DESIGN OF RETAINING WALLS, PYRMONT BRIDGE.

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The following paper gives the calculations on which is based the design of the retaining walls on the south side of the Pyrmont Bridge, to illustrate to our undergraduate members the application of theory in practice.

DESCRIPTION.

The retaining walls are built of concrete faced with sandstone masonry, and are founded on turpentine piles driven to rock through mud, clay and sand. Near the water this *made ground* is subject to tidal influence and could not sustain any load without piling.

In cross-section the piles are grouped closer together at the front of the wall, to sustain the resultant pressure from the earth thrust and weight of wall. In elevation they are spaced 3 ft. 0 in. apart, centre to centre longitudinally. Each row of piles has a 12 in. by 12 in. ironbark headstock mortised over heads of piles, and the rows are tied together, longitudinally, with four 12 in. by 12 in. ironbark girders. The girders and headstocks form a grill or platform, on which the concrete footings are built. The section and elevation of wall and plan of grill are shown on Plate.

The concrete is mixed in the proportion of six sandstone or gravel, 2 in. gauge, to two-and-a-half sand, to one cement; the masonry is rock faced sandstone laid in regular courses set in cement mortar, the rock face projecting not more than three inches beyond the pitch line of joints.

The section of wall proposed to be investigated is 31 ft. 6 in. high above top of footing, battered 1 in 12 for a height of 30 ft. on face, and stepped at back as shown. Rubble backing is provided for drainage purposes, and the filling behind the walls is earth.

METHOD OF PROCEDURE.

It is usual to consider a strip of wall and backing 1 foot wide, and the intensity of pressure investigated not only at the bottom of the wall, but at various points in its height. Only two sections are investigated here, viz., at top and bottom of footing. In proportioning the wall, the thickness at any section should be from $\frac{1}{4}$ to $\frac{1}{3}$ the height of the wall.

The calculations should be undertaken in the following order :—

(1) Find weight and centre of gravity of wall and backing.

(2) Find thrust due to earth pressure.

(3) Find resultant line of pressure by combining (1) and (2).

(4) Find the intensity of pressure.
  (*a*) Neglecting the tensile strength of concrete.
  (*b*) Considering the tensile strength of concrete.

CALCULATION OF WALL ABOVE TOP OF FOOTING

(Figures 1, 2, and 3.)

Weight and centre of gravity. — The weights adopted are as follows :—

Sandstone concrete	= 137 lbs. per cubic feet	= $1\frac{1}{4}$ cwts.
Rubble	= 118 „ „ „	= $1\frac{1}{20}$ „
*Earth filling	= 127 „ „ „	= $1\frac{1}{9}$ „

The following table gives the weight and moment about the point A, of the concrete and rubble backing :—

CONCRETE.		
WEIGHT.	LEVERAGE.	MOMENT.
Cwts.		ft. cwts.
1 ft. 6 in. x 31 ft. 6 in x 1 ft. x $1\frac{1}{4}$ = 59.6	3 ft. 3 in.	191·95
6 in. x 27 ft. x „ x „ = 16·88	4 ft. 3 in.	71·74
12 in. x 23 ft. x „ x „ = 28·75	5 ft. 0 in.	143·75
12 in. x 19 ft. x „ x „ = 23·75	6 ft. 0 in.	142·50
12 in. x 15 ft. x „ x „ = 18·75	7 ft. 0 in.	131·25
12 in. x 11 ft. x „ x „ = 13·75	8 ft. 0 in.	110·00
12 in. x 7 ft. x „ x „ = 8·75	9 ft. 0 in.	78·75
12 in. x 3 ft. x „ x „ = 3·75	10 ft. 0 in.	37·50
$\frac{2 \text{ ft. } 6 \text{ in.}}{2}$ x 29 ft.† x „ x „ = 45·31	1 ft. 8 in.	75·51
Total ... 218·75		982.95

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* This allows 7 lbs. per cubic foot for live load.

† This height as shown by Figures 1 and 4 is 30 feet, but to approximate to the actual quantity in the finished wall, 29 feet was adopted, an unnecessary refinement.

RUBBLE BACKING.		
WEIGHT.	LEVERAGE.	MOMENT.
Cwts.		ft. cwts.
12 in. x 28 ft. 6 in. x 1 ft. x $1\frac{1}{20}$ = 29·92	10 ft. 0 in.	299·20
12 in. x 24 ft. 6 in. x ,, x ,, = 25·72	9 ft. 0 in.	231·48
12 in. x 20 ft. 6 in. x ,, x ,, = 21·52	8 ft. 0 in.	172·16
12 in. x 16 ft. 6 in. x ,, x ,, = 17·33	7 ft. 0 in.	121·31
12 in. x 12 ft. 6 in. x ,, x ,, = 13·12	6 ft. 0 in.	78·72
12 in. x 8 ft. 6 in. x ,, x ,, = 8·93	5 ft. 0 in.	44·65
6 in. x 4 ft. 6 in. x ,, x ,, = 4·46	4 ft. 3 in.	18·96
Total ... 121·00		966·48

Let x = the longitudinal distance of centre of gravity of concrete and rubble from the point A.

Then x (218·75 + 121·00) = (982·95 + 966·48) *i.e.*, x = 5 ft. 9 in.

The weight of the concrete and rubble, *viz.*, 339·75 cwts., acts vertically through its centre of gravity, distant 5 ft. 9 in. from point A.

Thrust due to Earth Pressure.—The thrust due to the earth pressure at back of wall is given by the formula.

$$T = \tfrac{1}{2}\, wh^2\, tan^2 \left(\frac{\pi}{4} - \frac{\phi}{2}\right)$$ (Moseley and Romilly Allen)

and is identical with the formula of Rankine

$$T = \tfrac{1}{2}\, wh^2\, \frac{1 - sin\ \phi}{1 + sin\ \phi}$$

Where w = weight of earth = $1\frac{1}{9}$ cwts. per cubic ft.

h = height of wall subjected to pressure = 31 ft. 6 in.

ϕ = angle of repose = 34° for earth backing.

$\pi = 180° \ : \ \frac{\pi}{4} - \frac{\phi}{2} = 28° \ : \ tan\ 28° = \cdot 53$

substituting these values in the formula above

$$T = \tfrac{1}{2} \times \tfrac{10}{9} \times (31{\cdot}5)^2 \times (\cdot 53)^2 = 154 \text{ cwts.}$$

This thrust acts at $\frac{1}{3}$ the height of wall above surface of ground, *i.e.*, at 10 ft. 6 in. above top of footing.

Resultant line of pressure.—Combining the weight of wall and backing, 339·75 cwts., acting vertically through its centre of gravity, with the earth thrust of 154 cwts. acting 10 feet 6 inches above footing (Figure 1), the resultant line of pressure falls 12 inches within the toe of wall. The weight-thrust diagram is drawn to any convenient scale; in Figures 1 and 4, the scale is 200 cwts. = 1 inch.

Intensity of pressure.—Neglecting the tensile strength of concrete.— Assuming the line of pressure must fall within the centre third of a wall, then the weight of wall and rubble (viz., 339·75 cwts.) must be taken as distributed over an *effective* width of 3 × 12 in. = 3 ft. 0 in.

The maximum intensity of pressure (compression) = $\frac{2}{3} \times 339{\cdot}75 \times \frac{1}{12} \times \frac{1}{12} = 1{\cdot}572$ cwts. = 176·0 lbs. per square inch.

Taking the ultimate compressive strength of concrete (6 : $2\frac{1}{2}$: 1) as 64 tons per square foot = 8·88 cwts. = 996 lbs. per square inch.

Factor of safety against crushing = 996 ÷ 176·0 = 5·6.

Figure 2 shows the distribution of pressure decreasing from 1·57 cwts. per square inch at face of wall, to nil at a point 3 feet from face. The wall behind this point is not subjected to stress.

As a check to the above figures, maximum intensity = 1·572 cwts.; minimum intensity = 0,

$$\text{average intensity} = \frac{1{\cdot}572}{2} = 786 \text{ cwts. per square inch.}$$

This obtains over an area 36 in. × 12 in.

Total pressure = 36 × 12 × ·786 = 339·56 cwts., the weight of wall and rubble backing almost exactly.

Intensity of pressure.—Considering the tensile strength of concrete.— The maximum compression is given by the formula:—

$$\frac{2l - 3d}{l} \times \frac{2N}{l} = {\cdot}77 \text{ cwts.} = 86 \text{ lbs. per square inch.}$$

Where N = normal pressure over 1 inch × width of wall

$$= \frac{339{\cdot}75}{12} = 28{\cdot}31 \text{ cwts.}$$

l = width = 10 ft. 6 in. = 126 inches.

d = distance of line of pressure from point A = 12 inches.

$$\text{Hence factor of safety against crushing} = \frac{996}{86} = 11{\cdot}6$$

The maximum tension is given by the formula

$$\frac{3d - l}{l} \times \frac{2N}{l} = {\cdot}32 \text{ cwts.} = 36 \text{ lbs. per square inch}$$

Taking the ultimate tensile strength of concrete (6 : $2\frac{1}{2}$: 1) as 150 lbs. per square inch.

Factor of safety against tension = 150 ÷ 36 = 4

Figure 3 shows the maximum compression of 86 lbs. per square inch at face, and maximum tension of 36 lbs. per square inch at back of wall. They decrease towards and vanish at a point inside the wall. By similar triangles this point is 88·8 inches from the face of wall.

This concludes the calculations of Wall above top of footing.

WALL AND FOOTING. (FIGURES 4, 5 and 6).

Weight and Centre of Gravity.—The weight of a strip of footing 1 ft. wide = 1 ft. × 2 ft. 6 in. × 12 ft. 6 in. × $1\frac{1}{4}$ = 39·06 cwts. This acts at its centre of gravity 4 ft 9 in. from point A.

Let y = the longitudinal distance of wall and rubble, and footing from the point A.

$$\text{Then } y\,(339{\cdot}75 + 39{\cdot}06) = 1949{\cdot}43 + (39{\cdot}06 \times 4 \text{ ft. } 9 \text{ in.})$$
$$i.e.,\ y = 5{\cdot}63 \text{ ft.} = 5 \text{ ft. } 7\tfrac{1}{2} \text{ in.}$$

Thrust Due to Earth Pressure.—The earth thrust is the same as in previous case and acts at the same height above surface of ground.

Resultant Line of Pressure.—Combining the weight and earth thrust as before, the resultant line of pressure falls 1 ft. 10½ in. within toe of footing, as in Figure 4.

Intensity of Pressure.—Neglecting the tensile strength of concrete.—The weight of wall, footing and rubble 378·8 cwts. is taken as distributed over an effective width of 3 × 1 ft. 10½ in. = 5 ft. 7½ in.

The maximum intensity of pressure (compression)

$$= \frac{2}{3} \times 378{\cdot}81 \times \frac{1}{12} \times \frac{1}{22\frac{1}{2}''} = {\cdot}93 \text{ cwts. per square inch}$$

$$\therefore \text{ Factor of safety} = 8{\cdot}88 \div {\cdot}93 = 9{\cdot}6$$

Figure 5 shows this intensity of pressure decreasing from ·93 cwts. per square inch (maximum) at face to zero at a point 5 ft. 7½ in. from face. The piles are driven as close as possible over this width, they are here spaced 3 ft. apart, because it is difficult to drive a number of piles 18 in. diameter at the head closer together than this. The intensity of pressure at the points B and C can be found by proportion. The ordinates at these points divide the stress intensity diagram into three areas X, Y and Z. Vertical lines through the centres of gravity of the figures Y and Z and their distances from the points B and C respectively are shown.

The piles are spaced 3 ft. apart longitudinally, as shown by elevation and plan, so each of the areas X, Y and Z multiplied by 36 in., gives the stress intensity for a strip 3 ft. wide, *i.e.*, the load which has to be carried by each transverse row of piles at B, C, D and E.

$$\therefore\ X = \frac{{\cdot}93 + {\cdot}85}{2} \times 6 \text{ in.} \times 36 \text{ in.} = 192{\cdot}2 \text{ cwts.} = 9{\cdot}6 \text{ tons.}$$

$$Y = \frac{{\cdot}85 + {\cdot}35}{2} \times 36 \text{ in.} \times 36 \text{ in.} = 777{\cdot}6 \text{ cwts.} = 38{\cdot}9 \text{ tons.}$$

$$Z = \frac{{\cdot}35}{2} \times 25\tfrac{1}{2} \text{ in.} \times 36 \text{ in.} = 16{\cdot}6 \text{ cwts} = 8{\cdot}0 \text{ tons.}$$

Total, 56·5 tons.

As a check, the weight of wall footing and rubble for a strip 1 foot wide is 378·8 cwts., and for a strip 3 feet wide = 1136·4 cwts. = 56·82 tons which is practically identical with the 56·5 tons above.

The weights of Y and Z are distributed on the piles at B, C and D, in accordance with the law of the lever, but the whole weight of X is carried by pile B. The loads on the piles are as follows :—

$$\text{Pile B} = 9{\cdot}6 + 38{\cdot}9 \times \frac{20\frac{1}{2}}{36} = 9{\cdot}6 + 22{\cdot}2 = \quad 31{\cdot}8 \text{ tons.}$$

$$\text{,, } C = (38{\cdot}9 - 22{\cdot}2) + 8 \times \frac{27\frac{1}{2}}{36} = 16{\cdot}7 + 6{\cdot}1 = 22{\cdot}8 \text{ ,,}$$

$$\text{,, } D = 8 - 6{\cdot}1 \qquad = \qquad = 1{\cdot}9 \text{ ,,}$$

$$\text{,, } E = 0 \qquad = \qquad = 0 \text{ ,,}$$

Total, 56·5 ,,

These are the maximum loads on the piles. They are driven to rock and receive practically no support from skin friction. The heads of the piles are, in such cases, proportioned to carry a safe load of 300 lbs. per square inch. For a pile 18 inches diameter the safe permissible load = 254·47 square inch × 300 lbs. = 34 tons, so none of the piles are excessively loaded.

Intensity of pressure.—Considering the tensile strength of concrete.— The maximum compression is given by the formula :—

$$\frac{2l - 3d}{l} \times \frac{2N}{l} = {\cdot}65 \text{ cwts.} = 73 \text{ lbs. per square inch.}$$

Where l = 12 ft. 6 in. = 150 inches.
d = 1 ft. $10\frac{1}{2}$ in. = $22\frac{1}{2}$,,
N = 378·8 ÷ 12 = 31·57 cwts.

∴ Factor of safety against crushing = 996 ÷ 65 = 13·6.

The maximum tension is given by the formula :—

$$\frac{3d - l}{l} \times \frac{2N}{l} = {\cdot}23 \text{ cwts.} = 26 \text{ lbs. per square inch.}$$

∴ Factor of safety against tension = 150 ÷ 26 = 5·8.

Figure 6 shows the maximum compression and tension and the point where there is neither tension or compression. This concludes the calculations of the wall and footing.

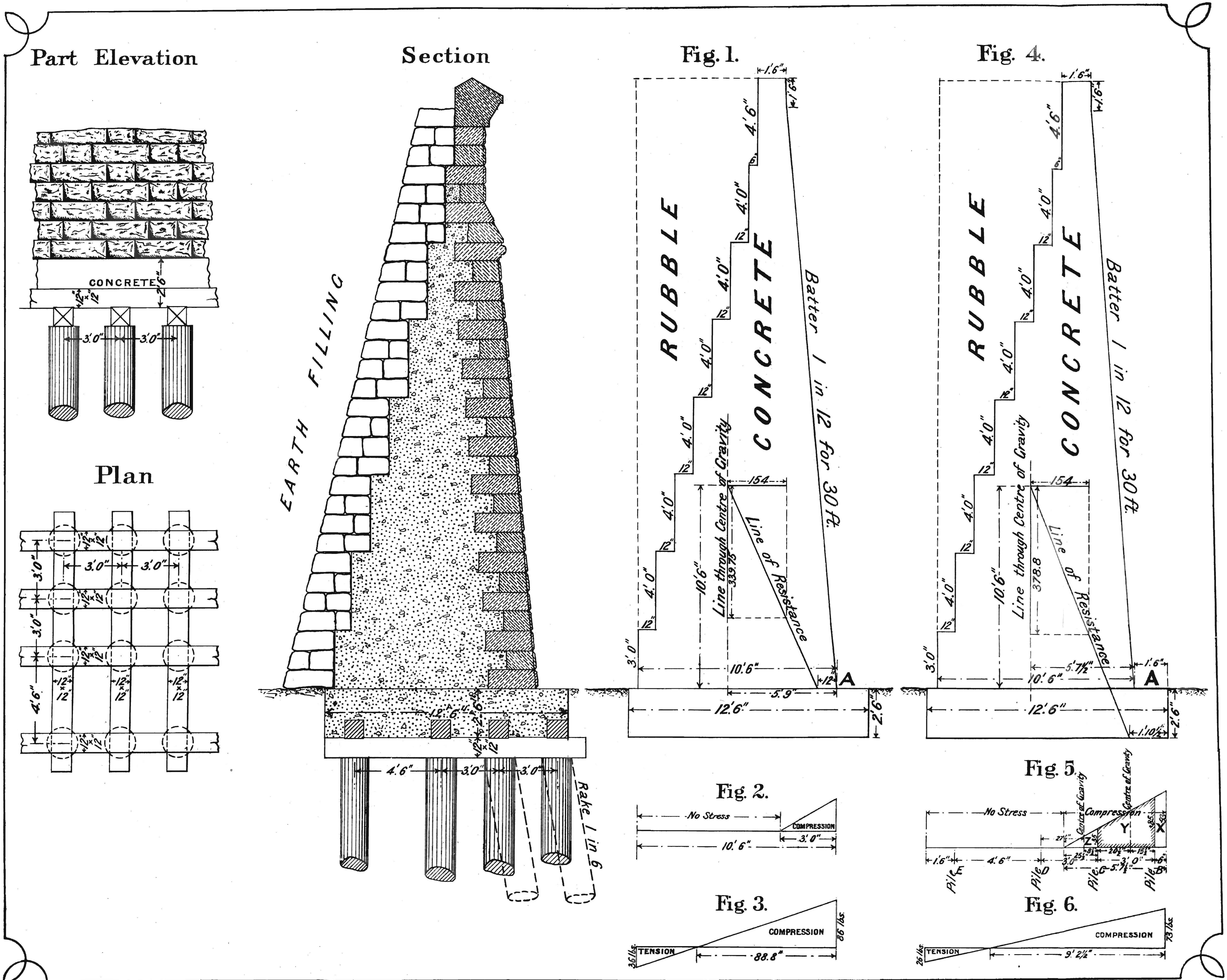
NOTES ON DESIGN AND CONSTRUCTION.

From papers on the strength of concrete read by Professor Warren before the Royal Society of New South Wales, on September 18th, 1901, and December 3rd, 1902, it will be seen that the compressive strength adopted of 64 tons per square foot, ultimate, is a fair one for 6 : $2\frac{1}{2}$: 1 concrete, but the tensile strength of 150 lbs. per square inch ultimate is high when compared with the strength of 6: 2: 1 and 6: 3: 1 concrete given therein, the average ultimate tensile strength being only 74 lbs. per square inch; but Professor Warren states that the results obtained can only be regarded as a rough indication of the tensile strength which is in every case below the real tensile strength.

The specified tensile strength of 3 to 1 Portland cement mortar, for the wall in question was 100 lbs. per square inch, seven days old, and 200 lbs., 28 days old, so 150 lbs. per square inch was taken as a fair average tensile strength for the concrete.

In carrying out the work, the ground, in places, was very soft, the pile driving did not harden it up, and the material between the driven piles had no carrying capacity, it was, therefore, evident that the total thrust would have to be carried by the piles, and to avoid them hinging on their toes and springing forward under it, timber ties were notched over the girders in the grill and carried across to the grill on to the other side of roadway. These ties prevented any movement which might have occured in the foundations.

In any pile foundation for a retaining wall, it would be better to drive the two front rows of piles with a batter, as indicated by dotted lines on section, this spreads the base and at the same time the piles are better placed to resist the resultant thrust, and would render the timber ties referred to above, unnecessary.



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