

A COMPARATIVE ANALYSIS OF COLUMN FORMULÆ.

TABLE 7, No. 1.

$$y = \frac{p}{f}$$

$$\theta = \sqrt{\frac{p}{q}} \times 90^\circ$$

$$x = \sqrt{\frac{f}{q}} = \left( \sqrt{\frac{f}{\pi^2 E}} \right) \frac{l}{r}$$

$$\frac{p}{f} \text{ against } \frac{l}{r}$$

$$y = \frac{\cos \theta}{\cos \theta + \varphi}$$

$$y x^2 = \frac{p}{q}$$

from  $\varphi = .05$  to  $\varphi = .60$ .

$$x = \sqrt{\left( \frac{p}{q} \frac{1}{y} \right)}$$

p	$\varphi = .05$		$\varphi = .10$		$\varphi = .15$		$\varphi = .2$		$\varphi = .25$		$\varphi = 30$		
	q	y	x	y	x	y	x	y	x	y	x	y	x
0	.952	0	.909	0	.870	0	.833	0	.800	0	.769	0	
.1	.946	.325	.898	.334	.854	.342	.815	.351	.779	.358	.746	.366	
.2	.939	.462	.884	.476	.836	.489	.792	.502	.753	.515	.718	.528	
.3	.929	.568	.867	.588	.813	.607	.765	.626	.723	.644	.685	.662	
.4	.916	.661	.845	.688	.784	.714	.732	.740	.686	.764	.645	.787	
.5	.899	.746	.816	.783	.747	.818	.689	.852	.640	.884	.597	.915	
.6	.874	.829	.776	.879	.698	.927	.634	.973	.581	1.016	.536	1.058	
.7	.835	.915	.717	.988	.628	1.056	.559	1.119	.504	1.179	.458	1.236	
.8	.768	1.021	.623	1.133	.524	1.236	.452	1.330	.398	1.418	.355	1.501	
.9	.617	1.208	.446	1.420	.349	1.605	.287	1.771	.244	1.922	.212	2.062	
.92	.562	1.280	.391	1.535	.299	1.753	.243	1.947			.176	2.286	
.94	.489	1.387	.324	1.705	.242	1.972	.193	2.207			.137	2.614	
.96	.388	1.572	.241	1.996	.175	2.345	.137	2.648			.096	3.168	
.98	.240	2.021	.136	2.681	.095	3.208	.073	3.660			.050	4.316	
.99	.136	2.698	.073	3.683			.038	5.137					
.993							.027	6.087					

p	$\varphi = .35$		$\varphi = .40$		$\varphi = .45$		$\varphi = .50$		$\varphi = .55$		$\varphi = .60$		
	q	y	x	y	x	y	x	y	x	y	x	y	x
0	.741	0	.714	0	.690	0	.667	0	.645	0	.625	0	
.1	.715	.374	.687	.381	.661	.389	.637	.396	.615	.403	.594	.410	
.2	.686	.540	.656	.552	.629	.564	.604	.575	.581	.587	.560	.598	
.3	.651	.679	.620	.696	.592	.712	.566	.728	.542	.744	.521	.759	
.4	.609	.810	.577	.833	.548	.854	.522	.875	.498	.896	.476	.917	
.5	.559	.946	.526	.975	.497	1.003	.470	1.031	.447	1.058	.425	1.084	
.6	.498	1.098	.464	1.137	.435	1.174	.409	1.210	.387	1.246	.366	1.280	
.7	.420	1.290	.388	1.343	.361	1.393	.337	1.442	.316	1.489	.297	1.535	
.8	.310	1.580	.292	1.655	.268	1.727	.248	1.795	.231	1.862	.216	1.926	
.9	.187	2.194	.118	2.318	.152	2.435	.139	2.547	.128	2.655	.118	2.758	
.92			.138	2.581			.114	2.845			.097	3.087	
.94			.107	2.967			.087	3.281			.074	3.568	
.96			.074	3.615			.060	4.011			.050	4.372	
.98											.025	6.183	
.99											.012	9.215	
.993													

$\varphi = 0$  Horizontal Line through (1,0) to (1, 1) and Curve as follows,

$$\text{if } m = \text{ordinate} = \frac{q}{f}$$

$$n = \text{abscissa} = \sqrt{\frac{f}{\pi^2 E} \cdot \frac{l}{r}}$$

$mn^2 = 1$ , which is the radical for the Euler Curve.

A COMPARATIVE ANALYSIS OF COLUMN FORMULÆ.

BY R. W. HAWKEN.

$$y = \frac{p}{f}$$

$$x = \varphi$$

TABLE 7, No. 2.

$\frac{p}{f}$  against  $\varphi$

$$\theta = \sqrt{\frac{p}{q}} \times 90^\circ$$

$$y = \frac{\cos \theta}{x + \cos \theta}$$

$$x = \varphi$$

x = 0	$\frac{p}{q}$								
	.1	.2	.3	.4	.5	.6	.7	.8	.9
.15	.854	.836	.813	.784	.747	.698	.628	.524	.349
.20	.815	.792	.765	.732	.689	.634	.559	.452	.287
.25	.779	.753	.723	.686	.640	.581	.504	.398	.244
.30	.746	.718	.685	.645	.597	.536	.458	.355	.212
.35	.715	.686	.651	.609	.559	.498	.420	.320	.187
.40	.687	.656	.620	.577	.526	.464	.388	.292	.168
.45	.661	.629	.592	.548	.497	.435	.361	.268	.152
.50	.637	.604	.566	.522	.470	.409	.337	.248	.139
.55	.615	.581	.542	.498	.447	.387	.316	.231	.128
.60	.594	.560	.521	.476	.425	.366	.297	.216	.118

$\varphi = 0$ . Two lines coincident with the axes.

$$y = \frac{f}{p}$$

$$x = \frac{f}{q} = \frac{f l}{\pi^2 E r^2} (-)^2$$

$$y = \frac{q}{p}$$

$$x = \frac{p}{q}$$

TABLE 7 NO. 3.

$\frac{f}{p}$  against  $(-)^2$

$$\theta = \sqrt{\frac{p}{q}} \times 90^\circ$$

$$y = \frac{\varphi + \cos \theta}{\cos \theta}$$

from  $\varphi = .20$  to  $\varphi = .60$

$$x = y - \frac{p}{q}$$

$\frac{p}{q}$	$\varphi = .20$		.30		.40		.50		.60	
	y	x	y	x	y	x	y	x	y	x
.1	1.227	.123	1.341	.134	1.455	.145	1.569	.157	1.682	.168
.2	1.262	.252	1.393	.279	1.524	.305	1.655	.331	1.786	.357
.3	1.307	.392	1.460	.438	1.613	.484	1.767	.530	1.920	.576
.4	1.366	.547	1.550	.620	1.733	.693	1.916	.766	2.099	.840
.5	1.450	.725	1.676	.838	1.901	.950	2.126	1.063	2.351	1.176
.6	1.577	.946	1.865	1.119	2.154	1.292	2.442	1.465	2.731	1.638
.7	1.788	1.252	2.182	1.528	2.576	1.803	2.970	2.079	3.364	2.355
.8	2.212	1.769	2.817	2.254	3.423	2.739	4.029	3.223	4.635	3.708
.9	3.484	3.135	4.726	4.253	5.968	5.371	7.210	6.489	8.452	7.606
.92	4.120	3.791	5.680	5.226	7.241	6.661	8.801	8.097	10.360	9.532
.94	5.181	4.870	7.272	6.835	9.362	8.800	11.453	10.766	13.543	12.731
.96	7.303	7.011	10.454	10.036	13.606	13.062	16.757	16.087	19.909	19.113
.98	13.669	13.396	20.003	19.603	26.338	25.811				
1.00	Infny.	Infny.	Infny.	Infny.	Infny.	Infny.	Infny.	Infny.	Infny.	Infny.

$\varphi = 0$  Horizontal line through (1,0) to (1, 1), and Curve as follows

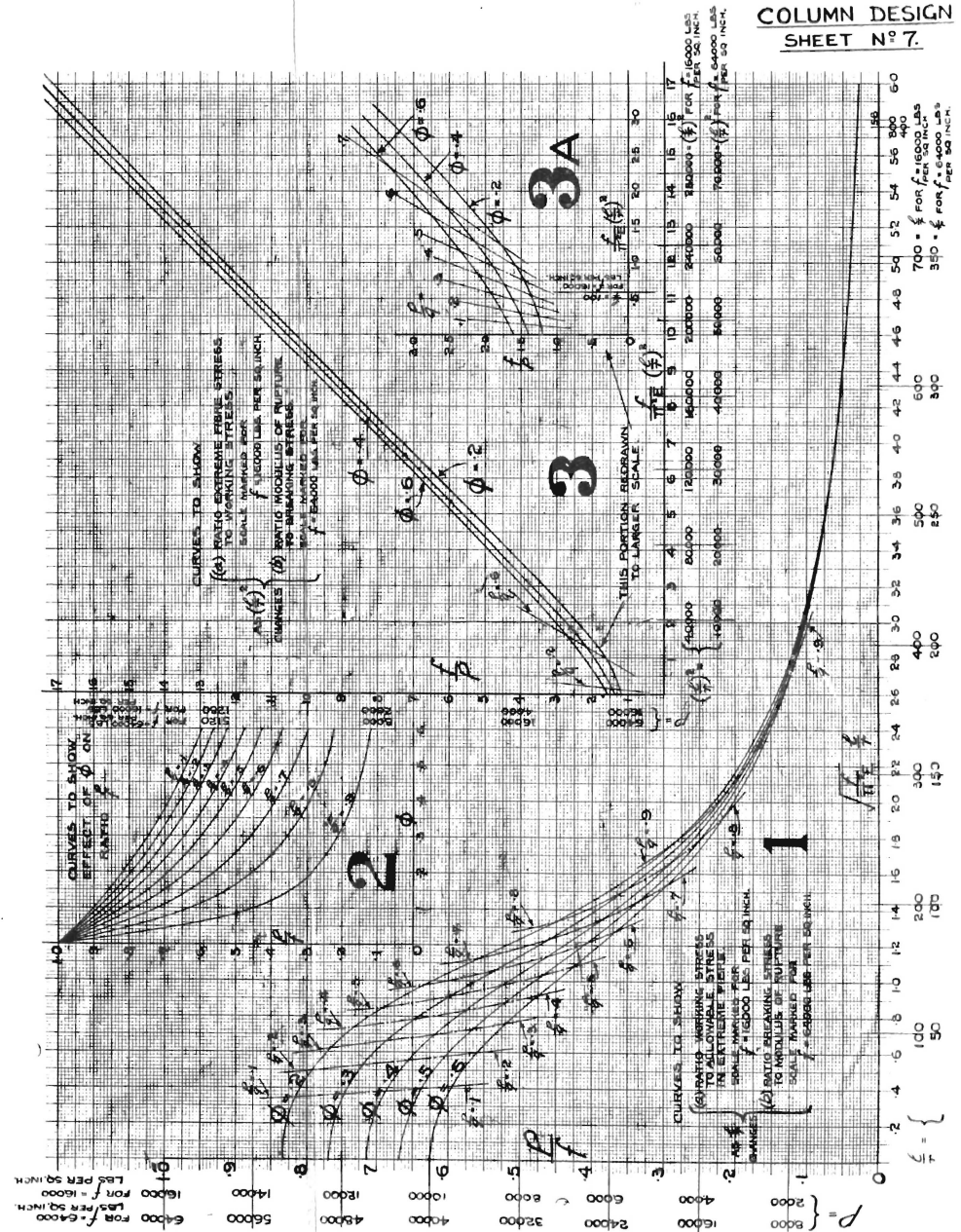
if  $m =$  ordinate =  $\frac{f}{q}$

$n =$  abscissa =  $\frac{f}{\pi^2 E} \left(\frac{l}{r}\right)^2$

$m = n$  which is a straight line at  $45^\circ$  through origin and (1, 0)

Rankine's assumption is explained in the text.

COLUMN DESIGN SHEET N<sup>o</sup> 7.



$$y = \frac{f}{p} = \frac{f}{P} (A)$$

TABLE 8 NO. 1.

$$\theta = \sqrt{\frac{p}{q}} \times 90^\circ$$

$$x = \sqrt{\frac{f}{q}} = \sqrt{\frac{f}{\pi^2 E} \frac{l}{r}}$$

A against  $\frac{l}{r}$

$$y = \frac{\varphi + \cos \theta}{\cos \theta}$$

$$\frac{x^2}{y} = \frac{P}{q}$$

from  $\varphi = .2$  to  $\varphi = .6$

$$x = \sqrt{\left(\frac{p}{y}\right) \frac{1}{q}}$$

$\frac{p}{q}$	$\varphi = .2$		$\varphi = .4$		$\varphi = .6$	
	y	x	y	x	y	x
.1	1.227	.351	1.455	.381	1.682	.410
.2	1.262	.502	1.524	.552	1.786	.598
.3	1.307	.626	1.613	.696	1.920	.759
.4	1.366	.740	1.733	.833	2.099	.917
.5	1.450	.852	1.901	.975	2.351	1.084
.6	1.577	.973	2.154	1.137	2.731	1.280
.7	1.788	1.119	2.576	1.343	3.364	1.535
.8	2.212	1.330	3.423	1.655	4.635	1.926
.9	3.484	1.771	5.968	2.318	8.452	2.758
.92	4.120	1.947	7.241	2.581	10.360	3.087
.94	5.181	2.207	9.362	2.967	13.543	3.568
.96	7.303	2.648	13.606	3.615		
.98	13.669	3.660				

$\varphi = 0$  Horizontal line through (1, 0) to (1, 1) and Curve as follows:-

If  $m =$  ordinate  $= \frac{f}{q}$  and

$$n = \text{abscissa} = \left(\sqrt{\frac{f}{\pi^2 E}}\right) \frac{l}{r}$$

$m = n^2$  which is a parabola through origin and (1, 0),

$$\frac{y}{p} = \frac{f}{P} (A)$$

TABLE 8 NO. 2.

$$\theta = \sqrt{\frac{p}{q}} \times 90^\circ$$

$$x = \sqrt{\frac{q}{f}} = \left(\sqrt{\frac{\pi^2 E}{f l^2}}\right) r$$

A against  $r$

$$y = \frac{\varphi + \cos \theta}{\cos \theta}$$

$$y x^2 = \frac{q}{p}$$

from  $\varphi = .05$  to  $\varphi = .6$

$$x = \sqrt{\frac{1}{y \times \frac{p}{q}}}$$

$\frac{p}{q}$	$\varphi = .05$		$\varphi = .10$		$\varphi = .15$		$\varphi = .2$		$\varphi = .4$		$\varphi = .6$	
	y	x	y	x	y	x	y	x	y	x	y	x
.1	1.057	3.076	1.114	2.996	1.171	2.922	1.227	2.854	1.455	2.622	1.682	2.438
.2	1.066	2.166	1.131	2.103	1.197	2.044	1.262	1.990	1.524	1.811	1.786	1.673
.3	1.077	1.760	1.153	1.700	1.230	1.646	1.307	1.597	1.613	1.437	1.920	1.318
.4	1.092	1.513	1.183	1.453	1.275	1.400	1.366	1.353	1.733	1.201	2.099	1.091
.5	1.113	1.341	1.225	1.278	1.338	1.223	1.450	1.174	1.901	1.026	2.351	.922
.6	1.144	1.207	1.288	1.137	1.433	1.079	1.577	1.028	2.154	.880	2.731	.781
.7	1.197	1.092	1.394	1.012	1.591	.948	1.788	.894	2.576	.745	3.364	.652
.8	1.303	.979	1.606	.882	1.909	.809	2.212	.752	3.423	.604	4.635	.519
.9	1.621	.828	2.242	.704	2.863	.623	3.484	.565	5.968	.431	8.452	.363
.92	1.780	.781	2.560	.652	3.340	.570	4.130	.514	7.241	.387	10.360	.324
.94	2.045	.721	3.091	.587	4.136	.507	5.181	.453	9.362	.337	13.543	.280
.96	2.576	.636	4.151	.501	5.727	.426	7.303	.378	13.606	.277	19.909	.229
.98	4.167	.495	7.334	.373	10.502	.312	13.669	.273	26.338	.197		
.99	7.350	.371	13.701	.272	20.051	.224	26.401	.196				
1.00	Infy	0	Infy	0	Infy	0	Infy	0	Infy	0	Infy	0

$\varphi = 0$  Two lines at right angles vertical and horizontal through (1, 0)

COLUMN DESIGN SHEET NO. 8.

