COLUMN ANALYSIS

— AND —

DESIGN.

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INTRODUCTION.

In the following papers the author studies the various standard formulæ and gives some new curves which may be used either for deducing data from breaking test results, or for the design of columns.

The deductions are made on a basis of the author's interpretation of the Euler Value for a column, which is assumed the maximum unit load; any other load is a fraction, and acts with some eccentricity.

The striking fact will be shewn that the curves are to be read to a scale according to the stress in the extreme fibre ; otherwise stated—*for analysing breaking test results over a certain range, curves different to those for design purposes should be used.* To this are due most of the anomalies shewn by different formulæ and the author believes that when the present curves* become familiar the treatment of column analysis will be remodelled.

Though the present papers deal with the primary column only, they should be of some assistance in the analysis of compound columns and their details, such as the design of latticing and joints, the proportion of thickness to length, and the effect of partial fixing : even the reinforced column may have some rational design. The problem is timeworn, yet the author trusts that the present treatment, while it closes some old paths, opens up others better and permanent.

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* The drawings from which the diagrams have been prepared are on squared paper of which each smallest sub-division is ·1 inch. Prints to full-size will be prepared if enough demand arises to defray the expense.

A COMPARATIVE ANALYSIS OF COLUMN FORMULAE.

In a paper and a supplementary paper* submitted to the Institution of Civil Engineers, the author gave an investigation of the Column Problem and curves showing how the results might be applied in the analysis of column stresses and to the design of columns.

In this paper it is proposed to analyse the various standard column formulae in the light of the theory stated in the papers mentioned.

The notation to be used is endorsed on the diagram accompanying called "Column Design Curves" (see Sheet No. 1), and will be stated here also.

Note—Tables and Curves are printed at the end of the paper..

Notation.

E = Modulus of Elasticity of the material.

I = Moment of Inertia of the cross-section.

l = Length of pin-ended column.

Q = Euler's value $= \dfrac{\pi^2 EI}{l^2}$ †

A = Area of cross section.

r = Radius of Gyration.

$$q = \frac{Q}{A} = \frac{\pi^2 E}{\left(\frac{l}{r}\right)^2}$$

P = Load on Column.

$$p = \frac{P}{A}$$

e = Eccentricity of loading

f_b = Stress at Extreme Fibre due to bending only.

*Attached as appendices B and C to this paper. These should be read before reading the present paper.

†See Appendix A.

*y = Distance of extreme fibre from neutral axis in formula $M = \frac{f_b I}{y} = P(e+a)$.

f = Total intensity of stress at extreme fibre $= p \pm f_b$

In this paper the upper sign only is used.

a = Deflection due to loading, where $(a+e) = e \sec \frac{p}{q} \sqrt{\frac{\pi}{2}}$

$$f = p + f_b = p + p\,(a+e)\,\frac{y}{r^2}$$

$$l' = \text{Virtual length} = \sqrt{\left(\frac{q}{p}\right)}\, l.$$

$$\varphi = \frac{ey^*}{r^2}$$

In the papers mentioned,

If $Q = \frac{\pi^2 EI}{l^2}$ = Euler's value for any column

it was pointed out (1) that Q is a mathematical quantity, so that we may speak of the Q of the column.

(2) that for a column to bend under any load other than Q, there must be some eccentricity of loading (whether due to curved neutral axis or to a definite measurable eccentric loading the effect is similar), and that if P is the load causing bending and 'e' the eccentricity and 'l' the length, then 'P' is the "Euler Load" of a column of virtual length l', where $l' = \sqrt{\frac{q}{p}}\,l$ and the column bends in a complete cosine curve of length l' = unity, the actual column being the central portion occupying length $\frac{l}{l'}$ of the cosine curve.

The curve of maximum deflection corresponding to this treatment is

$$y = \sec \sqrt{x}\,\frac{\pi}{2} \text{ where } x = \frac{p}{q}$$

$$\text{and } y = \frac{(a+e)}{e} = \frac{\text{total max. deflection}}{\text{eccentricity}}$$

The diagram entitled "Column Design Curves" (Sheet No. 1) has been prepared, which shows these curves. In addition to the original curves (No. 1 and No. 2), the curve $y = \sec \sqrt{\frac{p}{q}}\,\frac{\pi}{2}$ has been drawn (No. 3), which shows the maximum deflection of the column as the load varies (see Sheet No. 1)†.

*A better symbol would be the Greek γ which will be adopted in subsequent additions.

†Tables and Curves are printed at the end of the paper.

Then it has been shown that

$$f = p + f_b = p + p\left(\frac{a+e}{e}\right)\frac{ey}{r^2} = p + p\frac{ey}{r^2}\sec\sqrt{\left(\frac{p}{q}\right)}\frac{\pi}{2}$$

this does not allow of 'p' being deduced in terms of 'f' and various approximate formulae have been suggested to get an algebraic equation, though the formulae as deduced empirically do not indicate this fact directly. These sometimes take the form of deducing f_b, in other cases 'f' is deduced directly. The approximate forms by easy transference give quadratic equations in 'p,' no useful object is served by writing down the solutions of the quadratics in general terms.

It is the purpose of this paper to compare and contrast the approximations involved. The results, as usually quoted, appear in various forms; the original form will be given where thought necessary.

(1)* Johnson's Formula:—This appears as follows:—

If M_1=bending moment at point of maximum deflection, from cross-bending external forces and from eccentricity of position of longitudinal loading:—

v_1=maximum deflection of member from all causes acting simultaneously.

M_2=bending moment from the direct loading, P, into its arm, $v_1 = Pv_1$.

P=total direct loading on member, tension or compression.

f_1=unit stress on extreme fibre from bending alone at section of maximum bending moment, or of maximum deflection, as the case may be, in pounds per sq. in.

l =length of member.

y_1=distance from centre of gravity axis to the extreme fibre under consideration on which the stress from bending is f_1.

I =moment of inertia of the cross-section.

b =breadth of a solid rectangular section.

h =height of section, out to out, in the plane in which bending occurs $=2y_1$ for symmetrical sections.

f_2=unit stress in member from the direct loading, supposed to be uniformly distributed, $= \frac{P}{A}$.

f =total maximum unit stress on extreme fibre$=f_1+f_2$.

$$\text{then } v_1 = k\frac{f_1 l^2}{Ey_1} \qquad \text{(a)}$$

By approximating the condition of uniform loading we have the general relation between the deflection of the beam and the stresses on its extreme fibre.

$$v_1 = \frac{f_1 l^2}{10Ey_1} \qquad \text{(b)}$$

then

$$M_o = \frac{f_1 I}{y_1} = M_1 \pm Pv_1 \qquad \text{(c)}$$

*Theory and Practice of Modern Framed Structures, 7th Edition, 1903, pp. 154, 155.

putting v_1=value from (a)

$$f_1 = \frac{M_1 y_1}{I \mp \frac{Pl^2}{10E}} \quad \text{.......................................(d)}$$

Putting this into the present notation this becomes,—

$$f_b = \frac{pey}{r^2} \left\{ \frac{1}{1 - \frac{\pi^2}{10} \frac{p}{q}} \right\} \quad \text{.......................................(1)}$$

Johnson's formula in the quadratic form appears thus :—

$$\text{since } f = p + f_b = p + p\,\frac{(a+e)y}{r^2}$$

$$= p + \frac{pey}{r^2} \; \frac{1}{1 - \frac{\pi^2}{10} \frac{p}{q}}$$

$$\text{then } p^2 - p\left[f + q\,\frac{10}{\pi^2}\left(1 + \frac{ey}{r^2}\right)\right] + f\,q\,\frac{10}{\pi^2} = 0 \quad \text{..................(2)}$$

(2) *Fidler's Formula—

After discussing the intrinsic eccentricity and assuming a central load with a neutral axis following a 'slight curve.' Calling Δ eccentricity of loading, i.e., distance from curved line of neutral axis, and δ the distance of deflected curve from the original curve of neutral axis and resistant force of the column, and P load of column assumed central (see Fig. 1).

If R is Euler's value

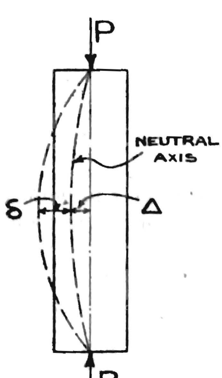

FIG. 1.

$$P = R\,\frac{\delta}{\Delta + \delta} \quad \text{and} \quad \delta = \Delta\,\frac{P}{R - P}$$

Putting this into the present notation, then—

$$P = Q\left(\frac{a}{a+e}\right) \quad \text{and} \quad a = e\,\frac{P}{Q - P}$$

$$i.e. \quad a = e\left(\frac{p}{q - p}\right) = e\left(\frac{1}{\frac{q}{p} - 1}\right)$$

$$\text{that is } (a+e) = e\,\frac{1}{1 - \frac{p}{q}} \quad \text{......................(3)}$$

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*A Practical Treatise on Bridge Construction, p. 162.

In other words, that Q (a)=P (a+e) which means that when the load P is applied with eccentricity 'e' and the column deflected amount 'a' from the original, then P (a+e) is the bending moment, and that this bending moment is equal to Q.a; we may imagine that the eccentric load P on lever arm (a+e) may be replaced by an axial load Q with lever arm 'a.' This seems a fairly reasonable approximate assumption when it is understood that Q is Euler's Value, and is thus the only central load that would keep the column bent, but it leads to a *cul de sac* for the reasons :—

Firstly: When we consider the fact that with Q on the column there would be a bigger direct stress than with P on the column, so that it is not correct to replace P eccentrically by Q centrally.

Secondly: By reference to our present assumption that with Q on the column, the column would bend in a complete cosine curve. whereas with P on column the actual column only occupies the central portion of the cosine curve.

Thirdly: To exclude the direct compressive stress due to Q may be a reasonable argument for very long columns, but with a short column (the deflection being necessarily small for the total stress to remain within the elastic limit) the direct stress is the preponderating stress.

The portion of the curve p=O to p=·5q, Sheet No. 2. shows the fairly close approximation to the curve of $\sec \sqrt{\left(\frac{p}{q}\right)}\,\frac{\pi}{2}$ of the "Johnson" curve $\dfrac{1}{1-\frac{\pi^2}{10}\frac{p}{q}}$ and "Fidler" curve $\dfrac{1}{1-\frac{p}{q}}$.

For this reason may they be used profitably for some investigations, but it does not seem to have been realised that Johnson's formula and Fidler's first assumption are almost the same. (π^2=9.87 and would be assumed 10).

Fidler's Second Assumption is, that Bending Moment=P (a) and since from his first assumption $a=e\dfrac{p}{q-p}$ in the present notation he gets

$$f=p+f_b=p+p\frac{ey}{r^2}\left(\frac{p}{q-p}\right)$$

and a quadratic writing $\dfrac{ey}{r^2}=\varphi=\cdot 4$ as an average safe value.

$$(\cdot 6)\ p^2-p\ (f+q)+fq=O \qquad \text{..........................(4)}$$

The solution is known as Fidler's Formula and used in tables of sections.

$$\text{that is } p=\frac{(f+q)-\sqrt{(f+q)^2-2.4\ fq}}{1\cdot 2} \qquad \text{................(4a)}$$

In his first assumption, Fidler assumes Q.a=P (a+e), that is that bending moment due to P is P (a+e). Why he should now write Bending moment as P (a) is not clear.

Looking at curve 3 (Sheet No. 1), we see how 'a' varies with ratio $\frac{p}{q}$

When p is about ·3q .. $a=\frac{1}{2}e$
When p is about ·6q .. $a=2e$
after this 'a' gets relatively large,
from $p=\cdot 9q$ .. .. $a=12e$
to $p=$nearly 1 .. .. $a=$nearly infinity.

As in practice it is with the former limits that the column would be designed, to neglect 'e' is quite inaccurate.

Perhaps Mr. Fidler considered that, as in his analysis, it was a central load with a bent neutral axis that was assumed, the lever arm of the load was the distance from the neutral axis, but if this is so, it can hardly be reconciled with his primary assumption that $Q.a=P\ (a+e)$.

It would seem that Fidler's formula gives a rough approximation to a breaking stress, but if applied with a factor of safety to deduce working stresses it is inaccurate.

The author some time ago, came to what is Fidler's first assumption, as follows :—

Assuming,

(1) That the load P with original eccentricity 'e' causes a bending moment P.e.

(2) That the couple P.e. deducts a certain amount of resisting power from the column which may be expressed as causing a negative moment of inertia, leaving a net moment of inertia and a central load P.

Then if I is moment of Inertia of the Column and I_n is the 'net moment of inertia.'

Our original column may now be considered to be equivalent to a centrally loaded column of moment of inertia I_n and P is the Euler value of the net column, *i.e.* :—

$$P=\frac{\pi^2 E I_n}{l^2} \qquad \ldots\ldots\ldots\ldots (a)$$

Now $\frac{\pi^2 E}{l^2}=\frac{q}{r^2}$ and

From (a) $\frac{1}{I_n}=\frac{\pi^2 E}{P l^2}$

Again $f=\frac{P}{A}+\frac{Pay}{I_n}=\frac{P}{A}+\frac{\pi^2 E}{P l^2}\ Pay=\frac{P}{A}+\frac{qay}{r^2}$

$$(f-p)=\frac{qay}{r^2}\ \therefore\ a=\frac{(f-p)\ r^2}{q\ y} \qquad \ldots\ldots\ldots\ldots (b)$$

Now $f_b = p\,(a+e)\,\dfrac{y}{r^2}$ and substituting for 'a'

$$= p\left\{\frac{f-p}{q}\,\frac{r^2}{y} + e\right\}\frac{y}{r^2}$$

$$= \frac{p}{q}(f-p) + \frac{pey}{r^2}$$

$$= \frac{p}{q}(f_b) + \frac{pey}{r^2}$$

$$\therefore\ f_b = \frac{pey}{r^2}\ \frac{1}{1-\dfrac{p}{q}} \qquad \ldots\ldots\ldots\ldots (6)$$

This, as pointed out by Mr. Ross, and shown above, is equivalent to the result got by equating Q.a.=P (a+e). It may also be obtained by assuming the curve 3 of sheet No. 1 as a rectangular hyperbola.

From this the quadratic would be

$$p^2 - p\left(f+q+q\frac{ey}{r^2}\right) + fq = 0 \qquad \ldots\ldots\ldots\ldots (7)$$

which is what Fidler's analysis would give if the Bending moment were assumed=P (a+e) instead of P (a), as he assumes. This has been tabulated under the name "Fidler (amended)."

Of the quadratic* forms this, being of maximum simplicity, is probably as good as any other for rough approximations for 'centrally' loaded columns, as in any case it is necessary for 'centrally' loaded columns to assume an empirical value of $\dfrac{ey}{r^2}$ (Fidler gives .4 and Moncrieff .15 to .6).

With 'f' in units of 1000 lbs. and $\varphi = .4$ we get

$$p^2 - p\,(f + 1{\cdot}4q) + fq = 0 \qquad \ldots\ldots\ldots\ldots (8)$$

which would be easily solved by using Curve 1 and a slide rule*. A better quadratic is given later (see No. 17).

(3) **Andrews' Formula.** This appears as a modification of Johnson's Formula, the coefficient 10 is replaced by 8, giving,

$$f_b = \frac{M_o y}{I - \dfrac{Wl^2}{8E}} \qquad \text{Let } M_o = Pe \qquad f_o = \frac{M_o y}{I}$$

$$M = P\,(a+e) \qquad f_b = \frac{My}{I}$$

*See P.I.C.E., 1916, for slide rule solution of quadratic.

It is deduced thus* :—Deflection of beam with uniform B.mt.

$$= \delta = \frac{M_o l^2}{8EI}$$

and $f_b = \frac{My}{I}$ then if 'x' is eccentricity ('e' of present notation).

$$\therefore f_b = \frac{W(x+\delta)y}{I} = \frac{Wxy}{I} + \frac{Wf_o l^2}{8EI}$$

Assuming $M_o = M$, *i.e.* $\frac{M_o y}{I} = \frac{My}{I}$ or $f_o = f_b$

then $f_b\left(1 - \frac{Wl^2}{8EI}\right) = \frac{Wxy}{I} \quad \therefore f_b = \frac{\frac{My}{I}}{1 - \frac{Wl^2}{8EI}}$ *i.e.*, in the present notation $f_b = \frac{pey}{r^2} \; \frac{1}{1 - \frac{\pi^2}{8}\frac{p}{q}}$ ....(9)

The deduction given assumes as an approximation that $M_o = Pe =$ original bending moment, and that this is equal to $M_1 = P\ (a+e) =$ subsequent bending moment. This as shown by the curve is only approximately true when $\frac{P}{Q}$ is small, and is quite wrong for large values of P.

Without these assumptions the method gives $f = \left(1 + \frac{\pi^2}{8}\frac{p}{q}\right)\frac{pey}{r^2}$ these bracketed being the first two terms of the expansion of sec $\sqrt{\left(\frac{p}{q}\right)}\frac{\pi}{2}$ or the expression may be got as the value of the fraction†. These are not approximately true unless $\frac{p}{q}$ is small.

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* "Further Problems in the Theory and Design of Structures." (E. Andrews) p, 217. There is a misprint in the book which is here corrected. The method is somewhat open to objection since if the approximations are made less rough, the results are further from the exact value.

†Thus $\delta = \frac{M_o l^2}{8EI}$ and $f_b = \frac{M_1 y}{I} = \frac{W(x+\delta)y}{I} = \frac{M_o y}{I} + \frac{W.M_o l^2}{8EI}\frac{y}{I}$

$$= \frac{M_o y}{I}\left(1 + \frac{Wl^2}{8EI}\right) = \frac{pey}{r^2}\left(1 + \frac{\pi^2}{8}\frac{p}{q}\right)$$

alternatively $f_b = \frac{pey}{r^2} + \frac{\pi^2}{8}\frac{p}{q}\frac{f_o \times f_b}{f_b} \quad \therefore f_b = \frac{\frac{pey}{r^2}}{1 - \frac{\pi^2}{8}\frac{p}{q} f_o / \frac{pey}{r^2} / \text{etc.}} = \frac{pey}{r^2}\left(1 + \frac{\pi 2}{8}\frac{p}{q}\right)$

It thus represents a rough approximation of the result got by assuming the deflection on reaching only the first stage (see under 5 for result got by successive deflections to the limit). This rough approximation, however, follows closely the correct curve up to $\frac{p}{q}$=about .3.

From this the quadratic would be,

$$p^2 - p\left(f + \frac{8q}{\pi^2} + \frac{8q}{\pi^2}\frac{ey}{y^2}\right) + fq\frac{8}{\pi^2} = O \qquad \ldots\ldots(10)$$

The objection to a formula of type $\dfrac{1}{1 - \text{constant}\,\dfrac{p}{q}}$ when the constant is greater than unity is that it reaches infinity before x=1 and thus is unsuitable for explaining effects when P is nearly equal to Q.

What would apparently be safe approximations in ordinary mathematics are misleading in the case of columns when the load P approximates to the Q of the column.

(4) Perry's Approximation.*

This is given as $\dfrac{pey}{r^2}\ \dfrac{1.2^*}{1 - \dfrac{p}{q}}$ $\qquad \ldots\ldots(11)$

This curve is plotted, and shows 20 % more than the Fidler Curve, it has a close agreement in the higher values of $\frac{P}{Q}$ with the second curve, and gives a good approximation for reading breaking tests, but as it is in the lower values that we are usually working, it is probably not so useful for design as those given above.

The quadratic corresponding to Perry's approximation is

$$p^2 - p\left(f + q + 1.2q.\frac{ey}{r^2}\right) + fq = O \qquad \ldots\ldots(12)$$

(5) Moncrieff's Formula—

That Δ (*i.e.*, 'a' of present notation)$=\dfrac{Pl^2E}{8EI - \dfrac{5Pl^2}{6}}$

Which may be deduced as follows:—

On the assumption that the curve of deflections is a parabola.

From properties of a parabola,

Area of half parabola $= \frac{2}{3}$ height $\times$ length.

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*Morley strength of Materials, p. 263.

Distance of c.g. $= \frac{5}{8}$ length.

Let e = original eccentricity.

$\Delta_1 \ \Delta_2$ etc. = induced deflections.

Then $\Delta_1 = \frac{Pl^2}{8EI}.e = ke$* say, then assuming this as the B. Mt. curve and as a parabola.

$\Delta_2 = (\frac{5}{6}ke \times \frac{l}{2})\frac{lP}{4EI} + ke = (\frac{5}{6}k^2e + ke)$, and again assuming a parabola.

$$\Delta_3 = \frac{5}{6}\,\frac{5}{6}(k^2e + ke)\ k + ke = ke\left\{1 + \frac{5}{6}k + \left(\frac{5}{6}k\right)^2 + \dots\right\}$$

and so on as a geometrical progression to infinity.

$$\therefore \text{Final } \Delta = \frac{ke}{1 - \frac{5}{6}k} = \frac{\frac{Pl^2}{8EI}e}{1 - \frac{5}{6}\frac{Pl^2}{8EI}} = \frac{Pl^2\ e}{8EI - \frac{5}{6}Pl^2}$$

which is the form in which it is quoted.†

Using our present notation,

this reduces to $a = e\,\dfrac{1}{\frac{8}{\pi^2}\frac{q}{p} - \frac{5}{6}}$

$$\therefore (a + e) = e\left(1 + \frac{1.2}{\frac{48}{5\pi^2}\frac{q}{p} - 1}\right)$$

$$(a + e) = e\,\frac{1 + \cdot 2\,\frac{5\pi^2}{48}\frac{p}{q}}{1 - \frac{5\pi^2}{48}\frac{p}{q}} \qquad \dots\dots\dots\dots(13)$$

Looking at the curve No. 5 of Sheet No. 2, we see that it is a fairly close approximation to the sec $\sqrt{\frac{p}{q}}\frac{\pi}{2}$ curve. The error in assuming a parabola instead of a sine curve gets serious as $\frac{p}{q}$ approaches unity, and thus for very small eccentricities may be unsuitable for reading breaking results. The quadratic corresponding is,

$$p^2\frac{5\pi^2}{48}(1 - \cdot 2\varphi) - p\left[\frac{5\pi^2}{48}f + q(1 + \varphi)\right] + q = fq = O \quad \dots(14)$$

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* Andrews' formula stops at this stage.

†Proceedings of American Soc. C.E., 1902, quoted in "Engineering Construction in Steel and Timber," Warren. page 267.

(6) Author's suggested approximation.

When studying the various curves of approximation, that of the form $\frac{1+\alpha x}{1-\beta x}$ where α and β are constants, and $x = \frac{p}{q}$ seemed to lead to the most suitable solution; the author after many trials adopted $\frac{1+\cdot 25x}{1-x}$ as the best on the whole. It has the advantage of,

(1) A simple fractional coefficient in the numerator, viz., $\cdot 25=\frac{1}{4}$ and of unity in the denominator.

Up to $x=.5$ it appears accurate to 1 in 500, at $x=.5$ to 1 in 1000, thence the accuracy diminishes somewhat, but even at $\cdot 9$ the accuracy is 1 in 70.

(2) It does not run out to infinity until $x=1$, thus has not the disadvantage of the $\frac{1}{1-\beta x}$ type where β is greater than unity.

We would have then $(a+e) = \dfrac{1+\cdot 25\,\frac{p}{q}}{1-\frac{p}{q}}\,e$ ....(15)

and the quadratic corresponding and endorsed on the diagram, viz.,

$$p^2\left(1-.25\frac{ey}{r^2}\right) - p\left[f + q\left(1+\frac{ey}{r^2}\right)\right] + fq=0 \qquad \text{..............(16)}$$

writing $\frac{ey}{r^2} = \cdot 4$. with Fidler*, this would be,

$$.9p^2-p\,(f + 1.4q) + fq = 0 \qquad \text{............................(17)}$$

which, if the assumption of a constant $\frac{ey}{r^2}$ is justified, gives a value of 'p' to an accuracy of 1 in 500 up to $p=\frac{1}{2}q$. which is the ordinary range of design.

A table† showing the results of various approximations is appended, and the curves are also shown, see Sheet No. 2.

Summary.

Summarising the results deduced, and writing $\frac{ey}{r^2}=\varphi$ for clearness we have as

‡*Approximations of exact form* $f=p\left\{1+\varphi \sec \sqrt{\left(\frac{p}{q}\right)}\,\frac{\pi}{2}\right\}$

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*Probably the values of φ given by Moncrieff and Fidler would be modified if deduced from the correct formula.

†Tables are printed on page opposite curves at end.

‡See Table 2 and Sheet No. 2. A curve of $\frac{f_b}{q}\,\frac{1}{\varphi}$ has been also plotted: this shows directly how the bending stress varies with the load.

In the quadratic form,

$$\text{Johnson } p^2-p\left[f+\frac{10}{\pi^2}q\,(1+\varphi)\right]+fq\,\frac{10}{\pi^2}=0 \qquad \ldots\ldots\ldots\ldots(1)$$

$$\text{Fidler } (1-\varphi)\,p^2-p\,(f+q)+fq=0 \qquad \ldots\ldots\ldots\ldots(2)$$

$$\text{Fidler (Amended) } p^2-p[f+q\,(1+\varphi)]+fq=0 \qquad \ldots\ldots\ldots\ldots(2a)$$

$$\text{Andrews } p^2-p\left[f+\frac{8}{\pi^2}q(1+\varphi)\right]+f\,q.\frac{8}{\pi^2}=0 \qquad \ldots\ldots\ldots\ldots(3)$$

$$\text{Perry } p^2-p[f+q\,(1+1.2\varphi)]+fq=0 \qquad \ldots\ldots\ldots\ldots(4)$$

$$\text{Moncrieff } p^2\,(1-\cdot 2\varphi)\,\frac{5\pi^2}{48}-p\left[\frac{5\pi^2}{48}f+q\,(1+\varphi)\right]+fq=0 \ldots.(5)$$

$$\text{Hawken } (1-.25\varphi)\,p^2-p[f+q\,(1+\varphi)]+fq=0 \qquad \ldots\ldots\ldots\ldots(6)$$

Of the formulæ tabulated above the correct secant curve is as easy of application as any of the others, and (it is thought) in every way the most satisfactory.

Of the others,

No. 1 (Johnson) is a rough approximation, and becomes inaccurate for high values of the load.

No. 2 (Fidler) would approximate to Johnson's if the B. Mt. were taken similarly; the existing formula is inaccurate.

No. 2a (Fidler Amended). The amended form 2a is an easily remembered rough approximation.

No. 3 (Andrews) is accurate for low values of the load, but is very inaccurate for high values and quite misleading for reading breaking test results.

No. 4 (Perry) is inaccurate at low values of the load, but accurate at high values, it is thus unsuitable for design purposes, as it is lower values of the load that one designs for.

No. 5 (Moncrieff) is a fair approximation and almost as easily used as the others; to some degree the remarks on (3) apply.

No. 6 (Hawken) is suggested as a very close approximation throughout, besides being fairly simple to handle. For all practical purposes it is exact up to $\frac{p}{q}=.9$.

Central Loading Formulæ. These are attempts to allow for the intrinsic eccentricity by formulæ connecting p, f, l and r.

(7). *Rankine* quoted in form $p=\dfrac{f}{1+C\left(\frac{l}{r}\right)^2}$ where C is a constant.