NOTES ON THE THEORY OF THE ALTERNATE CURRENT TRANSFORMER.

(*A Paper read before the Sydney University Engineering Society, on June 20th,* 1907).

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I feel that perhaps some explanation is needed as to why I should bring before this Society a subject which is treated of in ordinary text books on alternating currents and in special books on the same subject, but the fact is that I have found books very unsatisfactory when treating of this subject, and I think that by the methods that I have used, it can be made clearer than usual. Also I am convinced that many books are in error in at least one important point.

I have treated the subject all through by the use of vector or clock diagrams, and instead of simply drawing diagrams with no actual values, I have taken an imaginary transformer and worked out the actual values in each case.

In each case the results are checked by the following method :—The watts supplied to the transformer less the losses in the transformer should be equal to the watts given out, and, as will be seen from the diagrams, this is the case as nearly as can be expected. The primary and the secondary watts are always worked out from the voltage, current, and the power factor; while the losses are, except in one case, only the copper loss, which is easily determined. In one case, I introduced iron losses, to show their effect, but in all the other cases, the transformer has been taken as having no iron. This is done merely to simplify matters.

It will be noticed that no vectors to represent magnetic fluxes are included in the diagrams, as I am convinced that such lines are unnecessary and only complicate matters.

1.—LIST OF SYMBOLS USED IN THE PAPER.

f = Frequency in periods per second.

p = $2 \pi f$.

R_1 = Resistance of primary of transformer in ohms.

R_2 = Resistance of secondary of transformer in ohms.

n_1 = Total number of primary turns.

n_2 = Total number of secondary turns.

A = Cross sectional area of transformer core in square centimetres.

l = Mean length of magnetic circuit of transformer in centimetres.

E_1 = E.M.F. supplied to the primary in volts.

E_2 = E.M.F. available outside the secondary in volts,

E_{o2} = E.M.F. of secondary on open circuit in volts.
E_{t2} = Total E.M.F. induced in the secondary in volts.
$C\mu$ = Primary magnetising current in amperes on any load.
$C_o\mu$ = The same, no load.

Note.—This current is 90° in front of the counter E.M.F. due to self-inductiion.

$C\eta$ = Primary iron losses current in amperes.
C_h = Primary hysteresis losses current in amperes.
C_e = Primary eddy current losses current in amperes.

Note.—$C\eta = C_h + C_e$ and $C_o = \sqrt{(C\mu)^2 + (C\eta)^2}$ where

C_o = Primary no load current in amperes.
C_1 = Primary resultant current on load in amperes.
C_2 = Secondary current in amperes.
$C_2\frac{n_2}{n_1}$ = Primary load current, *i.e.*, current which has to be supplied to the primary so that C_2 may flow in the secondary.
L_1 = Co-efficient of self-induction of primary in Henrys.
L_2 = Co-efficient of self-induction of secondary in Henrys.
M = Co-efficient of mutual induction of primary and secondary in Henrys.
v = The leakage co-efficient between primary and secondary

$$= \frac{\text{total magnetic flux through primary due to } C\mu}{\text{total magnetic flux through secondary due to } C\mu}$$

k = The ratio of the total E.M.F. induced in the secondary to the counter E.M.F. in the primary due to $C\mu$, which is, on open circuit, practically $= \frac{E_1}{E_{t2}}$ for a transformer in which R_1 is small.

$+ p L_1 C\mu$ = Counter E.M.F. of self-induction in primary due to $C\mu$, in volts.

{ This E.M.F. is 90° behind $C\mu$. }

$- p L_1 C\mu$ = Impressed E.M.F. necessary in the primary to overcome $+ p L_1 C\mu$.

$+ p L_1 C_2 \frac{n_2}{n_1}$ = Primary counter E.M.F. due to primary load current in volts.

$+ p L_2 C_2$ = Secondary counter E.M.F. due to secondary current in volts.

$+ p M C_2$ = Primary E.M.F. of mutual induction in volts, due to C_2 in the secondary.

$+ p M C_2 \frac{n_2}{n_1}$ = Secondary E.M.F. of mutual induction in volts due to $C_2 \frac{n_2}{n_1}$ in the primary.

$p\,L_1\,C_2\,\frac{n_2}{n_1} - p\,M\,C_2 =$ Primary counter E.M.F. in volts due to leakage.

$$= p\left\{L_1 - M\,\frac{n_1}{n_2}\right\} C_2\,\frac{n_2}{n_1}$$

Note.—This E.M.F. is 90° behind the primary load current.

$$L^1_1 = \left\{L_1 - M\,\frac{n_1}{n_2}\right\}$$

= "Equivalent self-induction of primary" in Henrys.

$- p\,L^1_1\,C_2\,\frac{n_2}{n_1} =$ E.M.F. which must be supplied from outside to overcome the counter E.M.F. due to leakage.

Note.—This E.M.F. is 90° in front of the primary load current.

$p\,L_2\,C_2 - p\,M\,C_2\,\frac{n_2}{n_1} =$ Counter E.M.F. due to leakage in the secondary in volts.

$$= p\left\{L_2 - M\,\frac{n_2}{n_1}\right\} C_2$$

$$L^1_2 = L_2 - M\,\frac{n_2}{n_1}$$

= Equivalent self-induction of secondary in Henrys.

$- p\,L^1_2\,C_2 =$ E.M.F. in volts used up in secondary because of leakage.

$$L_1 = \frac{4\pi}{10}\;\frac{n_1}{l}\;A\;\frac{n_1}{10^8}\;\mu \text{ Henrys.}$$

$$L_2 = \frac{4\pi}{10}\;\frac{n_2}{l}\;A\;\frac{n_2}{10^8}\;\mu \text{ Henrys.}$$

$$M = \frac{4\pi}{10}\;\frac{n_1}{l}\;A\;\frac{n_2}{10^8} \times \frac{1}{v}\;\mu \text{ Henrys.}$$

$$M = \frac{1}{v}\;\sqrt{L_1\,L_2}\text{ Henrys.}$$

$$E_{t2} = p\,M\,C\mu \text{ volts.}$$

$$= p l_1\,C\mu\,\frac{n_2}{n_1} \times \frac{1}{v}\text{ volts.}$$

ϕ_o = Angle between E_1 and C_o, open circuit.
ϕ_1 = Angle between E_1 and C_1, on load.
ϕ_{t2} = Angle between E_{t2} and C_2.
ϕ_2 = Angle between E_2 and C_2.

$W_1 = E_1\,C_1 \cos\phi_1$ in watts or $E_1\,C_o \cos\phi_o$.
$W_2 = E_2\,C_2 \cos\phi_2$ in watts.
$C^2\,R$ = Total copper loss in transformer in watts.

METHOD OF DRAWING THE DIAGRAMS.

In each case except VI, IX and X, the following are taken as known :—

The voltage supplied to the primary $= 100$ volts $= E_1$.

The secondary current $= C_2$.

The angle between the secondary current and the total secondary voltage E_{t2}.

The following is the method :—

1. When there is no leakage.

(*a*) Draw a vertical line to show the direction of $- p\, L_1\, C\mu$ and E_{t2}.

(*b*) Take a point on it about the centre of the sheet and mark off $C\mu$ horizontally towards the left.

(*c*) Draw a line of the required length to represent C_2 at the given angle from E_{t2}, starting from O.

(*d*) Produce this line through O and mark off on it $C_2 \frac{n_2}{n_1}$ to the same scale of amperes as $C\mu$.

(*e*) Find the resultant of $C\mu$ and $C_2 \frac{n_2}{n_1} = C_1$

(*f*) On this resultant, starting from O, mark off $C_1\, R_1$ to the same scale of volts as E_1.

(*g*) Through the upper end of $C_1\, R_1$ draw a perpendicular line.

(*h*) With centre O and radius 100 volts describe an arc of a circle to cut this perpendicular in B.

(*i*) Draw $O\, B_1$ which is the position of E_1 on this load.

(*j*) From B draw a line parallel to $C_1\, R_1$ to cut the vertical line through O at D. Then OD is $- p\, L_1\, C\mu$.

(*k*) Perpendicularly downwards from O is $E_{t2} = - p\, L_1\, C\mu \times \frac{n_2}{n_1}$

(*l*) On C_2 mark off $C_2\, R_2$.

(*m*) Join the lower end of $C_2\, R_2$ to the lower end of E_{t2}. This line is E_2.

2. When there is leakage.

As in 1 for *a*, *b*, *c*, *d*, but leave the length of $C\mu$ unmarked.

(*e*) Through O draw a line at right angles to C_2 and mark off on it to the primary and secondary scales of volts respectively $- p\, L^1{}_1\, C_2 \frac{n_2}{n_1}$ and $- p\, L^1{}_2\, C_2$, each being 90° in front of its current.

(*f*) From the outer end of $- p\, L^1{}_1\, C_2 \frac{n_2}{n_1}$ draw a line parallel to $C_2 \frac{n_2}{n_1}$ and equal to $C_2 \frac{n_2}{n_1} R_1$.

(*g*) Through the end of $C_2 \frac{n_2}{n_1} R_1$ draw a perpendicular line.

(*h*) With centre O and radius E_1 draw an arc of a circle to cut this perpendicular in F.

(*i*) Join O.F. which is the position of E_1 on this load.

Note.—This is not quite correct as it omits $C\mu\ R_1$ which has yet to be determined but is very small.

(*j*) The length of this perpendicular gives $- p\ L_1\ C\mu$.

(*k*) Determine $C\mu$ from the length of $- p\ L_1\ C\mu$ and mark it off.

(*l*) Find the resultant of $C\mu$ and $C_2 \frac{n_2}{n_1} = C_1$.

(*m*) E_{t2} is $- p\ L_1\ C\mu \times \frac{n_2}{n_1} \times \frac{1}{v}$

(*n*) Mark off $C_2\ R_2$ on the C_2 line from O.

(*o*) Through the lower end of E_{t2} draw a line parallel to $- p\ L^1{}_2\ C_2$ and equal to it.

(*p*) From the end of $C_2\ R_2$ draw a line to the end of the one drawn in (O).

This last drawn line is E_2 on this load.

Details of imaginary transformer dealt with in the paper.

$A = 100$ square centimetres.
$n_1 = 10{,}000$ turns.
$n_2 = 1{,}000$ turns.
$l = 100$ centimetres.
$R_1 = 2{\cdot}0$ ohms.
$R_2 = {\cdot}020$ ohms.
$f = 50\ v$.
$E_1 = 100$ volts,

Note.—Except in one case, No. II, the transformer is considered to have an endless magnetic circuit, and to contain no iron, consideration of that being omitted for the sake of brevity.

$$L_1 = \frac{4\pi}{10} \times \frac{100 \times (10^4)^2}{100 \times 10^8} = 1{\cdot}25 \text{ Henry.}$$

$$p = 6{\cdot}28 \times 50 = 314.$$

As there is no iron $C\eta = 0$ and so $C_o = C\mu$.

$$C\mu = \frac{100}{\sqrt{4 + ({\cdot}314 \times 1{\cdot}25)^2}} = \frac{100}{\sqrt{4 + (392)_2}} = 255 \text{ amperes.}$$

$$p\ L_1\ C\mu = 314 \times 1{\cdot}25 \times {\cdot}255$$
$$= 100 \text{ volts.}$$

$$\phi_o = \tan^{-1} \frac{p\ L_1}{R_1}$$
$$= \tan^{-1} \frac{314 \times 1{\cdot}25}{2{\cdot}0} = \tan^{-1} 196 = 89{\cdot}\dot{7}^\circ$$

The watts lost in the primary at no load

$= 100 \times \cdot 255 \times \cos 89 \cdot 7^{\circ}$ or $(\cdot 255)^2 \times 2 \cdot 0$

$= 25 \cdot 5 \times \cdot 0052 \qquad = \cdot 065 \times 2 \cdot 0$

$= \cdot 13 \qquad = \cdot 13$

If there is no magnetic leakage or $v = 1$

$M = 1 \cdot 25 \times 10^4 \times 10^3 \times 10^2 \times 10^{-2} \times 10^{-8}$

$= \cdot 125$ Henry.

$E_{02} = 314 \times \cdot 255 \times \cdot 125 = 10$ volts.

$E_1 \times \frac{n_2}{n_1} = 100 \times \frac{10^3}{10^4} = 10$ volts.

Description of the Diagrams.

I. This is simply the transformer, without magnetic leakage, on open circuit.

II. This diagram serves two purposes.

Firstly.—It shows the effect of the iron losses, which are given quite arbitrary values.

In working out the iron losses, the total iron losses current, taken as usual, as being in phase with $-p\,L_1\,C\mu$ is multiplied by the voltage and by the cosine of the angle between them, which is 1°.

Secondly.—It shows why a transformer which has no leakage acts as a non-inductive circuit to the secondary current and the primary load current.

The secondary current C_2 produces in the secondary a counter E.M.F. of self-induction $= p\,L_2\,C_2$, but this is neutralised by an E.M.F. of mutual induction $= p\,M\,C_2\,\frac{n_2}{n_1}$ produced by the primary working current $= C_2\,\frac{n_2}{n_1}$

Similarly in the primary, there is a counter E.M.F. $= p\,L_1\,C_2\,\frac{n_2}{n_1}$ but it is neutralised by an E.M.F. of mutual induction $= p\,M\,C_2$.

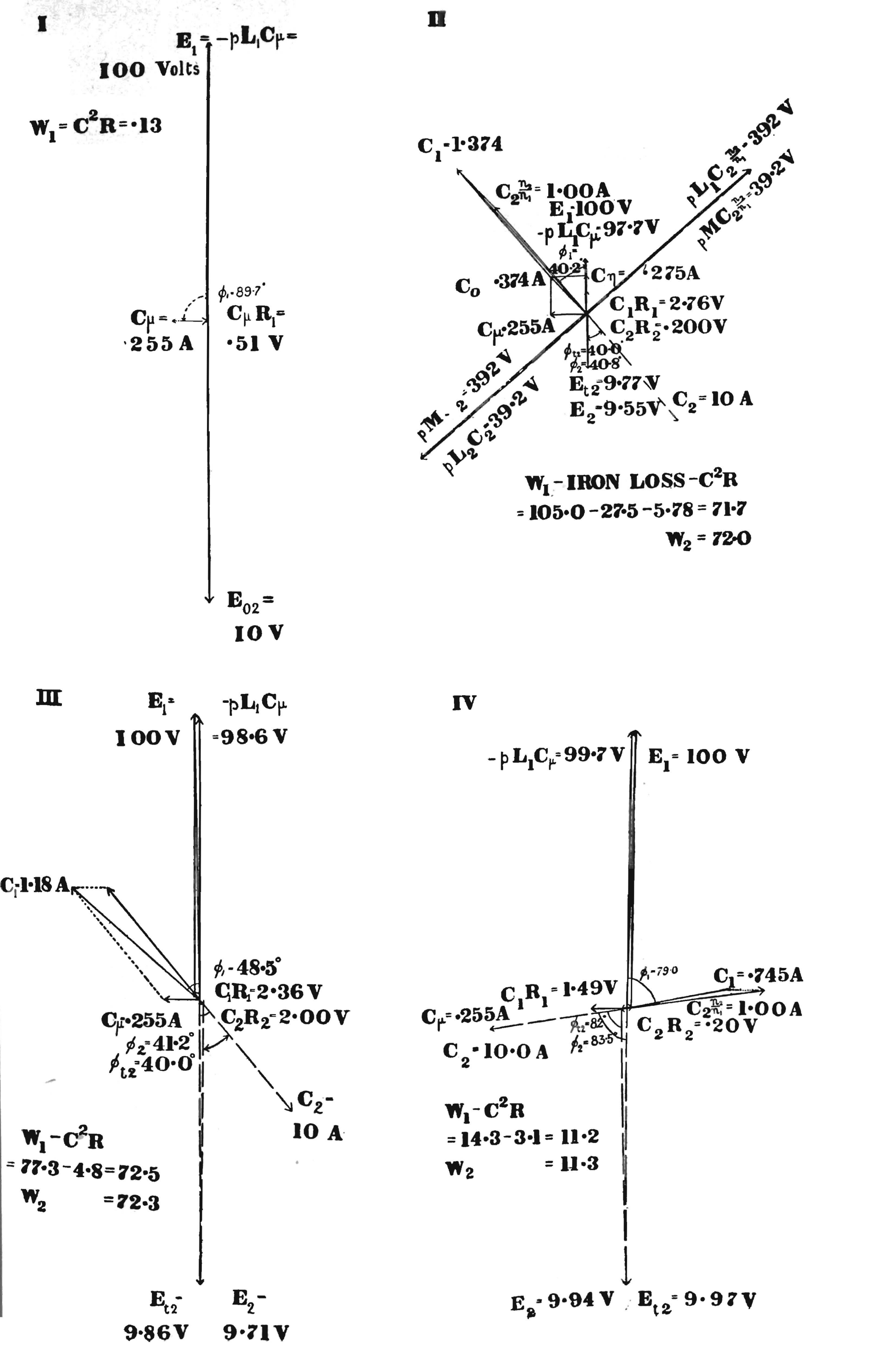
III. Shows a transformer without leakage working on an inductive load.

IV. Shows a transformer working with a leading secondary current.

V. Shows a transformer working on short circuit, where there is no leakage. In this case the voltage is reduced in the primary to that necessary to get the same secondary current of 10 amperes, as in the other diagrams. It will be noticed that $C\mu$ is negligibly small.

VI. Shows a transformer, with $v = 1 \cdot 1$ working on an inductive load.

It will be noticed that E_{02}, the secondary voltage on open circuit, is no longer $= p\,L_1\,C\mu \times \frac{n_2}{n_1}$ but is $p\,L_1\,C\mu\,\frac{n_2}{n_1} \times \frac{1}{v}$ or $p\,L_1\,C\mu\,k$.



I
E1 = -pL1Cμ = 100 Volts
W1 = C²R = ·13
φ1 = 89·7°
Cμ = ·255 A
CμR1 = ·51 V
E02 = 10 V
II
C1 = 1·374
C2 n2/n1 = 1·00 A
E1 = 100 V
-pL1Cμ = 97·7 V
φ1 = 40·2°
pL1C2 n2/n1 = 392 V
pMC2 n2/n1 = 39·2 V
Co ·374 A
Cη = ·275 A
C1R1 = 2·76 V
C2R2 = ·200 V
Cμ = ·255 A
φt2 = 40·0
φ2 = 40·8
Et2 = 9·77 V
E2 = 9·55 V
C2 = 10 A
pM . 2 = 392 V
pL2C2 = 39·2 V
W1 - IRON LOSS - C²R
= 105·0 - 27·5 - 5·78 = 71·7
W2 = 72·0
III
E1 = 100 V
-pL1Cμ = 98·6 V
C1 = 1·18 A
φ1 = 48·5°
C1R1 = 2·36 V
C2R2 = 2·00 V
Cμ = ·255 A
φ2 = 41·2°
φt2 = 40·0°
C2 = 10 A
W1 - C²R
= 77·3 - 4·8 = 72·5
W2 = 72·3
Et2 = 9·86 V
E2 = 9·71 V
IV
-pL1Cμ = 99·7 V
E1 = 100 V
φ1 = 79·0
C1 = ·745 A
C1R1 = 1·49 V
C2 n2/n1 = 1·00 A
Cμ = ·255 A
φt2 = 82
φ2 = 83·5
C2R2 = ·20 V
C2 = 10·0 A
W1 - C²R
= 14·3 - 3·1 = 11·2
W2 = 11·3
E2 = 9·94 V
Et2 = 9·97 V

There are two points about this diagram which demand attention.

(*a*) The first is that as $p\ L_1\ C\mu$ is now considerably smaller on load than an open circuit, $C\mu$ should be proportionately reduced. This has been done in the diagram, but it is a point neglected in most books.

Really $C\mu$ should be reduced in all cases, because $-\ p\ L_1\ C\mu$ is reduced by the resultant along its line, of $C_1\ R_1$, but this is generally very small, and, in the cases where there is no leakage this has been neglected.

(*b*) The second is that in the secondary $p\ L_2\ C_2$ is now larger than $p\ M\ C_2\ \frac{n_2}{n_1}$ so that the secondary has some self-induction, and there is in it an E.M.F. of self-induction $= p\ L^1{}_2\ C_2$ where $L^1{}_2$ is the "equivalent self-induction" $= L_2 - M\ \frac{n_2}{n_1}$ and there is similarly on "equivalent self-induction" in the primary $L^1{}_1 = L_1 - M\ \frac{n_1}{n_2}$

(*c*) The primary working current is not $C_2\ \frac{1}{k}$ but $C_2 \times \frac{n_2}{n_1}$. It is generally given as $C_2\ k$, but cannot be as $p\ L_1\ C_2\ \frac{1}{k} = 357$ which is the same as $p\ M\ C_2$, when there would be no counter E.M.F. due to leakage in the primary.

And similarly in the secondary $p\ L_2\ C_2$ is $39{\cdot}2$ just as $p\ M\ C_2\ \frac{1}{k}$ is.

That my method of drawing this diagram is right, is shown by the usual test $W_1 - C_2\ R = W_2$, when there is only a small error of ·25 in 44·05 or ·56 %.

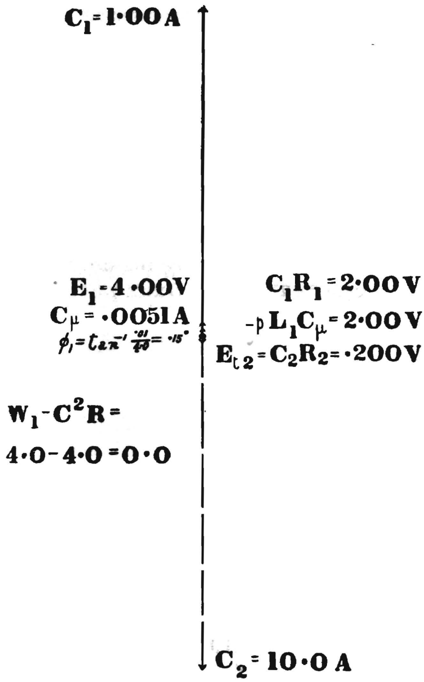
VII. Shows the ordinary method, as given in books, of drawing this same diagram and, in this case alone amongst the set, $W_1 - C_2\ R$ is not equal to W_2. The error is 3·3 in 40 = 8·25 %.

There was a difficulty in drawing this as the books do not give any method of finding the primary and secondary counter E.M.Fs. due to leakage.

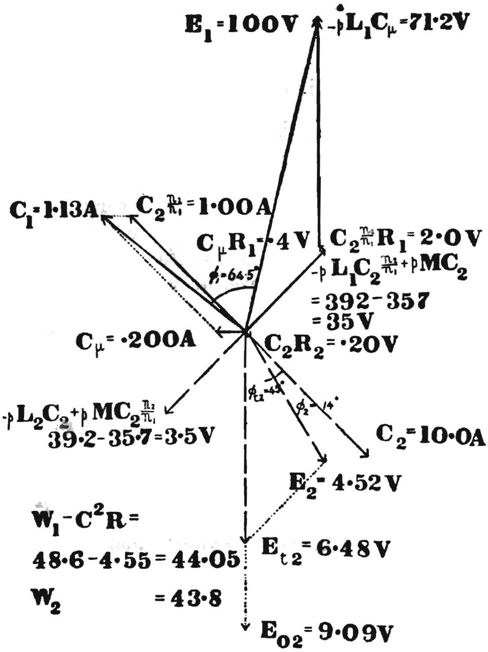
Prof. Sylvanus Thompson, for instance, merely says (Dynamo Electric Machinery, 1896 edition, p. 710), "we must take into account the unbalanced self-induction (if any) etc.

C. E. Lamb (Alternating Currents) gives a method, but it is very cumbrous, and his is the only book in which I have found a correct diagram (p. 64).

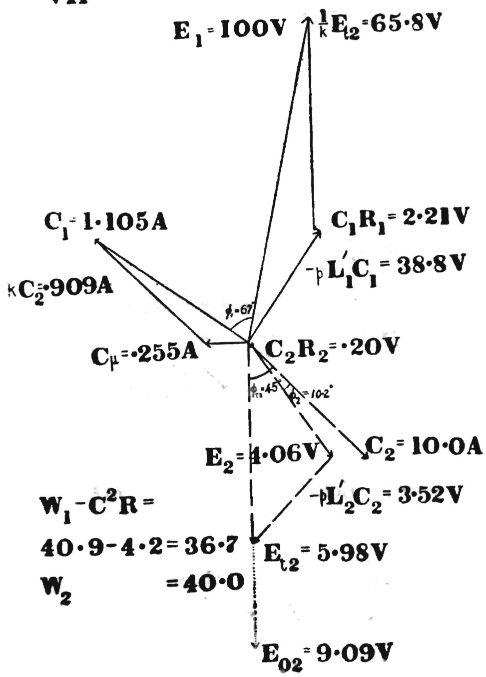
V


C1 = 1·00 A
E1 = 4·00V
Cμ = ·0051 A
φ1 = tan⁻¹ ·01/·60 = ·15°
C1R1 = 2·00 V
-pL1Cμ = 2·00 V
Et2 = C2R2 = ·200 V
W1 - C²R =
4·0 - 4·0 = 0·0
C2 = 10·0 A

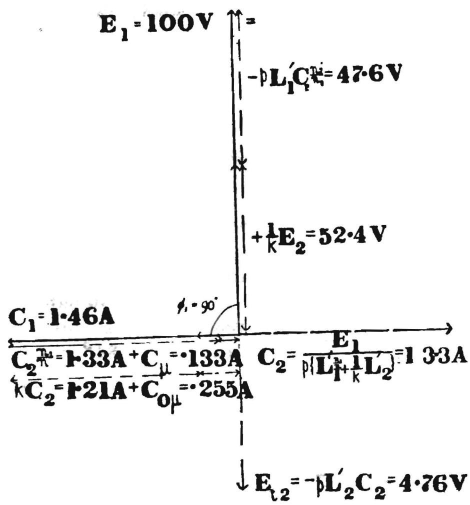
VI


E1 = 100V
-pL1Cμ = 71·2V
C1 = 1·13A
C2 n2/n1 = 1·00A
CμR1 = ·4 V
C2 n2/n1 R1 = 2·0 V
φ1 = 64·5°
-pL1C2 n2/n1 + pMC2
= 392 - 357
= 35 V
Cμ = ·200A
C2R2 = ·20V
-pL2C2 + pMC2 n2/n1
39·2 - 35·7 = 3·5V
φt2 = 45°
φ2 = 14°
C2 = 10·0A
E2 = 4·52 V
W1 - C²R =
48·6 - 4·55 = 44·05
W2 = 43·8
Et2 = 6·48 V
E02 = 9·09V

VII


E1 = 100V
1/k Et2 = 65·8V
C1 = 1·105A
kC2 = ·909A
C1R1 = 2·21V
-pL′1C1 = 38·8V
φ1 = 67°
Cμ = ·255A
C2R2 = ·20V
φt = 45°
φ2 = 10·2°
E2 = 4·06V
C2 = 10·0A
-pL′2C2 = 3·52V
W1 - C²R =
40·9 - 4·2 = 36·7
W2 = 40·0
Et2 = 5·98V
E02 = 9·09V

VIII


E1 = 100V
-pL′1C1 n2/n1 = 47·6V
+1/k E2 = 52·4V
C1 = 1·46A
φ1 = 90°
C2 n2/n1 = 1·33A + Cμ = ·133A
kC2 = 1·21A + C0μ = ·255A
C2 = E1 / p(L′1 + 1/k L′2) = 1 33A
Et2 = -pL′2C2 = 4·76V

For the reason given in VI. (*c*) above, if the primary working current is taken as $C_2 \frac{1}{k}$, there is no counter E.M.F. due to leakage, and yet this is the value usually taken.

So I simply took $L^1{}_1$ and $L^1{}_2$ as the same values as in VI.

VIII. Shows the transformer working on short circuit with leakage but with no resistance. The secondary angle of lag inside the transformer is then 90° and the current is the maximum possible with $E_1 = 100$ volts.

In this case it should be nated that:—

(*a*) $C_1 = C_2 \frac{n_2}{n_1} + C\mu$, where $C\mu$ is the reduced value.

(*b*) $C_1 = k\,C_2 + C_0\mu$, where $C_0\mu$ is the no load value.

(*c*) $C_1 = k\,C\mu$ where $C\mu$ is the reduced value.

IX. Is the same transformer with leakage and resistance, working on short circuit with the normal secondary current of 10 amperes. The secondary angle of lag, inside the transformer, is then fixed as its tangent must be $\frac{p\,L^1{}_2}{R_2}$

X. Shows a simplified diagram, everything being transferred to the primary side. A method which can be adopted in each of the other cases. $C\mu$ is neglected in this diagram, being very small, and the primary part of the drawing is the triangle which Dr. D. R. Morris and Mr. G. A. Lister (Journal Inst. E.E., vol. 37, p. 265), call the "Characteristic Triangle" of the transformer.

XI. Shows how the characteristic triangle can be used to determine approximately the regulation of the transformer for any current and angle of phase difference.

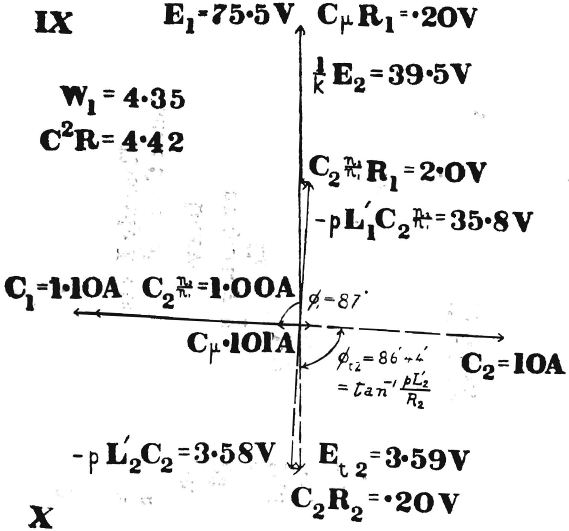
The primary voltage is taken as 100 volts. Circles for secondary current 10 amperes, 5 amperes, and 0 amperes are drawn, and the values of E_2 for 10 amperes, and the following angles of phase difference are shown: 0° lag, 14·2° lag, (the same as in VI., giving practically the same result. It is not quite the same because $C\mu$ is here neglected), 37° lag (power factor ·80) and 37° lead.

The last shows how the secondary voltage may rise when working a leaky transformer on a circuit containing capacity.

I have just lately been able to do a test which completely confirms my theories.

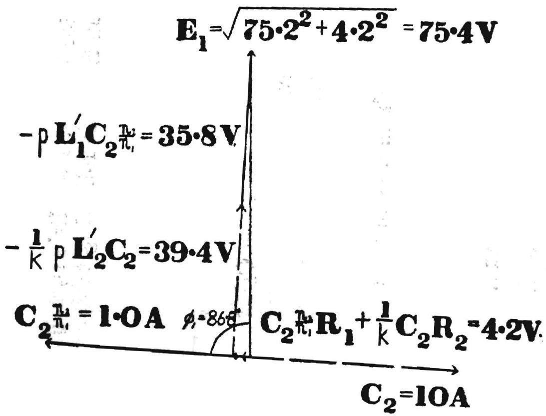
The two coils before you form a leaky transformer which is very suitable for this purpose, and I drew out the diagram for them working in short circuit with the fixed coil as the primary some time ago.

IX



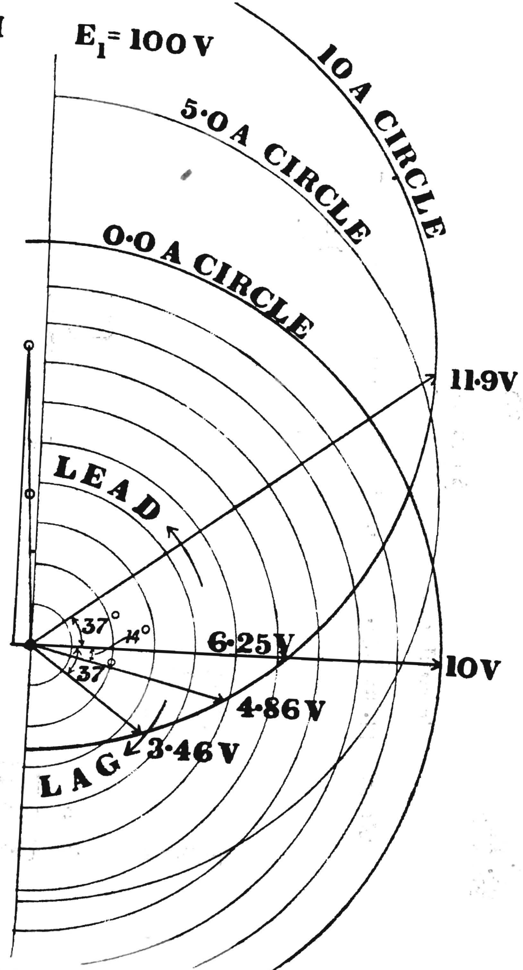
E1=75·5V
CμR1=·20V
1/k E2=39·5V
W1=4·35
C²R=4·42
C2 n2/n1 R1=2·0V
-pL'1C2 n2/n1=35·8V
C1=1·10A
C2 n2/n1=1·00A
φ1=87°
Cμ·101A
φt2=86°+4'
=tan⁻¹ pL'2/R2
C2=10A
-pL'2C2=3·58V
Et2=3·59V
C2R2=·20V

X

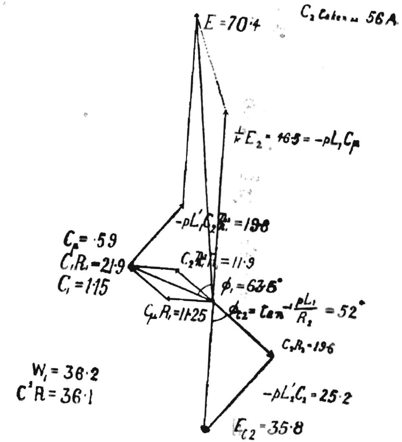


E1=√(75·2²+4·2²)=75·4V
-pL'1C2 n2/n1=35·8V
-1/k pL'2C2=39·4V
C2 n2/n1=1·0A
φ1=86·8°
C2 n2/n1 R1+1/k C2R2=4·2V
C2=10A

XI



E1=100V
10A CIRCLE
5·0A CIRCLE
0·0A CIRCLE
11·9V
LEAD
37°
14°
6·25V
10V
37°
4·86V
3·46V
LAG



E=70·4
-pL'2C2 n2/n1=19·8
Cμ=·59
C1R1=21·9
C1=1·15
C2 n2/n1=11·9
φ1=63·5°
CμR1=11·25
φt2=tan⁻¹ pL1/R1=52°
C2R2=19·6
W1=36·2
C²R=36·1
-pL'2C2=25·2
Et2=35·8

Recently I was able to test them under the same conditions, and found the result to agree admirably as seen from the following table.

	Drawing.	Test.
Frequency	50	51·3
E_1	70·4	68
C_1	1·15	1·15
C_2	·56	·56 (worked out from $W_1 - C_1^2 R_1$)
$C_1^2 R_1 + C_2^2 R_2$	36·1	(36·1 not measured)
Watts, W_1	36·2	(36·1 lag wattmeter)
ϕ_1	63·5°	62·8°

Note.—The frequency was 2·6% too high in the test and the voltage was 3·3% too low.

Particulars of Experimental Transformer.

	Coil used as primary.	Coil used as secondary.
Resistance ..	19·0 ohms.	35·0 ohms.
Wire ..	733 turns. No. 20.	820 turns. No. 22.
Co-efficient of Self-Induction	·250 Henry.	·334 Henry.
Co-efficient of Mutual Induction	·171. Coils close together.	

$$\text{Leakage co-efficient} = \frac{\sqrt{L_1 L_2}}{M} = 1{\cdot}69.$$

M is obtained, the mean of three experimental valves for $p\,M\,C\mu$.

The leakage co-efficient obtained experimentally as the ratio of $-p\,L_1\,C\mu$ to $p\,M\,C_2$ came out practically the same as the above, viz., 1·67.

$$L_1{}^1 = ({\cdot}250 - {\cdot}171 \times \tfrac{733}{820}) = ({\cdot}250 - {\cdot}153) = {\cdot}097 \text{ Henry.}$$

$$L_2{}^1 = ({\cdot}334 - {\cdot}171 \times \tfrac{820}{733}) = ({\cdot}334 - {\cdot}191) = {\cdot}143 \text{ Henry.}$$

On open circuit, with 68 volts on the primary, the primary current $C_0\mu$ is ·85 amperes, and ϕ_1 is 76°.

On short circuit with 70·4 volts on the primary, the primary current $C_1 = 1{\cdot}15$, and $C\mu$ is reduced to ·59 amperes, C_2 being ·56 amperes.

Conclusions.

I think that I have proved conclusively that :—

(1) In a leaky transformer $E_{t2} = -p\,L_1\,C\mu \times k$ and not $-p\,L_1\,C\mu \times \frac{n_2}{n_1}$ (as given in many books, including Kapp.)

(2) In a leaky transformer the primary load current $= C_2\frac{n_2}{n_1}$ and not $C_2\,k$.

(3) In a leaky transformer $C\mu$ should be reduced on load, as $p\ L_1\ C\mu$ is reduced.

(4) It is quite unnecessary to introduce any lines in the diagrams to show magnetic fluxes, which lines only complicate the diagram.

List of Books Consulted.

Transformers—Kapp.
The Alternate Current Transformer—Fleming.
Alternate Current Transformer—Baum.
Alternating Currents—Franklin and Williamson.
Alternating Currents and Alternating Current Machinery—Jackson
Alternating Currents—Lamb.
Practical Alternating Current—Smith.
Alternating Currents—Hay.