THE CAPACITY OF RAPID TRANSIT RAILWAYS.

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All the main items in the make-up of a rapid transit railway have some influence on the carrying capacity of the railway, capacity meaning the number of passengers that can be carried in a given time. For the purpose of this paper, however, any influence of road-bed, location, grades, curves, switches and crossings, etc., type of power supply, variation of line voltage, detail equipment of cars, etc., will not be considered, but all deductions will be based on the assumption that the railway and its equipment in detail are capable of supplying all demands of traffic or other conditions assumed.

Obviously the capacity on an hourly basis is the product of the number of trains per hour, the cars per train, and the passenger carrying capacity per car. Of these three factors the first has the predominating influence and is the least determinate.

In setting out a rapid transit railway, the first consideration must be the safety of the passengers, and the provision of conditions that are quite safe frequently bring about conditions, as will be shown later, that have a very important bearing on the train capacity.

If what is termed in America "Tail-light Signalling," that is running a train by looking out for the tail-end of the train in advance, were a safe system, train capacities would reach high figures, for trains could follow one another much as the city trams do. With modern speeds and weights of trains, however, no risks of any kind can be taken, and even the ordinary system of signalling, where a semaphore or light indicates to a driver where he must stop owing to hazardous conditions, is not sufficiently safe. The risk of non-compliance with a signal, with trains on short headway, is too great to be justifiable, particularly with electric traction when only one man is available to watch for signals. To take care of possible non-observance of signal indication the automatic train stop has been introduced. The pressure air brake is universal on rapid transit railways, and is brought into operation by a reduction of the pressure in the train pipe. With an automatic train-stop equipment branches from the train pipe are brought to the sides of the car at the ends of the bogies on the left leading sides for left-hand operation. In the usual form these are terminated by plug cocks near the rails, which open to atmosphere causing a sudden reduction in brake pipe pressure resulting in an "emergency" application of the brakes. The automatic train-stop is simply an arm operated synchronously with an adjacent signal, and lowered when the signal indicates clear or caution, but raised to engage with the cock on the train when the signal indicates that the train should stop.

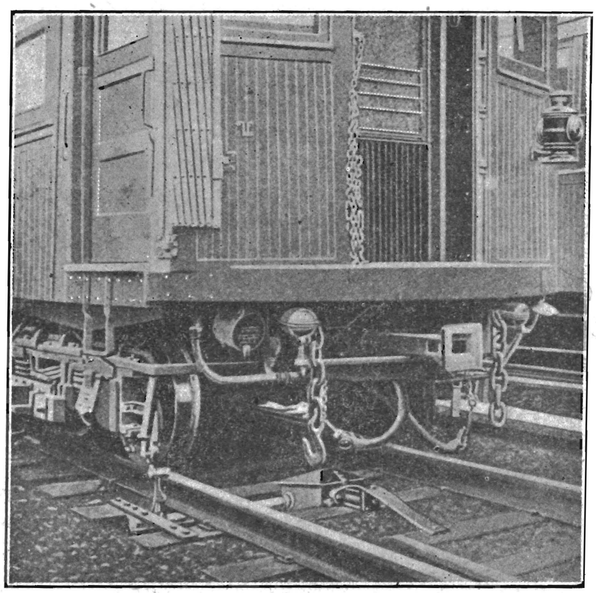
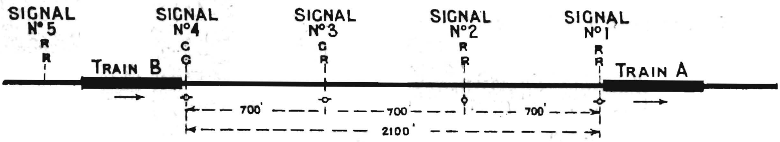


Diagram No. 1.

Diagram No. 1 shews a near view of a typical automatic train-stop and the train-stop cock on a carriage. The adoption of automatic train-stops would be useless unless provision were made for the train being stopped to have a clear braking distance between the train-stop and the hazard at any speed. This necessity brought about a system of signalling known as the "automatic continuous overlap" system, shewn by Diagram No. 2. Each signal can give three indications—clear, caution, and stop. An automatic train-stop is located adjacent to each signal and is raised when a train should stop



BLOCK SYSTEM WITH CONTINUOUS OVERLAPS.
DISTANCE BETWEEN SIGNALS 700 FEET.
DISTANCE BETWEEN TRAINS 2100 FEET.
HEADWAY AT 35 M.P.H & 40 SECONDS STATION STOP. 114 SECONDS.
-o- S REPRESENTS AUTOMATIC TRAIN STOP. LOWERED.
ϕ S REPRESENTS AUTOMATIC TRAIN STOP. RAISED.

Diagram No. 2.

at that signal, automatically stopping the train should the driver fail to do so. The minimum spacing of signals is usually 50 per cent. greater than the safe braking distance at maximum speed, and their operation is controlled automatically by the trains on the track. The signals and train-stops may be moved either electrically or pneumatically, but in either case the control is electrical. The two signals immediately behind a train always indicate stop, the next behind caution, and the next to the rear of that indicates clear. An oncoming train may then travel at full-speed to the caution signal, where speed should be reduced to stop at the stop signal. Should the driver fail to stop, the train-stop cock is tripped, resulting in a compulsory stop before reaching the next signal. It will be seen on the diagram that the automatic stop associated with the signal immediately in the rear of a train is shewn depressed; this is because the signal goes to danger immediately the head end of the train passes the signal, and the remainder of the train must pass without any of the cocks being tripped on cars following the leading car. Protection is provided by the train-stop of the signal second behind the train. It will be clear from the diagram that the minimum distance between trains running under clear conditions is three times the signal spacing, or $4\frac{1}{2}$ times the safe-braking distance if the usual 50 per cent. margin be provided.

The spacing of trains in accordance with the braking distance has given rise to an expression, "The lower the maximum speed, the greater the train capacity," obviously implying a lower limit to the speed which will have associated with it the maximum number of trains per hour.

If L be the train length in feet, and D the minimum distance between trains under free running conditions, V the speed of the train in feet per second, I the interval in seconds between trains, fb the braking deceleration in feet per second per second,

$$I = \frac{D + L}{V}$$

For automatic signalling described above

$$D = 4\tfrac{1}{2} \times \frac{V^2}{2\ fb}$$

$$\text{giving } I = \frac{4\frac{1}{2}\frac{V^2}{2\ fb} + L}{V} = \frac{9}{4} \cdot \frac{V}{fb} + \frac{L}{V}$$

If fb and L be constant, the minimum value of I will be given when

$$\frac{d\ I}{d\ v} = 0$$

$$\frac{d\ I}{d\ V} = \frac{9}{4\ fb} - \frac{L}{V^2} = 0$$

$$V = \frac{2}{3}\sqrt{fb\ L}$$

This indicates that any increase or decrease in maximum velocity from this critical value will result in less trains per hour passing any point if no stops are to be made. The above expression for a deceleration of 2 miles per hour per second, and a train length of 500 feet, gives a velocity of 17.4 miles per hour for maximum train density, with an interval of 39 seconds, corresponding to 92 trains per hour.

On account of the length of train being a factor in the spacing of trains, it may be thought that a limit exists to the length for greatest carrying capacity, in fact the statement is sometimes made that short trains at more frequent intervals can deal with more traffic than long trains at greater intervals. The statement when used to compare steam railways with electric railways frequently appears to be correct, but in general, if the maximum speed and braking deceleration are constant, the greater the train length, the higher the carrying capacity of the railway, for as before the interval between trains is

$$I = \frac{9}{4}\frac{V}{fb} + \frac{L}{V}$$

The capacity is proportional to $\frac{L}{I}$

$$\text{Capacity} = C = k \frac{L}{\frac{9\ V}{4\ fb} + \frac{L}{V}} = k_1 \frac{L}{K + L}$$

For maximum capacity

$$\frac{d\,C}{d\,L} = \frac{k_1\,K}{(K + L)^2} = O$$

or L=infinity, indicating a greater capacity with increased train length.

Practical location and construction considerations will generally determine economical length of train, greater traffic demand resulting in justifiable expenditure on longer stations.

Treating an ordinary single track of railway for traffic in one direction and with stations located at intervals, it is apparent that the movement of one train through a station will determine the minimum interval between it and a following train if neither train is to be checked for safety considerations. The movement through a station is probably most easily followed by reference to the distance time curves on Diagram No. 3. The movements of the head and tail end of a train are shewn by two curves whose horizontal distance apart is equal to the train length. The ordinates represent time and the abscissae distance.

TIME-DISTANCE CURVES. 40 TRAINS PER HOUR AND 31 TRAINS PER HOUR

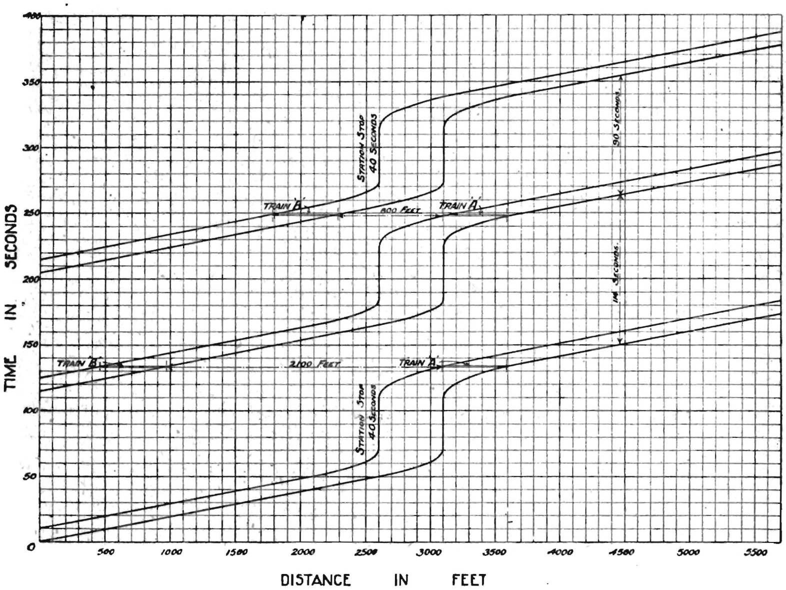


Diagram No. 3.

During constant speed running the distance time graph is two parallel straight lines, the tangent of their inclination to the ordinates being a measure of the speed. During uniform deceleration the graph becomes two parabolas, concave upwards; during a stop (at a station for example), the lines again become parallel but vertical, and during acceleration, if uniform, they again become parabolic but convex upwards.

In all calculations for this paper, except where otherwise stated, acceleration and deceleration are assumed uniform, and maximum speed constant. The influence of variations can easily be allowed for in any expressions where the assumptions made would give results appreciably in error.

Analysing the effect of station stop with a signalling system normally maintaining a minimum of three overlaps between trains running freely, referring to Diagram No. 4, if

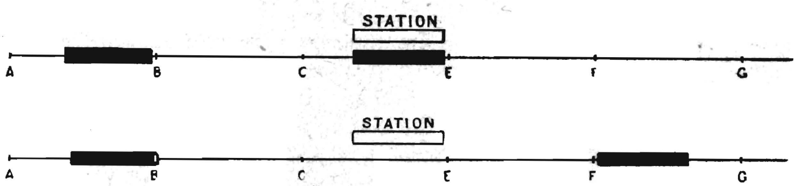


Diagram No. 4.

a train remain in the station a following train is blocked, as shewn on the upper sketch, at signal B. With the leaving train just not clear of F, the entering train may advance to C, and when the leaving train has cleared F, signal C will go to caution, clearing the automatic train-stop, and the entering train may advance to E.

Since the train-stop at C will not clear and signal C will not go from stop to caution until the leaving train has cleared F, the entering train for free running conditions must be behind C sufficient distance to allow it to stop at C if required, i.e. it must be practically at B or three overlaps distant, as in the case of free running on open track.

If k be the ratio of safe braking distance to braking distance at deceleration fb, fa the acceleration in feet per second per second, other symbols being used as previously, the time for the leaving train to pass from the station to clear F

$$= \frac{V}{fa} + \left(L + \frac{k\,V^2}{2\,fb} - \frac{V^2}{2\,fa}\right)\frac{1}{V}$$

$$= \frac{V}{fa} + \frac{L}{V} + \frac{k\,V}{2\,fb} - \frac{V}{2\,fa}$$

$$= \frac{V}{2 . fa} + \frac{L}{V} + \frac{k V}{2 . fb}$$

Time for entering train to go from B and stop at E

$$= \frac{k V^2}{2 . fb} \cdot \frac{1}{V} + \frac{V}{fb} + \left(\frac{k V^2}{2 fb} - \frac{V^2}{2 fb}\right) \frac{1}{V}$$

$$= \frac{k V}{2 fb} + \frac{V}{fb} + \frac{k V}{2 fb} - \frac{V}{2 fb}$$

$$= \frac{k V}{fb} + \frac{V}{2 fb}$$

The interval I between trains will be the sum of these two times and the station stop ts seconds

$$= \frac{V}{2 fa} + \frac{L}{V} + (3.k + 1) \frac{V}{2 fb} + ts$$

If n be the number of overlaps between trains, this becomes a general formula

$$I = \frac{V}{2 fa} + \frac{L}{V} + (n . k + 1) \frac{V}{2 . fb} + ts$$

If N = Number of trains per hour

$$N = \frac{3600}{(k . n + 1) \frac{V}{2 fb} + \frac{V}{2 . fa} + \frac{L}{V} + ts}$$

The Diagram No. 5 shews this formula in graphical form for a maximum speed of 35 m.p.h., fa, fb and ts varying through wide limits. The dotted lines shew examples with 30 seconds station stop with various values for fa and fb. With fa and fb each equal to 1 m.p.h.p.s., N=23.4; doubling the acceleration to 2 m.p.h.p.s. and keeping the brake equipment constant gives N=24.9, a gain of 6.3 per cent., while keeping the acceleration constant, but improving the brake equipment to give 2 m.p.h.p.s. gives N=34.1, a gain of 45.8 per cent.

The great influence of the length of station stop will be apparent by taking any horizontal line through the right hand half of diagram, any such line representing a number of combinations of fa and fb.

One great disadvantage of the continuous overlap system of signalling, or of any block system, is due to the fact that trains at any speed are required to maintain equal minimum space intervals; and to allow for possible neglect to obey a

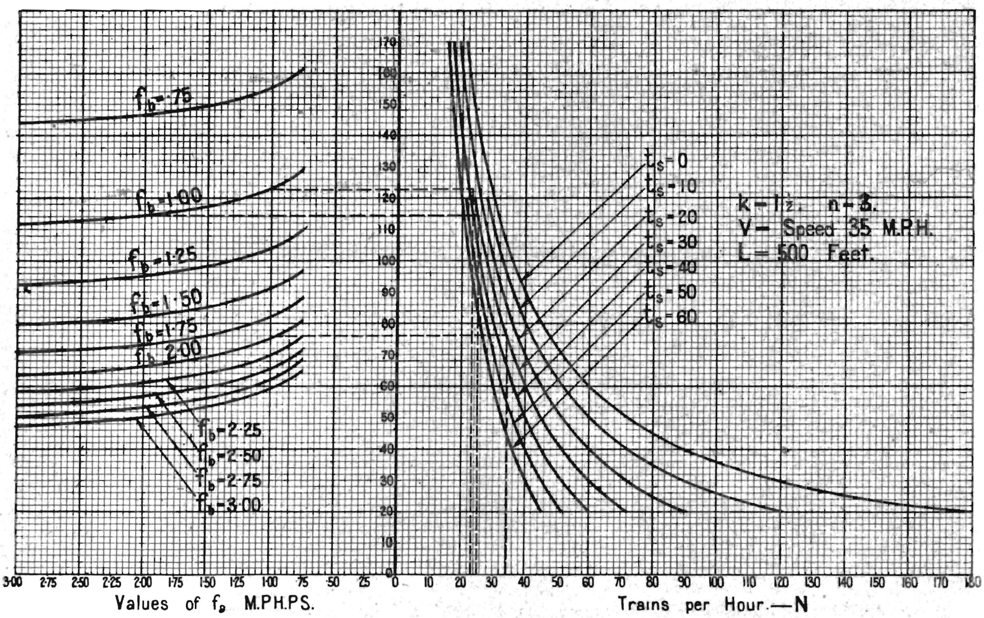


Diagram No. 5.

signal a distance equal to a complete safe braking distance at maximum speed must be maintained between trains. The ideal manner of operating trains would be to automatically ensure having at any free running speed a minimum of only one safe braking distance between the head of one train and the tail end of the next train in advance. Such a system would just take care at all times of the worst possible hazard. A following train would have its possible speed automatically adjusted in accordance with the movements of a train in advance. Should the train in advance stall practically instantaneously, the following train would stop before reaching it.

Many engineers are working to try and perfect such a system of automatic speed control, and it is not unreasonable to expect a satisfactory solution in the not distant future.

Last year on a section of one of the New York subways, a system of automatic speed control that did not aim at reaching the final perfect scheme was successfully in operation, and would have been installed on a considerable mileage of new subways but for some legal difficulties brought about principally by the influence of the war on prices. The system depended for its operation on the division of the track into sections in a similar way to that already described for automatic signalling with train stops, but enforced obedience to caution signals, enforcing stopping at a stop signal and thus cutting down the free running distance between trains to two overlaps instead of three as in the case previously outlined. In addition the system was arranged to allow of various predetermined maximum running speeds with corresponding block lengths in accordance with the braking equipment and the grade on which a train was running.

Direct driven from an axle in each car was a centrifugal speed-measuring device or governor, spring-loaded through a cam to give a straight line relationship between the speed of the car and the height of the sleeve.

Contact ramps were arranged at intervals along the track outside the running rails. These contact ramps were energised with high voltage, or medium voltage, or were in a de-energised condition. High voltage was used to cut out the speed control element in the train, enabling it to continue at maximum speed, in cases where the control mechanism had been in operation in the previous block.

Medium voltage was the normal condition for free running up to maximum speed.

No voltage put in operation the speed control device in the car. The speed control device in the car consisted of a cam (operating relays), and cut so that its roller traced out a braking curve with speed distance axes. Providing the driver kept the speed below that allowable by the cam, the control of the train was entirely in the hands of the driver. If, however, the speed of the car exceeded the allowable speed, an automatic brake application ensued, preventing the train reaching the end of the block. A whistle was arranged to sound whenever the speed of the train was close to the maximum for the running conditions.

In addition to the speed control ramps, selector ramps were located on the track between the rails. The function of these was to alter the maximum allowable running speed and block lengths. The combinations used were 21-480, 21-700, 33-900, 40-1200, 48-1600, where the first number of each combination designates the allowable maximum speed in miles per hour, and the second number the length of block in feet.

If the same braking safety be insisted on with a system enforcing obedience to a caution signal the minimum distance between trains becomes twice the distance between signals, giving from the general formula with $n=2$, and $k=1\frac{1}{2}$

$$N = \frac{3600}{\dfrac{V}{2\,fa} + \dfrac{L}{V} + \dfrac{2\,V}{fb} + ts}$$

This formula is shewn graphically on the Diagram No. 6 with the same examples shewn by dotted lines as were shown previously with three overlaps distance between trains. The possible number of trains per hour now equals 28.3, with fa and fb each equal to 1 m.p.h.p.s. With fa=2 m.p.h.p.s. and fb=1 m.p.h.p.s. N=30.4, a gain of 7.4 per cent., whilst with fa=1 m.p.h.p.s. and fb=2 m.p.h.p.s. N=39.0, or 37.8 per cent. increase by doubling the braking deceleration.

In order to complete the series of deductions the case of perfect speed control will be considered to shew the maximum possible capacity of a rapid transit track if minimum safety conditions of one braking distance from a hazard be obtained and accepted.

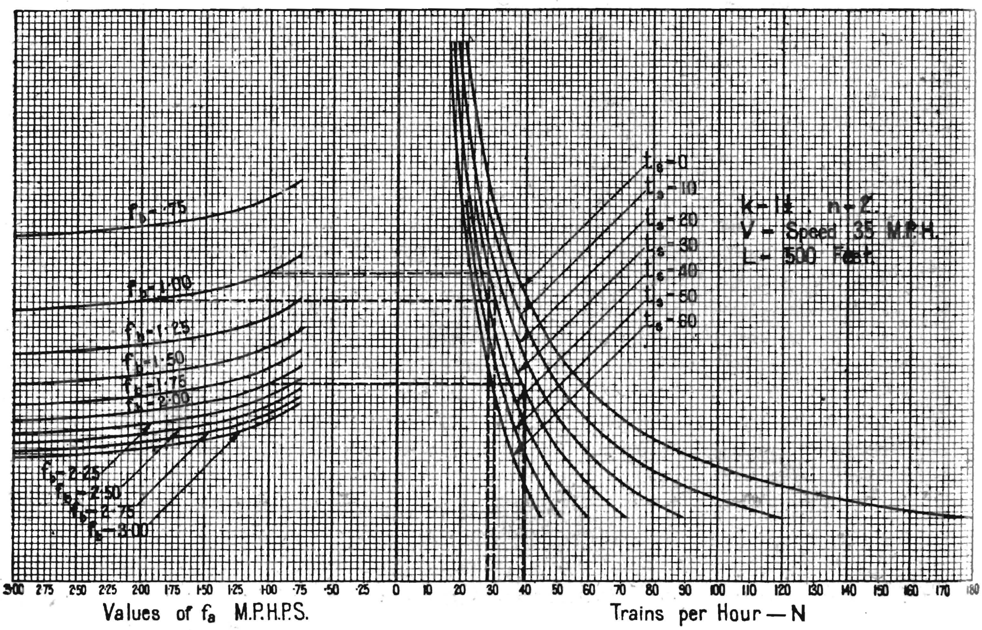
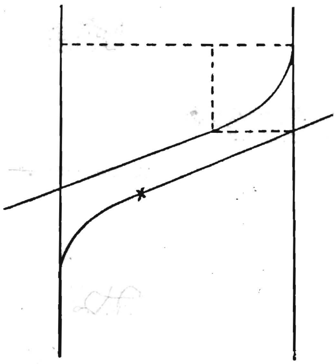


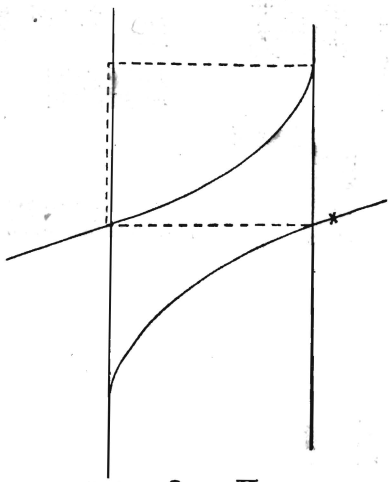
Diagram No. 6.

As in previous cases the train movement at stations will limit the track capacity.

Referring to the Diagram No. 7 two cases present themselves, one in which a leaving train accelerates to maximum velocity before reaching the end of the platform, and the other where the leaving train is still accelerating at the platform end. In either case the minimum interval for free running is clearly obtained when the entering train commences to decelerate at the instant the leaving train is passing the platform



— CASE I —



— CASE II —

end. The intervals, however, are different. The former case becomes the general case previously discussed, with k and n each equal to 1, giving

Diagram No. 7.

$$N = \frac{3600}{\frac{V}{2\,fa} + \frac{V}{fb} + \frac{L}{V} + ts}$$

In the second case the interval is $\frac{V}{fb} + \sqrt{\frac{2\,L}{fa}} + ts$

$$\text{giving } N = \frac{3600}{\frac{V}{fb} + \sqrt{\frac{2\,L}{fa}} + ts}$$

Diagram No. 8 has been prepared based on a train length of 500 feet and a maximum speed of 35 miles per hour as in the previous two cases. Similar examples are shewn by the dotted lines; the trains per hour being 37.8 with acceleration equal to 1 m.p.h.p.s. and deceleration of 1 m.p.h.p.s., which increase to 43.1 trains per hour on doubling the acceleration, equivalent to 14 per cent. increase, and to 48.9 trains per hour on doubling the deceleration, this increase being 29.2 per cent.

From the foregoing it is evident that taking account of the effect of station stops there is a limiting value of V that will give a maximum number of trains per hour.

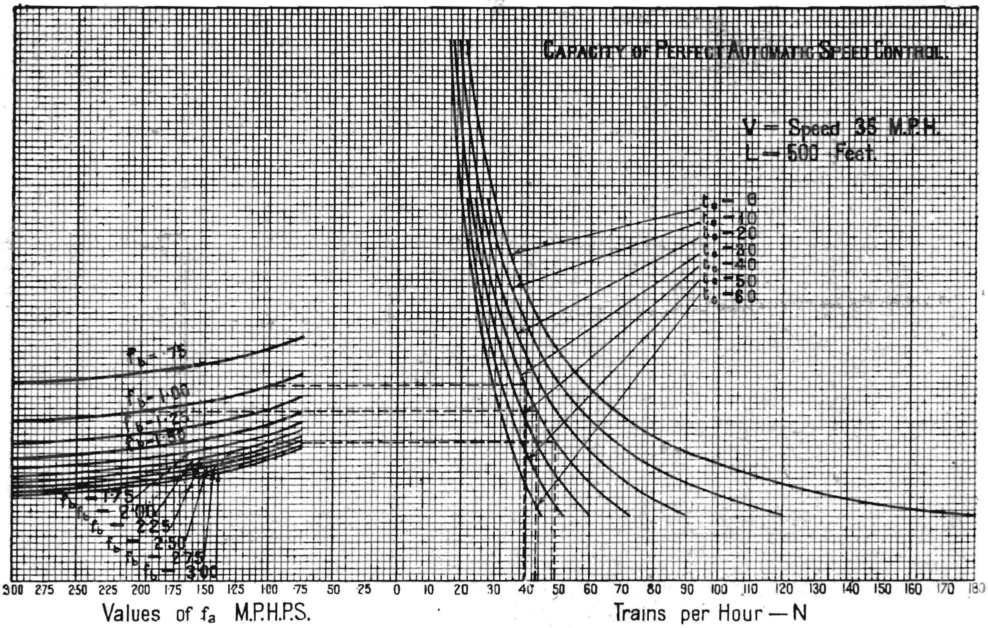


Diagram No. 8.

Taking the general expression

$$I = (k.n. + 1)\frac{V}{2\ fb} + \frac{V}{2\ fa} + \frac{L}{V} + ts$$

For minimum I

$$\frac{d\ I}{d\ V} = (k.n. + 1)\frac{1}{2\ fb} + \frac{1}{2\ fa} - \frac{L}{V^2} = O$$

$$\text{or } V^2 = \frac{L}{(k\ n + 1)\frac{1}{2\ fb} + \frac{1}{2\ fa}}$$

$$\text{Minimum } I = V \left\{ (kn + 1)\frac{1}{2\ fb} + \frac{1}{2\ fa} \right\} + \frac{L}{V_1} + ts$$

$$= V\frac{L}{V^2} + \frac{L}{V} + ts$$

$$= \frac{2\ L}{V} + ts \qquad N = \frac{3.600}{\frac{2\ L}{V} + ts}$$