and we see that any eccentrically loaded column of length l bends as portion of a centrally loaded column of 'virtual length' $\sqrt{\frac{q}{p}}\,l$. The eccentricity may be large or small, but the action is similar in each case. In any case, every column is bent to the shape of a portion of a cosine curve, the complete Cosine curve from 0° to 90° being for the virtual length of the column as given by (2) above.

Again referring to Fig. 3, and considering the bending of the locus of the neutral axis

Let 'a' be deflection caused by load.

'e' be eccentricity of the load (including 'intrinsic eccentricity').

Then treating O as origin, the total deflection of the virtual column is (a+e) and the ordinate at the point A is, since OAB is a cosine curve.

$$a = (a+e)\left[1 - \cos x\frac{\pi}{2}\right] \text{ where } x = \text{ in this case } \frac{\frac{l}{2}}{\frac{l'}{2}} = \sqrt{\frac{p}{q}}$$

$$= (a + e)\left[1 - \cos\sqrt{\frac{p}{q}}\,\frac{\pi}{2}\right]$$

$$\therefore a \cos\sqrt{\frac{p}{q}}\,\frac{\pi}{2} = e\left(1 - \cos\sqrt{\frac{p}{q}}\,\frac{\pi}{2}\right)$$

$$\therefore a = e\left(\sec\sqrt{\frac{p}{q}}\,\frac{\pi}{2} - 1\right) \qquad \ldots\ldots\ldots\ldots(3)$$

$$\therefore (a + e) = e \sec\sqrt{\frac{p}{q}}\,\frac{\pi}{2} \qquad \ldots\ldots\ldots\ldots(4)$$

The B. Mt. at the centre is P (a+e) and if f_b be stress in the extreme fibre caused by B. Mt. and y "the distance of the extreme fibre from the neutral axis then $f_b = \frac{P(a+e)}{I}y = \frac{pey}{r^2}\sec\sqrt{\frac{p}{q}}\,\frac{\pi}{2}$ ....(5)

Let f be total stress in the extreme fibre.

$$f = p \pm f_b = p \pm p\frac{ey}{r^2}\sec\sqrt{\frac{p}{q}}\,\frac{\pi}{2}$$

$$\therefore \text{Max. Stress in extreme fibre} = p\left[1 + \frac{ey}{r^2}\sec\sqrt{\frac{p}{q}}\,\frac{\pi}{2}\right] \qquad \ldots\ldots(6)$$

A result to be obtained by writing (a+e) for 'a' in the ordinary Euler analysis as has been done in modern text books.*.

The method given above of deducing the results (2) to (6), the writer considers, visualises what is happening in any column, gives an exact result that replaces the ordinary approximation correct and otherwise, and supplies the explanation of the diagram† herewith attached. The diagram allows of the design of a column on a rational and correct basis, the necessary trial and error being simplified and guided, and gives to practically any desired accuracy the intensity of stress in the extreme fibre which is the main criterion of safety.

It will be seen that the diagram consists of,

Upper figure showing the curve of 'q' with varying $\frac{1}{r}$ (See 1 Sheet No. 1.)

Lower figure showing a cosine curve a $= \cos x \frac{\pi}{2}$ (See 2 Sheet No. 2.)

where $x = \sqrt{\frac{p}{q}}$ drawn for a total length unity, this unit being the virtual length in each case.

The vertical end ordinate is a+e again represented by unity.

We see from this diagram that for any $\frac{p}{q}$ the actual length of the column is represented by $x = \sqrt{\frac{p}{q}}$ a ratio, and the virtual length to the same scale by unity.

For example, looking at the diagram for $\frac{p}{q} = .3$ the length of the actual column is $\sqrt{\cdot 3} = \cdot 55$ of the virtual column: if the column were 11 feet long with an eccentricity of 3¼ inches, then the virtual length of the column is $\frac{11}{\cdot 55} = 20$ft.

The deflection at the centre caused by the load 'p' is,

$$\frac{\cdot 35}{\cdot 65} \times 3\frac{1}{4} \text{ inches} = 1\cdot 75 \text{ inches.}$$

---

*c.f. "Theory of Structures," and "Strength of Materials," Morley.

"Applied Mechanics and Mechanical Engineeiing," Jamieson.

(Where the formulæ is referred to as Professor Smith's formula.)

Writing under date 6/1/18 from Aldwych, England, Mr. Ross says:—"At a week end lately, Alan Burn, talking about columns, mentioned the method of deducing the secant formula from Euler's by means of the equivalent length method as you had done. He wrote some students essay, while at Sydney in which he gave the method."

†Sheet No. 1. The original diagram did not include Curve 3.

Total deflection $=3\cdot25+1\cdot75=5\cdot00$ (check $\frac{3\cdot25}{\cdot65}=5\cdot00$)

If $\frac{ey}{r^2}=5$in. and we measure $\sec\sqrt{\frac{p}{q}}\,\frac{\pi}{2}=\frac{1}{\cdot65}$

$f=p+p\times5\frac{1}{.65}=8\cdot8$ p nearly, or $p=\frac{f}{8\cdot8}$

It is not proposed in this paper to discuss the details of values of the intrinsic portion of 'e' but as a result of a comparison of various working formulae and the opinions of authorities the value of intrinsic eccentricity given on the diagram is suggested, viz.

$$\text{intrinsic } e = \frac{y}{20} + \frac{1}{600}$$

This has been deduced by Mr. C. N. Ross, M.Sc., B.M.E., who as Senior Demonstrator under the direction of Professor A. J. Gibson, is engaged in experimental research with columns in the laboratory of the University of Queensland. Two typical results taken from the tests are given to show the agreement of experimental results† with theory. It is proposed to give a complete description of these, together with others subsequently, so that no details are given here except to state that the measurements have been made with care and accuracy, and that the results may be accepted with confidence. The arithmetical results quoted are deduced from the observed slopes measured by the movement of mirrors attached to the column.*

The cosine curve is drawn through the point of greatest deflection (treating centre of column as origin), and the positions of the other two points noted as shown.

The author believes that he method shown and the diagram given herewith clear away many misconceptions regarding the primary phenomena taking place in columns under load, that incorrect ideas exist is shown by the chapters on columns even in the more recent text books on Engineering.

More particularly does this apply to the analysis of long struts such as the members of roof trusses, to the connecting rod of steam engine, etc., and also the struts loaded with a measured eccentricity such as bracket loaded struts; for struts of ordinary ratios of $\frac{l}{r}$ of say 80 to 100 the working formulae such as Straight Line, Rankine-Gordon, and others, have, by a long process of empirical selection, come to give fairly trustworthy results, but even in this case any out-of-the-way method of loading causes doubt.

*Described in a paper "An Experimental Investigation of the Strains in Unsymmetrical Riveted Joints, S. H. Barraclough, A. J. Gibson, H. W. May, E. P. Norman (Proc. Engineering Association of N.S.W., Vol. XVI., 1910, p. 45.

†Sheet No. 6.

APPENDIX C.

A SUPPLEMENTARY PAPER TO "A PRACTICAL COLUMN DIAGRAM WITH PROOF."

"COLUMN DESIGN CURVES."

An additional curve No. 3 of 'Column Design Curves' (Sheet No. 1), showing the variation of the maximum deflection with changes in the load has been added to those of the previous paper. This curve is very convenient, enabling the induced deflection to be visualised as the load varies, and the complete sheet entitled "Column Design Curves" should be of service to the designing draftsman.

In curve 3 it will be seen that the induced maximum deflection increases somewhat slowly with P up to a value of P about ·5 of Q, in fact, the induced deflection equals the eccentricity at P=·45 of Q. After P=·5 it rises very rapidly, for instance, when P=·8, 'a' the induced deflection=5 times the eccentricity and when P=·9 of Q=11½ times the eccentricity and then runs quickly into infinity.

For 'centrally' loaded columns the designer has to assume the eccentricity of loading, the suggested value, it will be seen from example, approximates very closely to that allowed for by ordinary Straight Line or by Rankine formulae, and from curve 3 the deflection can be scaled with an accuracy well within that of any assumptions that have to be made*, and the extra bending moment caused by deflection quickly deduced.

Many interesting facts may also be observed from this curve :—

Firstly : It is seen that in columns 'centrally' or eccentrically loaded to neglect the induced deflection becomes the more serious as P gets larger. Such a result is not shown so clearly by any ordinary formula.

Secondly : The need of designing on the basis of a factor of safety on the *load* is shown ; for instance, if the load were considered to be ·4 of Q and that by sudden application or some exceptionally increased load it were raised to ·8 of Q, the bending stress* would be increased approximately 6½ times. It will be seen from this that to take a factor of safety on the stress in the extreme fibre, is not logical.

Thirdly : As the writer reads the meaning of this curve it is seen that the putting of Q on the Column (like representing a point or a straight line, from their definition in mathematics) is impossible practically. What really happens is that when P approaches very near Q, 'a' approaches infinity, but for such a load to be held, the eccentricity must be infinitely small. It is the nearest approach only that can be obtained in experimental work. In this case the stress in the extreme fibre is still what is got from our previous formula. In columns of very short $\frac{l}{r}$ the Q of the column is so large that the material fails before anything approaching 'q' comes

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*The scale of the original is $\frac{P}{Q}$ = ·05 = 1 inch.

†The variation of total stress is shown by Sheet No. 5.

upon it. For steel this limit is approximately $\frac{l}{r} = 80$ but this does not mean, as it is sometimes expressed, that Euler's value is absurd, but that the phenomenon is the same, failure of the column occurring simply from the properties of the material.

Fourthly: From the curve it will be seen that the neglect of direct compression for very long columns may not cause very serious errors, only, however, because in that case Q is very small, and with any appreciable eccentricity the bending stress alone would cause overstraining. If the column is short 'q' is very large and $\frac{P}{Q}$ probably small for stresses to be within the elastic limit. For instance at $\frac{l}{r} = 40$ then 'q' = 180,000lbs. per sq. in., the greatest load that one would think of putting on the coulmn would be, say, 22,500lbs. per sq. in., in that case $\frac{P}{Q} = .125$ and the induced deflection is .18 of 'e' and the bending stress is comparatively small. A bracket on a short column illustrates this.

The amount of load and eccentricity corresponding, for an allowable fibre stress is readily calculated from the diagram.

It will thus be seen that the same action goes on in every column whether long or short, it is a matter of position of the ratio $\frac{P}{Q}$ on the curve which of the two, Direct Stress or Bending stress, may be neglected in comparison with the other. In every case both are present and in the ordinary cases 'centrally' loaded columns where P is say .4 of Q both are to be taken into consideration: this is done approximately by various empirical formulae such as Johnson's, Moncrieff's, etc.

Fifthly: The ordinary plotting of strengths against $\frac{l}{r}$ does not allow, unless some such facts are kept in mind, of interpolation with any accuracy. It would be desirable to plot experimental results on a basis of deducing the 'intrinsic eccentricity' (as defined by the author) and a mean could be assumed. Such experiments should consist of measurements within the elastic strength of the material, breaking tests have been done 'ad nauseam.' Thus with the aid of a diagram such as that shown for 'centrally' loaded columns the designer could at once see the effects of variations in his assumptions, and design the column with a fair knowledge of what is taking place; this applies also where the loading may come on different portions of the column or in different forms, whereas with only the formulae before him, the computation of the results and effects would be tedious.

The writer in a separate paper* has compared and contrasted the analyses of previous investigations and shown each in the quadratic form.

*A Comparative Analysis of Column Formulæ.

As a result the following approximation to the curve of $y = \sec \sqrt{x}\,\frac{\pi}{2}$ is suggested viz.,

$$y = \frac{1 + 0{\cdot}25x}{1 - x}$$

This curve has a close agreement and is fairly simple to handle, it gives as the quadratic, writing φ for $\frac{ey}{r^2}$ for clearness

$$(1 - {\cdot}25\varphi)p^2 - p[f + q(1 + \varphi)] + fq = 0$$

this equation has been endorsed on the diagram.

From this quadratic 'p' may be deduced in terms of 'f'; for many cases 'p' as thus deduced will be accurate enough for practical purposes: the diagram may be used for checking and for seeing rapidly the effect of varying the assumptions for 'e' and other constituents as explained previously.

In conclusion, every column, except the mathematical conception, is eccentrically loaded, and the phenomena occurring are similar; the purpose of these papers is to show how the exact theory may be easily handled, and any justifiable approximations quickly detected.

APPENDIX D.

ADDITIONS AND CORRECTIONS.

Notation (page 4). *The Greek γ will be used hencefoward instead of* 'y' *for the Distance of Extreme Fibre from the Neutral Axis.* The use of 'y' and 'x' for coordinates of curves and for other variables is so universal that it was found confusing to use 'y' for a constant though it followed usual Australian practice. It is hoped that the resemblance, yet distinctiveness of 'γ' and 'y' will minimise the trouble caused by the change.

Pages 6 and 9. Fidler and Fidler (Amended). The Author's approximation may be written $a=1.25\ e\dfrac{p}{q-p}$ whereas Fidler writes $a=e\dfrac{p}{q-p}$ the induced deflection is thus 25% different.

E. Andrews, in "Concrete," March, 1918, obtains Fidler's (Amended) formula *de novo*, by assuming an originally bent 'column.' On this basis he deduces curves resembling Sheet No. 7 No. 2 for a certain 'f' and values of $\dfrac{l}{r}$. Such curves will have the difference mentioned above, and the method used does not indicate the existence of the radical curves having the properties mentioned in the author's 'Introduction.'

Possibly with further experiment and research an empirical coefficient 'k' in the formula $a=ke\dfrac{p}{q-p}$ will be decided on.*

Page 12. Moncrieff's formula runs to infinity at $\dfrac{p}{q}=.973$, and for values over this is negative, over $\dfrac{p}{q}=.8$, it begins to become misleading.

Page 19. Professor R. H. Smith†, so far as can be discovered, first stated, and very forcibly, the essential eccentricity of loading.

He does not mention Euler but refers to $Q=\dfrac{\pi^2\ EI}{l^2}$ as having been stated by Redtenbacher, Grashof and Reuleaux, and calls it the 'German rule,' to which he has a strong objection.

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*If $\dfrac{p}{q}=\mu$ then $\dfrac{p}{q-p}=\dfrac{\mu}{1-\mu}$ which is more easily visualised when speaking in terms of $\dfrac{p}{q}$.

†Proc. Edinburgh and Leith Engineering Society, 1878, kindly lent by Mr. F. L. Kier, Assoc. M. Inst. C.E., Engineer for Bridges, Queensland Railways.

He classes as 'grotesque'* the idea of using $Q = \frac{\pi^2 EI}{l^2}$ with a factor of safety, and says his (Grashof) mistake was in assuming $e=O$. . . . . "its slightest variation from absolutely O altogether destroys the validity of the conclusions drawn."

He then solves the differential equation for an eccentric load (neglecting $\frac{dy}{dx}$) getting (see Fig. 4) the equation $(y+e)=(a+e) \cos x \sqrt{\frac{P}{EI}}$ .... (1)

Professor Smith apparently did not recognise that his result is the cosine curve with $(a+e)$ written for 'a'; and in the limit when $e=O$, P becomes Q (the Euler Value) and thus in the treatment of the present series putting

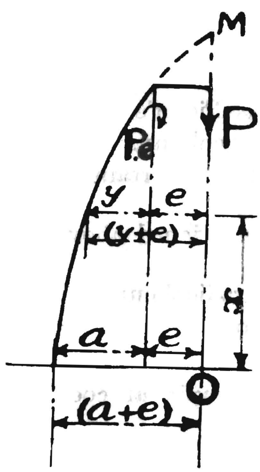

Fig. 4.

$EI=\frac{Ql^2}{\pi^2}$ when $y=O$ and $x=\frac{l}{2}$

$e=(a+e) \cos \sqrt{\frac{p}{q}} \frac{\pi}{2}$

or $(a+e)=e \sec \sqrt{\frac{p}{q}} \frac{\pi}{2}$ ....(1a).,

which is the basis of the present investigations and curves. The author sees no difficulty in reconciling the Euler result with the Smith result; one is the natural deduction from the other, as has been shewn in Appendix B.

He also deduces that the two points of intersection of the curves with the line of thrust are at a distance apart $l'=\pi\sqrt{\frac{EI}{P}}$..(2)

After investigating the meaning of this when $l=l'$ and $e=O$, he says "these last two cases have no bearing upon practice. There is never any indeterminateness in any actual physical phenomena, and there is an infinitely high probability that 'e' is never equal to 'O' with the mathematical exactness in order to make the rule which I have called the 'German rule' applicable."

Professor Smith did not realise that though physically we cannot obtain mathematical accuracy yet to assume such is the only way to deduce or anticipate physical results.

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*Similar views are still held: *vide.* Discussion by E. Godfrey on Report of Special Committee on "Steel Columns," Am. Soc. C.E. April, 1918.

"The Euler load, with a factor of safety of five, is used by European designers. This is an absurdity, for the reason that where the Euler formula has practical application, namely in slender columns, clear outside of the range of good structural design, no factor of safety of five is needed. There are long wooden derrick booms, being used every day, which have a factor of safety of less than two based on the Euler formula. The use of the Euler formula for short columns is worse than a guess. At 60 radii that formula shows an ultimate unit stress of 80,000lbs. The Committee's tests show 19,200 to 31,100. At 25 radii the Euler formula shows an ultimate strength of 460,000lbs. per square inch. No comparison is needed. What meaning could a factor of safety of five have where the ultimate strength shown in the formula bears no relation whatever to the real ultimate strength of the column? The saving feature of European specifications is the upper limit, which, the author believes, is about 14,000lbs. per square inch."

Result (2) of Professor Smith becomes, putting $EI = \frac{Ql^2}{\pi^2}$ then $l' = \pi\sqrt{\frac{EI}{P}} = \sqrt{\frac{q}{p}}\,l$, or what has been called in the present series the 'virtual length.'

The author's present papers are the result of his interpretation of the Euler (or Grashof) and Smith results (though deduced otherwise). Professor Smith's curves for $\frac{f}{p}$ for various values of $\frac{l}{r}$ for round and square sections are suggestive and probably correct; the other portions of the paper shewing the errors of the Gordon and Rankine formulae, and also the analysis of flat ended struts are not considered for the present.

Probably the effect of $\left(\frac{dy}{dx}\right)^2$, which is neglected throughout, would, though negligible, provide the element of determinateness that Smith found lacking.

More Column Design Curves and Sheets Nos. 3 to 5.

Curves for all values of φ may be deduced from the formulae shewn below. The limits when $\varphi = 0$ are mentioned also.

Writing $\theta = \sqrt{\frac{p}{q}}\,90^\circ$ and y and x for the co-ordinates of curves*.

Sheet No. 3. $y = \frac{\varphi + \cos\theta}{\cos\theta}$; $x = \frac{p}{q}$

$\varphi = 0$. Two lines at right angles, vertical and horizontal through (1, 1).

Sheet No. 4. $y = \frac{\cos\theta}{\varphi + \cos\theta}$; $x = \frac{p}{q}$

$\varphi = 0$. Two lines at right angles, vertical and horizontal through (1, 1).

Sheet No. 5. $y = \frac{\varphi + \cos\theta}{\cos\theta} \cdot \frac{p}{q}$; $x = \frac{p}{q}$

$\varphi = 0$. Two lines one through 0 and (1, 1); other vertically through (1, 1).

More Column Design Curves. *Factor of Safety.* *p.* 20.

The term 'factor of safety' in Column Design is somewhat difficult to define. The curves given above show what ratio the load P bears to the Euler load Q (by the ratio $\frac{p}{q}$ shown throughout) but if for a certain 'e' and consequent φ, the breaking load, say B, is deduced (in the sense that it will cause the stress 'f' *as a modulus of failure*† in the extreme fibre), then $\frac{B}{P}$ is the factor of safety *so far as load is concerned*; it would then

* The use of 'y' as distance to extreme fibre from neutral axis is somewhat unfortunate; in future reprints the Greek γ will be used.

†A term suggested here as it may differ considerably from the 'modulus of rupture,' it probably approximates the limit of elasticity.

be necessary to see that the load P does not cause more than 'f' in the sense of the maximum allowable fibre stress, what proportion of the limit of elasticity the 'f' shall be will have to be decided by experience, the proportion may approach unity for dead load and occasional stress. Similarly, if design is on the basis of fibre stress, then $\frac{P}{B}$ must be investigated to see that it is not beyond what is considered safe. The Curves of Sheet No. 7 No. 1 and Sheet No. 8 No. 1 would give the necessary data.*

It looks as if the factors of safety will become "factors of ignorance," so far as φ and 'f' are concerned, and a 'factor of safety' so far as $\frac{B}{P}$ is concerned. In repetition or reversal work, the fibre stress being necessarily small is usually the dominating factor.†

Method of Failure. Under the Euler load, or approaching it for comparatively long columns, the direct stress is small compared to the bending stress, for certain materials failure will occur by tension.

Values of φ. In addition to those mentioned the author's attention has been drawn‡ to that of Professor Basquin, who gives $\varphi = 0.1 + .001\frac{l}{r}$ and that of Johnson, viz. $\varphi = .001\frac{l}{r}$, these are on the reasonable assumption that 'e' varies as 'l', since $e = \text{constant} \times l \times \frac{r}{\gamma}$.‖ However $\frac{\gamma}{r}$ changes with the type of cross section from 2 for a solid circle, down to nearly 1, for a thin hollow circle. The author thinks that it is simpler and more accurate to have a formula for 'e' which will not involve separate tables for different cross sections: this especially applies in design where the length and approximate width are fixed by the conditions of the structure. That suggested by Mr. Ross, viz. $e = \frac{l}{600} + \frac{\gamma}{20}$ is direct, simple, and allows of accuracy though experimental results may alter the constants and possibly the form.

The Primary Centrally Loaded Column.**

Considering the action in a column relative to the centre cross section (or fixed end of half column) and for a certain load P, the variations of initial stress, of E, of alignment, and of other sources of eccentric action, probably cause an irregular line of application both positive and negative to the straight line joining the point of application and the centre of the column.

*The Tables and Curves give the data for any Elastic Material.

†A certain limited amount of eccentricity of loading may be beneficial by assuring that the columns bends always in the one direction and thus limits the range of stress or tendency to reversal.

‡Quoted from 'Modern Framed Structures,' Johnson, Part III, p. 32, by W. J. Doak, B.E., in his criticism.

‖γ is now used instead of 'y' for the distance from neutral axis to the extreme fibre.

**Columns of varying cross section will probably be designed as stress analysis becomes better known. The "ellipse of elasticity" used in arch analysis would be useful.

Each portion tends to bend in accordance with its relative eccentricity and the Q of its length (with zero length at the centre and consequent infinite Q), so that the effects of eccentricity get larger as the points get further from the centre ; the bending of each portion will be proportional to the amount of its eccentricity, but depends also, and chiefly, on the ratio of P to the Q of portion considered.

As the load is applied, the irregularities cause ripples. The net effect of these, is by their deflections, to provide the amount 'e' as a distance of the line of the application of the load from the mean line of bending*. From this also it would appear that 'e'† will change with P. It will be interesting to see if experiment will confirm this.

The effects of inequalities of E etc., so far as *moments of resistance* are concerned, are comparatively small, consequently the longitudinal axis of bending will assume a mean line, not necessarily through the neutral axes of all cross sections, the deflections of this mean line,‡ which is probably portion of a cosine curve, have been taken as the basis of column stresses.

||**Continental Practice** is to allow a factor of safety on the Q with a limiting value for 'p'. The usual figures are, factor of safety 5, and limiting value 14,000 lbs. per square inch.

This is equivelant to fixing the value of $\frac{p}{q}$, and to lowering the value of 'f', the allowable fibre stress inversely as $\left(\frac{l}{r}\right)^2$ thus :—

Referring to Sheet No. 7 No. 1 (other curves of Sheet No. 8 may be used as a check). For values of $\varphi = .2$ to $.4$.

$$\text{When } \frac{p}{q} = .2 \text{ then } \sqrt{\frac{f}{\pi^2 E}}\,\frac{l}{r} = .5 \text{ to } .6$$

$$\text{or after substituting } f = \frac{(\cdot 25 \text{ to } \cdot 36)\times 30\times 10^7}{\left(\frac{l}{r}\right)^2} \quad \ldots\ldots\ldots\ldots (1)$$

$$\text{or } \frac{l}{r} = .5 \text{ to } .6 \times \sqrt{\frac{30\times 10^7}{f}} \quad \ldots\ldots\ldots\ldots (2)$$

From (1) For $\frac{l}{r} = 100$ then $f = 7{,}500$ to $10{,}000$ lbs per square inch.

$$p = \frac{q}{5} = \frac{30{,}000}{5} = 6{,}000 \text{ lbs per sq. in.} \quad \ldots (3)$$

---

*Possibly it may be found that 'e' varies as $\left(\frac{l}{r}\right)^2$ since $\frac{P}{Q} = \frac{P}{\pi^2 EA}\left(\frac{l}{r}\right)^2$.

†The Curve 2 of Sheet No. 1 is for (a+e)=unity, as 'e' changes the actual deflections will be read to corresponding scales.

‡This mean line apparently also varies, though probably very little.

||See footnote p. 36. Godfrey in Am. Soc. C.E., April, 1918, gives the figures used.

From (2) For $f = 14{,}000$ then $\frac{l}{r} = (\cdot 5 \text{ to } \cdot 6) \times 146 = 73 \text{ to } 88$

$$\text{and } p = \frac{q}{5} = \frac{56{,}500 \text{ to } 39{,}000}{5}$$

$$= 11{,}000 \text{ to } 8{,}000 \quad \ldots\ldots\ldots\ldots (4)$$

There seems no special reason why 'f' should be so drastically reduced for extra lengths, nor for the small working stresses that the rule gives.

In addition to those mentioned previously, the author wishes to thank Messrs. J. F. Burgess, B.E., of the Queensland Railways, N. C. Aitken, C. B. Mott, fourth year students; and Mr. W. Poole, of the British Westinghouse Co., who assisted with proofs; also Mr. H. S. Mort, B.E., who calculated and checked the numbers for the Tables.