THE FLOW OF WATER THROUGH SHARP-EDGED ORIFICES.

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INTRODUCTORY.

Early in 1898, during the preparation of apparatus for other experiments, the author made some determinations of the co-efficient of discharge of two sharp-edged circular orifices, and in working up the results for publication it has been found that the variation with the head of the co-efficient of discharge can be represented by a simple formula which is also applicable to the results obtained by other experimenters. A study of all the results available throws some light on the manner in which the co-efficient varies with the size and shape of the orifice.

In this paper it is proposed to give an account of the author's experiments, and of the development of the formula, together with a general discussion of the subject of the flow of water through sharp-edged orifices. As this discussion covers ground not usually given in the ordinary text books on hydraulics, it is hoped that it will be useful to the student members, while the formulæ may be of value to graduates in practice, or at all events form a guide to futnre workers in the same field. A rather lengthy description of the apparatus is given in the hope that it will prove useful to any who may have facilities for continuing an investigation on the same subject which, though well-worn, still affords scope for accurate research work.

GENERAL.

Before proceeding with an account of the experiments it will be as well to briefly describe the phenomena of jets issuing from sharp-edged orifices, and to outline those portions of the theory generally given in text-books, which are necessary for a proper understanding of the experiments.

When, under the action of gravity, water issues from a sharp-edged circular orifice placed in a vertical plane, the jet so formed springs clear of the orifice, and, owing to the inertia of the particles approaching the orifice from points remote from the axis, converges for a distance of about half the diameter from the plane of the orifice, and then continues with a practically uniform circular section, the axis of which follows an approximately parabolic path until the jet is checked or till the friction of the air breaks it up into spray.

The jet at its point of greatest contraction is known as the *vena contracta* or contracted vein, and the ratio of the area of the jet at this point to the area of the orifice is known as the co-efficient of contraction—generally written C_c—the value of which is generally taken as ·62 though it varies slightly with the size of the orifice as well as with the head.

By the well-known theorem of Torricelli, the velocity of the stream-lines on the free surface of a jet, discharging into the air under a head h, is given by the formula $V^2 = 2gh$

It is found however that, from various causes which will be referred to later, the actual velocity is somewhat less than that given by Torricelli's theorem, and may be expressed by

$$V = C_v \sqrt{2gh}$$

where C_v is a co-efficient, termed the co-efficient of velocity, and has an average value of about ·98.

This value of the velocity cannot however be applied at once to calculate the discharge from the orifice by multiplying it by the area, for two reasons. In the first place the stream lines at the orifice are not at every point perpendicular to the plane of the orifice, but become more oblique as one passes from the centre to the circumference, the central stream line being perpendicular to, and the outside stream lines in the same plane as the orifice. In the second place this convergence must make the pressure in the interior of the jet at the orifice greater than at the circumference where it is equal to the atmospheric pressure, and, by Bernouilli's theorem, the velocity at the centre must therefore be less than that given by the above formula, as indeed has been proved experimentally. The rate of discharge must therefore be less than that calculated from the formula

$$q = Av = AC_v \sqrt{2gh}$$

where A = the area of the orifice and q is the quantity discharged in unit time.

At and beyond the "*vena contracta*," however, the stream lines are practically parallel; and it is generally assumed that there is little or no variation of pressure and consequently of velocity across the contracted section of the jet, and that therefore the rate of discharge may be calculated from the formula

$$q = C_c\,Av = C_c\,A\,C_v \sqrt{2gh}$$

where C_c A is the area of the jet at the *vena contracta*, C_c being the co-efficient of contraction.

This is generally written $q = C_d\,A \sqrt{2gh}$

C_d being termed the co-efficient of discharge, and is equal to the product of the co-efficients of contraction and velocity.

If Q be the total quantity discharged in a time T we have from the above

$$Q = q\,T = C_d\,AT \sqrt{2gh}$$

It is evident that the co-efficient of discharge as thus expressed is easily determined by experiment by measuring the total quantity of water discharged in a given time from an orifice of known area under a measured constant head.

The two other co-efficients can also for a circular orifice be determined fairly simply though with less accuracy; C_c by measuring the diameter of the jet at the *vena contracta*, and C_v by determining the parabolic path of the jet; in the case of jets from non-circular orifices however these two determinations are rendered more difficult by the fact that the shape of the jets, at and beyond the *vena contracta* is not similar to that of the orifice from which they issue.

The water on issuing from a square orifice converges until at a short distance from the orifice it reaches a point corresponding to the *vena contracta* of the circular jet. At this point the section of the jet is approximately octagonal. The water issuing from the corners has a greater convergent tendency than that from points of the square nearer the centre, and consequently this water, in seeking the centre of the jet, squeezes out the intermediate particles of water. The water thus squeezed out takes the form of thin sheets, hereinafter termed rays, which lie in planes at right angles to and bisecting the four sides of the square. The water thus displaced gradually recedes from the centre of the jet against the retaining action of the surface-tension, which preserves a connection between these sheets and the central portion of the jet. The surface-tension gradually overcomes the outward tendency of the sheets of displaced water, and then begins to bring them back towards the centre of the jet. After a time the rays subside into the central stream and, as under a constant head the stream always travels a constant distance longitudinally during the same time, the meeting of the rays takes place at a fixed point, and the distance from the contracted section to that point, called a wave-length, is constant.

But this is not all. The effect of the meeting together of the rays at the end of the wave-length is somewhat similar to that of the convergence of the water from the corners of the orifice, and the result is to squeeze out a second set of rays in planes bisecting the angles between those of the first set. The second set appears before the complete disappearance of the first set, so that at the end of the wave-length, or at what may be termed the node, there are twice as many rays as there are sides of the orifice. The secondary rays are subject to the same action of surface-tension as the primary, and they consequently spread out and then converge again in the same manner as the first. Similarly a third and fourth or indefinite number of sets of rays may be formed. Practically the effect of atmospheric friction and other causes is to break up the stream into spray sooner or later; the distance over which the stream retains its perfect form depends on the head, and on the size of the orifice which is being used.

This is the effect noticeable in the case of a jet issuing from a square orifice. When the orifice is triangular similar phenomena may be observed, but the number of rays is three, corresponding to the number of sides in the triangle. Generally the number of rays corresponds to the number of sides in a polygonal orifice; the extreme case being that of the circle, or polygon of infinite number of sides when the waves merge into a single circular jet. The rays in this case are also indefinitely reduced in section, and practically no rays are formed.

Description of Apparatus.

The foregoing phenomena are displayed in a very perfect form in the hydraulic laboratory at McGill University, where the author's experiments were made, and the following is a description of the apparatus there used.

The experimental tank in the side of which the orifices are placed is of cast-iron, twenty-eight feet in height and five feet square inside, the interior surfaces being perfectly flush. In order to minimise the disturbance created by the inflow, the water is admitted into a three inch chamber extending across the bottom of the tank, and having perforations on the lower surface through which the water flows into the tank near the bottom. Twelve inches above the bottom is a baffle plate perforated with three-eighths inch holes, and six inches higher there is a second baffle plate, also perforated with three-eighths inch holes, the pitch of which was determined by the projections on a horizontal plane of equal distances on a sphere of five feet radius, with its centre at the centre of the orifice which is seven feet above the bottom of the tank.

There are two main inlet pipes, one three inches, and one one and a half inches in diameter, connected to the City water supply, and there are also two main outlets connected to the bottom of the tank.

The tank is provided with a gauge glass one and a half inches in diameter, placed near one corner of the tank by the wall of the laboratory. On one side of the gauge there is a brass scale graduated from zero point in the same horizontal plane as the centre of the orifices which is always a fixed point. Behind the gauge-glass is a mirror, while a carrier with a horizontal wire passing in front of the gauge is arranged to slide up and down the brass scale which is marked in feet. Any required head can therefore be obtained by bringing the necessary scale graduation, the surface of the water in the gauge, the wire and its reflection in the mirror into the same plane. On the wall by the tank is a ladder extending to the level of the top of the tank, and beside this is a vertical spindle provided with handles at intervals; this spindle operates a three-way valve through which the water may be admitted to the tank or allowed to escape. The observer standing on the ladder could thus readily adjust the water to the desired level. To obviate the necessity of constantly climbing the ladder during an experiment, a second indicator is provided consisting of a float with a water-proof silk fishing cord attached passing over a large light pulley and vertically downwards in front of the tank. The cord is kept taut by a weight at the bottom and carries a friction-tight pointer which can be adjusted, whatever the height of water in the tank, to point to a line on a brass plate fixed in a convenient position near the inlet valves, which could therefore be regulated as required; near at hand is a small quarter inch tube with a tap which is useful for fine adjustment.

The head available on the supply pipe was 280 feet, and the range of head might have been therefore greatly extended had the tank been arranged so that it could be closed in at the top, and built strong enough for the full pressure. This would necessitate the provision of a mercury gauge.

At a later date in connection with some other experiments, arrangements were made for admitting steam to the tank, so that experiments could be made with hot water, and as this is always desirable and as the brass scale, when placed as in this case at the side of the tank, would be affected by the rise in temperature, the author would suggest the desirability of placing the gauge-glass and scale on a wall at some distance from the tank, care being taken to so fix it that the relative positions of the zero line and the centre of the orifice would not be affected by the rise in temperature. A suitable vernier arrangement with scales on two metals, with different temperature co-efficients, would enable one to determine the true value of the head with great accuracy. Greater accuracy could be obtained by the use of a hook gauge which could be arranged for use in small auxiliary vessels attached to the tank at suitable heights and each provided with a valve to admit the water, when readings were being taken at that head.

The most noteworthy feature of the tank is the orifice valve which is a gun-metal disc three-quarter inch in thickness, and twenty-four inches in diameter, fitted into a recess of the same dimensions in a cast-iron cover plate, with gun-metal bearing faces forming a water tight joint for the face of the disc.

This cover plate or body is bolted to an opening in the front of the tank, and the inner faces of the cover plate or disc are flush with the inner surface of the tank.

In the disc and on opposite sides of the centre there are two screwed openings, the one three inches and the other seven inches in diameter. By rotating the disc each opening can be made concentric with a screwed seven and a half inch opening in the body of the valve. The disc is rotated by means of a spindle through its centre, passing through a gland in front of the valve body and operated by a lever on the outside. Gun-metal bushes, with the required orifices, are screwed into the disc openings, and when screwed home have their inner surfaces flush with the valve surface, and therefore with the inside surface of the tank. By means of a simple device, these bushes can be readily removed and replaced by others without the loss of more than a pint of water. A cap with a central gland is screwed into the seven and a half inch opening of the valve body and forms a practically water-tight cover. The valve is rotated so as to bring the bush opposite the opening, and it is then unscrewed by means of a special key projecting through the cap gland. The valve is now turned back until the opening is closed when the cap is unscrewed, the bush taken out and another put in its place. The cap is again screwed into position, the valve rotated until the openings in the disc and tank side are concentric, when the bush is screwed home by the key.

Fig. 1 shows the arrangement as viewed from inside the tank.

In opening the orifice the lever is rotated through a quarter-turn till it reaches a stop formed by a set screw fixed in a projection on the

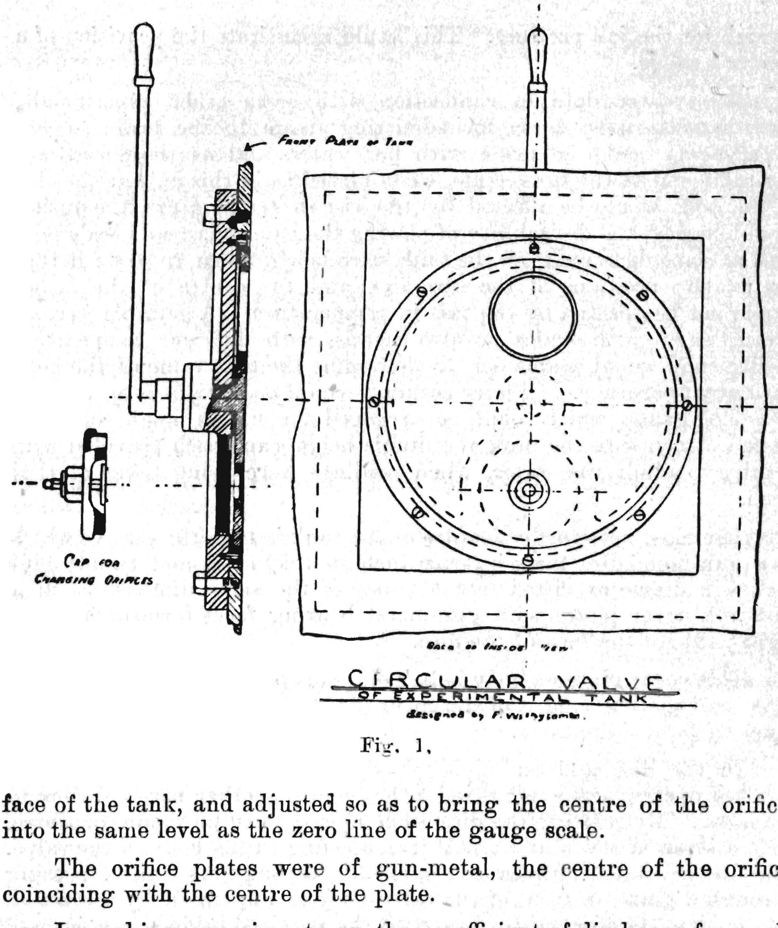

Fig. 1.

face of the tank, and adjusted so as to bring the centre of the orifice into the same level as the zero line of the gauge scale.

The orifice plates were of gun-metal, the centre of the orifice coinciding with the centre of the plate.

In making experiments on the co-efficient of discharge for small orifices the jets are discharged through a bifurcated tube into a calibrated vessel; in the author's experiments however, the jets were discharged into a flume at the end of which is a weir, beyond which is a flap-door with bevelled edges along the centre lines of which runs a piece of indiarubber cord, so that when the door is closed and pressed home there is a perfectly water-tight joint. This door is opened and shut by a lever with a spring clasp actuating a system of links acting as a toggle joint, so that the door can be rapidly and firmly shut and locked in position, each movement of the lever being recorded on the chronograph.

When this door is open the water runs to waste, but on closing the door the water flows over it into one or more of a set of five 1,000 gallon tanks of cast-iron, fixed firmly in concrete below the floor level. Each of these tanks is connected to a header by a valve so that they can be used separately or in any combination. To each tank is connected a vertical four inch brass pipe forming a float chamber; the float is attached to a brass rod with a pointer at the upper end moving over a brass scale and attached to a counter balancing weight by a fine cord passing round a frictionless pulley. The scale of each tank is marked in 100 gallons up to 1,000 gallons, and then by 10 gallons up to 1,080 gallons.

As indicated above, the duration of each experiment was recorded on a chronograph connected to a standard clock in the adjacent testing laboratory, a mark being made on the record every second by a fountain pen. The time of opening and closing the flap door was recorded by a glass stylus the point of which followed closely in the track of the fountain pen, and derived sufficient ink from the moist track to make its mark when the lever was moved. The mark indicating the movement of the door was on the opposite side of the line to that indicating the seconds, so that there could be no possible confusion between the two marks, even when occurring at the same time or place. A stop watch was also used as a check against large errors and also to indicate during the experiment the time which had elapsed since the start.

Experimental Work.

The water in the flume had of course to be kept up to the crest of the weir; before an experiment, therefore, the orifice was always opened and the water run to waste for ten to fifteen minutes to allow the water to attain a steady head above the crest of the weir, so that at the opening and closing of the flap-door the flow at every point would be steady. During this time the inlet valve was adjusted, the chronograph record placed in position, and the measuring tanks read. The temperature of water in the flume did not differ by more than a few degrees from that of the water issuing from the orifice, or as finally measured in the tank, so that no appreciable error was introduced, especially as the quantity of water measured was so large and the temperature very nearly that of the maximum density of water, the experiments being made during the winter months.

During the course of each experiment the head was kept under constant observation, and the temperature of the water issuing from the orifice taken at frequent intervals. The temperature was also taken in the measuring tanks when the quantity was read, but no correction has been made for the difference as it was within the limits of errors in the other measurements. This introduces the subject of the probable accuracy of the determinations.

Q.—The total quantity was never less than 1,000 gallons, and in some cases was 2,000 gallons, these latter being in the experiments on the two inch orifice at high heads. On the scale of the tank 0·87 inch corresponded to ten gallons; the position of the pointer was referred to the nearest ten gallon mark by means of a steel scale graduated in hundredths of an inch, so that the combined error of the initial and final readings would probably not exceed one in 5,000 or ·0002 as far as the relative values of the co-efficient in these experiments are concerned. The absolute values would depend upon the