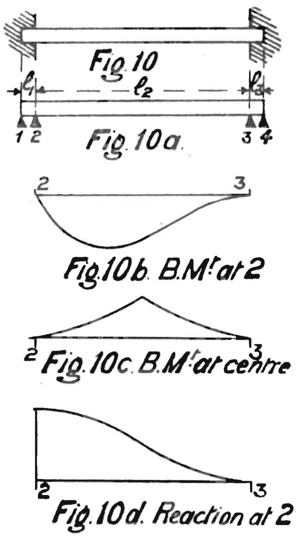

Fig. 10. Fig. 10a. Fig. 10b. B.M.t at 2. Fig. 10c. B.M.t at centre. Fig. 10d. Reaction at 2.

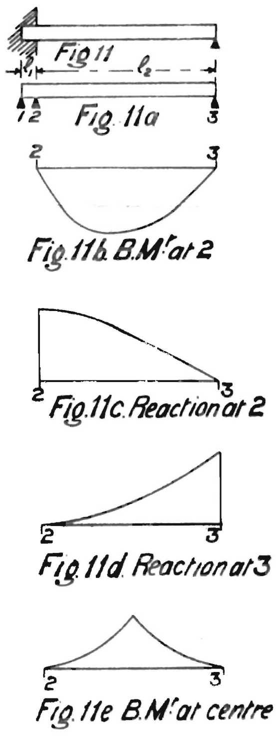

Fig. 11. Fig. 11a. Fig. 11b. B.M.t at 2. Fig. 11c. Reaction at 2. Fig. 11d. Reaction at 3. Fig. 11e. B.M.t at centre.

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