TABLE III.—CO-EFFICIENT OF DISCHARGE FOR CIRCULAR ORIFICES.

Head in Feet.	·48 in. DIAMETER.		·84 in. DIAMETER.		1·20 in. DIAMETER.	
	By Experiment.	By Formula. $C_d = ·594 + \frac{·028}{\sqrt{H}}$	By Experiment.	By Formula $C_d = ·593 + \frac{·020}{\sqrt{H}}$	By Experiment.	By Formula $C_d = ·592 + \frac{·016}{\sqrt{H}}$
0·4	·637	·637	·624	·624	·618	.617
0·6	·630	·630	618	·619	·613	·613
0·8	·626	·625	·615	·615	·610	·610
1·0	·623	·622	·612	·613	·608	·608
1·5	·618	·617	·608	·609	·605	·605
2·0	·614	·614	·607	·607	·604	·603
2·5	·612	·612	·605	·606	·603	·602
3·0	·611	·610	·604	·604	·603	·601
4	·609	·608	·603	·603	·602	·600
6	·607	·606	·602	·601	·600	·599
8	·605	·604	·601	·600	·600	·598
10	·603	·603	·599	·599	·598	·597
20	·599	·600	·597	·597	·596	·596

With the results given as above only to the third place of decimals, there is not sufficient variation in the values for the larger orifices given by Merriman to deduce any accurate values of m and n, while the values for the smallest orifice cannot all be fitted to a curve of the required form though the experimental values are not inconsistent with the general formula given hereinafter.

It will be noticed that the values given by Merriman for heads of 50 and 100 feet are not included, as the formulæ are only offered as representing the values up to heads of 20 feet. To include higher heads the values of m have to be somewhat decreased with a corresponding increase in the value of n, and the values would not then be comparable with those obtained at McGill.

It remains to be seen whether an expression can be formulated covering the variation of C_d with the area of the orifice; tabulating the values of m and n for the different diameters, it will be seen that while m decreases as the diameter increases, the variation is so slight that it cannot be accurately expressed ; the following table shows that the value of n is given by the equation $n = \dfrac{k}{\sqrt[3]{d^2}}$ the average value of k being about ·018.

TABLE IV.

d inches.	n observed.	$\dfrac{\cdot 018}{\sqrt[3]{d^2}}$
·24	—	·0465
·48	·028	·0294
·84	·020	·0202
1·00	·019	·0180
1·20	·016	·0160
2.00	·011	·0113
2·40	—	·0100

As the probable error in the values of n as determined from the experiments is not less than ·0005, the agreement may be considered good.

The general formula for the co-efficient of discharge for sharp edged circular orifices may therefore be written

$$C_d = m + \frac{k}{\sqrt{h}\ \sqrt[3]{d^2}}$$

where h is expressed in feet and d in inches
and m has an average value of ·5925 increasing slightly as the diameter decreases.
and k has an approximate value of ·018.

Seeing the data from which this formula is derived, it is desirable that a series of accurate experiments should be carried out by one observer with one set of apparatus under uniform conditions, when it is probable that the values of m and k may be somewhat modified ; it is offered as a first approximation only, for orifices of diameter up to 3 in., and for heads up to 20 feet ; it has the merit of being fairly simple in form, and given the values of m and k the values of C_d may be obtained very quickly by the use of a slide rule.

It will now be useful to examine the table of values of C_d for square orifices, given on page 81, of Merriman's treatise ; taking as before the orifices of medium size, the approximate values of m, n and k, are as follow :—

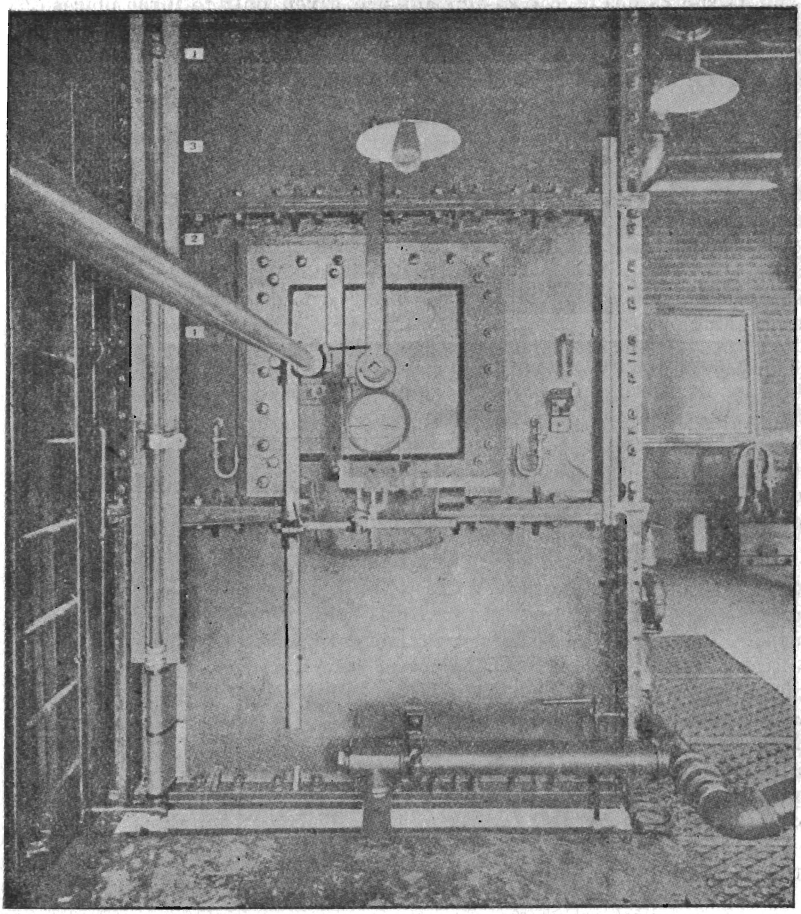
l inches.	m	n	$k = n\ \sqrt[3]{l^2}$
·48	·598	·029	·0178
·84	·598	·020	·0178
1·20	·598	·015	·0170

As the values of the co-efficient are given only to three places of decimals, the values of m and n are necessarily approximate; but it may be stated that as before m is approximately constant, but has a higher value then for circular orifices, while the figures in the last column show that as before $n = \frac{k}{\sqrt[3]{l^2}}$ where the average value of k may be taken as ·0175, or practically the same as the value of k for circular orifices; this agreement is in favour of the theory that the co-efficient is affected by the ratio of the perimeter to the area of an orifice; this is supported also by the value of n for the only other equilateral orifice for which a value is available; this orifice being triangular in form.

A series of experiments is needed on a set of equilateral triangular orifices of different areas in order to determine the values of m and n for such orifices; the dependence of n upon the ratio of area to perimeter could then be verified, and some idea be obtained of the variation of m with the number of sides of the regular polygon. Experiments on other regular polygonal orifices would also be useful in throwing light on these points.

Fortunately, however, in practice the variation of m and n with the shape of the orifice is of no great importance in the case of regular polygons as circular and square orifices are the only ones generally used. It is therefore better for practical purposes to define n in terms of the diameter or length of side respectively, and to consider m as a separate constant for each case. Generally it may be stated that both m and n and consequently C_d are greater, the greater the departure from the circle or the greater the ratio of perimeter to area, and this also holds good for rectangular orifices, which are frequently used in practice. Sufficient accurate data are not however available to determine a general law for rectangular orifices, though it is certain that the law $C_d = m + \frac{n}{\sqrt{h}}$ holds equally well for such orifices, and what evidence there is in favour of assuming that n varies as $\left(\frac{\text{perimeter}}{\text{area}}\right)^{2/3}$. It is of course easy to see that the the value of C_d would be greater for elongated rectangles than for squares, as the end contractions produce a relatively smaller effect. As rectangular orifices are much used it is desirable that a series of careful experiments should be made on such orifices, preferably on sets of orifices of the same perimeter but of different areas, or of the same area with different perimeters.

As indicated before, however, the first step towards the discovery of a general expression should be to ascertain the variation of the co-efficients of contraction and velocity, and a description will now be given of a piece of apparatus which may be used for making accurate measurements of the shape and size of the jet; this jet measurer is fixed to the face of the tank as shown in the photograph, which also gives some idea of some of the other apparatus already described.



A horizontal bar 2 in. in diameter and 7 feet in length is carried at each end in iron supports above and a little to one side of the jet. A split sleeve slides on this, carrying a second bar in a second split sleeve at right angles to the first. This second bar is thus capable of being set at any point along the first and can be set at any angle in a plane at right angles to the first bar. The split sleeve is provided with a set screw by means of which it and the bar it carries can be clamped in any desired position. This second bar is usually set vertically.

The second bar carries a split sleeve, and a third bar can be attached to it in the same way as the second bar to the first, and can be clamped in the same way. This bar is usually set horizontally and at right angles to the jet, at a point on the second vertical bar, a short distance below the jet.

The jet measurer proper is attached to this third bar by means of a split sleeve. The measurements are read on a vernier calliper, which is arranged to hold suitable capstan heads in its jaws. These capstan heads carry suitable measuring points, which can of course be brought together or separated by the ordinary motion of the vernier calliper. The arm of the calliper is mounted in guides attached to a frame. It can be clamped in the guides or can be clamped by a separate sleeve, which sleeve is, by means of a nut and screw, movable with respect to the frame. Thus the arm of the calliper has a longitudinal adjustment with respect to the frame, and by means of this adjustment the pointer connected to the fixed jaw of the calliper can be brought up until it just touches the surface of the jet of water which it is desired to measure. The pointer attached to the movable jaw is then adjusted into bare contact by means of the ordinary screw adjustment of the calliper. The frame in which the calliper arm is seated is not fixed to the sleeve on the third bar. It is rotatably mounted on a pin at right angles to the axis of the sleeve, and can be clamped by means of a milled nut on the screwed end of the pin. The guides which carry the calliper arm are capable of a small motion at right angles thereto. They are drawn towards the frame by a milled nut on a screwed arm, projecting through part of the frame, and when the nut is unscrewed are drawn back by a spring placed between a shoulder on the frame and the guides. The frame carrying the guides and calliper can be rotated in a second direction if desired. Including the rotation of the sleeve to which the calliper-bearing frame is attached, the calliper is rotatable in every direction. Independently of this, the arm on which it is supported has freedom of movement in every direction both rectilinear and rotational.

The manner in which the jet measurer may be used to determine the co-efficient of contraction by measuring the cross section of the jet at the *vena contracta* is sufficiently obvious, though the observations are somewhat difficult to take, principally on account of slight fluctuations in the position and shape of the jet, caused doubtless by irregularities in the motion of the water in the tank. For this reason it is preferable to take such measurements under a dropping head, the water being run into the tank to a somewhat higher level and the inlet valve then turned off, and the motion allowed to subside, after which the orifice may be opened and the reading taken as the head reaches the required value.

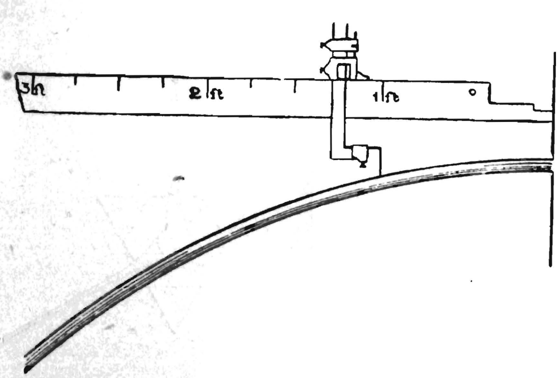


Figure 4.

For determining the path of the jet for calculating the co-efficient of velocity, a needle point clamped to a vernier calliper may be used, the calliper being mounted on a horizontal straight-edge, placed at right angles to the face of the orifice plate above the centre of the orifice, and graduated from the plane of the orifice, the arrangement being shown in Fig. 4.

The description of the jet measurer given above has been taken from a paper describing a series of measurements made with it on jets from square and triangular orifices, and giving an account of the phenomenon, referred to before, known as "inversion of the vein."

By these experiments it was shown that :—

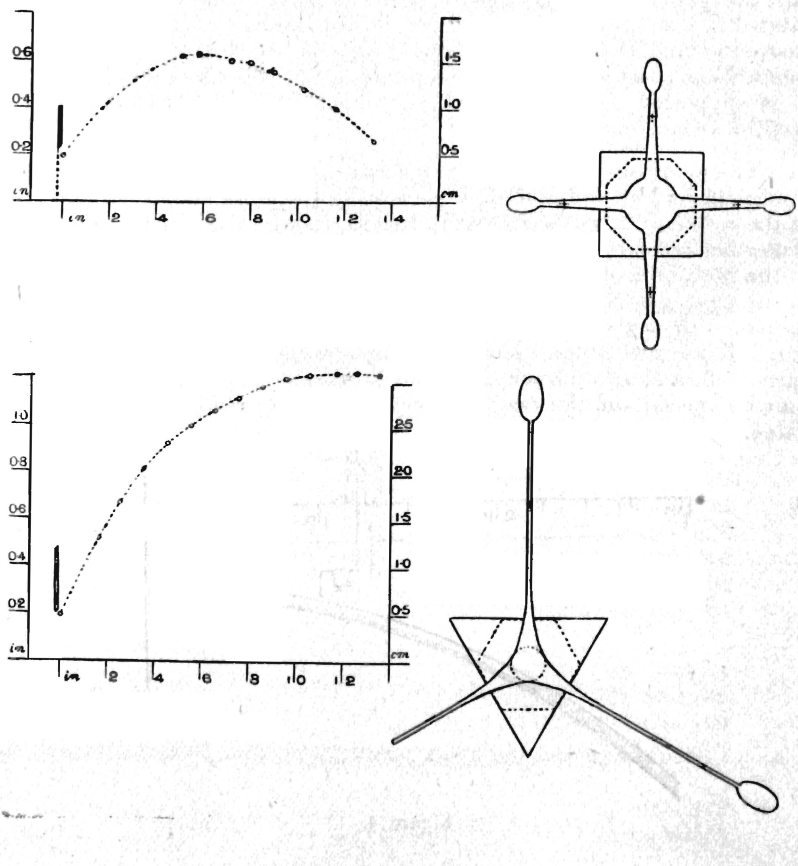
(1) the wave length for any orifice varied directly as the head

(2) ,, ,, ,, similarly shaped orifices varied directly as the area.

(3) the spread of the rays for similarly shaped orifices varied directly as the area.

(4) the maximum width of the jet in any orifice varied directly as the head.

Fig. 6



The mathematical theory which was developed, and was found to include the above results, was based on the assumption that the causes of the phenomena were as outlined in the description already given on page 47, the assumption being subsequently justified by the agreement of the experiments with the theory, and by the fact that the value of the co-efficient of surface tension obtained by inserting the experimental values in the theoretical formula agreed very well with the values usually accepted for water.

The remarkable shape of the jets is shown in Fig. 5, which is reproduced from the paper referred to. It might here be remarked that the form of the jets is substantially the same whether they are projected horizontally or vertically, and this fact effectually disposes of the theory usually advanced which ascribes the phenomena to the impact of stream lines emerging from different portions of the orifice under different heads.

It will be evident from the above figure that the determination of the area of the jet in the case of any but circular orifices is a matter of considerable difficulty; for while in the case of the circular jet, one or two measurements suffice to define the area, in all other cases a large number of measurements must be taken under exactly similar conditions. As mentioned before, any irregularity in the motion of the water in the tank, or any slight variation in temperature produces a corresponding variation in the position and shape of the jet. It is not uncommon, for instance, for the web to break, leaving an opening between the beaded rim and the central core; this has been found to be due to variation in temperature of the water, while the effect of irregular motion in the tank is to bring about an alteration in the position of the plane of the web, this making it very difficult to measure the thickness of the web accurately.

Although a vast number of experiments have been made on the discharge of jets from the various orifices, it will have been seen that, like those described in this paper, they are of a disconnected nature, and it is therefore desirable that further experiments should be carried out on a connected scheme. The author has indicated points on which further experiments are particularly required, and has described his own experiments and offers the formulæ derived therefrom, not on account of any merit they may possess, but rather as a guide to future work.