THE ALTERNATING CURRENT REPULSION MOTOR.

AN ANALYTICAL DISCUSSION.

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In dealing with the phenomena connected with the operation of alternating current motors of the commutator type, it must constantly be borne in mind that the machine posseses simultaneously the electrical characteristics of the direct current motor and the stationary alternating current transformer. This statement must not be confounded with a somewhat similar statement often made concerning polyphase induction motors, since only with regard to its mechanical characteristics can an induction motor be said to have any resemblance to a shunt-wound, direct current motor, its electrical characteristics being equivalent in all respects to those of a stationary transformer.

Before discussing the performance of the repulsion motor it will be well to investigate the properties common to all commutator type alternating current machines. It will be remembered that when current flows through the armature of a direct current machine, magnetism is produced by the m.m.f. of the armature current, such magnetism tending to distort the flux from the main field poles. In the familiar representation of the magnetic circuit of machines—the two-pole model—the armature magnetism is at right angles to the field magnetism, the armature current producing poles in line with the brushes, as generally arranged at the centre of the interpolar arc. The strength of the magnetism depends directly on the value of the armature current and the permeability of the path.

When alternating current is used we have introduced the transformer effect, viz., an alternating e.m.f. is produced proportional to the rate of change of magnetism, and hence in lagging time quadrature with it, and the current producing it.

Under whatsoever conditions the armature conductors be subjected to an alternating flux, a corresponding e.m.f. will be generated in mechanical line with the flux and in time quadrature to it.

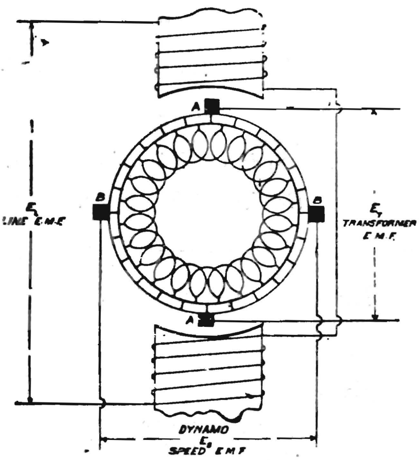

FIG. 1 - ELECTROMOTIVE FORCES PRODUCED IN AN ALTERNATING FIELD

Figure 1 represents a direct current armature situated in an alternating field, having two sets of brushes on the commutator, one in mechanical line and the other in mechanical quadrature with the alternating flux.

When the armature is stationary and an alternating e.m.f. is applied to field terminals, a corresponding e.m.f. will be induced in the armature between the brushes A.A., while between the brushes B.B. there will be no e.m.f. induced. Any change in position of the armature will not have any effect on the value of the e.m.f. induced between A.A.

When the armature is in motion there is an e.m.f. induced between the brushes B.B. by the armature conductors cutting the field flux, which is proportional to the speed and the flux and in time phase with the flux.

At a certain speed this speed e.m.f. will equal that due to the transformer action, and we call this speed synchronous speed. This is the same speed as that so termed in other machines. It is characterised by the fact that any movement of the brushes around the commutator will not alter the e.m.f. between them in value. This is on the assumption of sine distribution of flux in the air gap.

The maximum transformer e.m.f. is induced in the coils short-circuited by the brushes B.B., while the max speed e.m.f. is induced in those short circuits by the brushes A.A. Calling the e.m.f. in coil under A.E., the value of the e.m.f.

in any coil at an angle a from the line A.A. due to transformer action

$$= \pm \quad E \sin a$$

Similarly that due to motor action

$$= \pm \quad E \cos a$$

The total voltage induced in this coil is hence

$= E$ (since transf. and motor E. M. F. are introduced.)

And is the same for any coil being independent of the value of the angular displacement from A.

The time phase relation will, however, vary with the mechanical position of the coil.

From these facts it is seen that at synchronous speed the effective value of the e.m.f. in every coil is the same, and that there is no neutral e.m.f. position on the commutator.

In a repulsion motor, as commercially constructed, the secondary consists of a direct current armature, on the commutator of which brushes are placed in positions 180 degrees (electrical) apart, and directly short-circuited upon themselves, as illustrated in the two-pole model, figure 2. The stationary

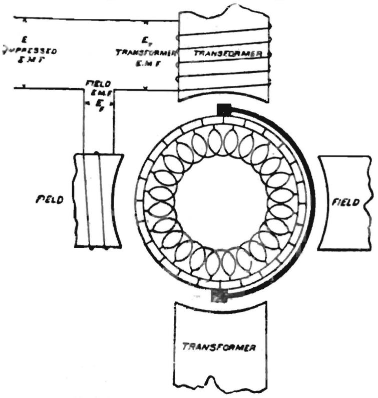

FIG. 2 — TWO-POLE MODEL OF IDEAL REPULSION MOTOR

primary member consists of a ring core, containing slots more or less evenly spaced around the air gap. In these slots are placed coils so inter-connected that when current flows through them definite poles will be produced upon the field core. The brushes on the commutator are given a location some 15 degrees from the line of the primary magnetism, or, more correctly expressed, are given a lead of 15 degrees ahead of the proper transformer position. That component of magnetism which is in line with the brushes produces current in the armature by transformer action, and this current gives rise to a torque, due to the presence of the other component of magnetism, in mechanical quadrature, with the secondary current.

The simplest method of treatment involves the assumption that the stator winding is in two parts, one in mechanical line and the other in mechanical quadrature with the transformer axis, as shown in figure 2, with the brushes set in the true transformer axis.

Owing to the fact that the iron must be worked at very low magnetic densities, we are quite justified in assuming that the permeability of the magnetic path will be constant, and hence that the m.m.f. and flux will be proportional.

If we call ϕt the transformer flux and ϕf the main field flux, the effective angular displacement of the brush axis from the transformer axis is given by the relation

$$\text{Cotan. } \delta = \phi t, \div \phi f.$$

where δ = angular displacement of the brushes from the transformer axis.

Now, if we represent the ratio of turns on the transformer coil to those on the field coil by N

$$\text{Cotan. } \delta = N$$

and is independent of the current in the coils.

Passing now to the repulsion motor conditions at starting, with an e.m.f. impressed on the primary = E.

The armature acts as short-circuited secondary of a transformer, of which the transformer coil of the stator winding is the primary.

For the sake of simplicity in treatment here we assume that there is no resistance or leakage reactance in the armature of the motor.

The ampere turns of the armature will in this case exactly neutralise those of the primary, and the full applied e.m.f. is impressed on the field coil. That is to say—

$$Et = O \text{ and } Ef = E.$$

Hence If = E $\div$ X, where X is the inductive reactance of the field coil.

Now consider the effect of speed on the e.m.f. distribution. Its primary effect is to introduce into the circuit of the armature an e.m.f. proportional to the speed S, and the field flux ϕf in time phase with the field flux.

This e.m.f. will tend to force a current through the path provided by the short-circuited brushes, which will be limited only by the self-induction of the armature, since we have assumed negligible resistance and leakage. In other words, the flux produced by this current will be just sufficient to produce by its rate of change an e.m.f. equal and opposite to that due to rotation. The direction of this flux will obviously be in line with the short-circuited brushes, and will thus thread the transformer coil, and its time phase will be 90 degrees behind the stator current. At synchronous speed the transformer flux thus produced must be equal in magnitude to the field flux, since the frequency of alternation is in each case the same, and the e.m.f. produced by the transformer flux must exactly neutralise the speed e.m.f. The rate of cutting of the flux by rotation is, at synchronous speed, equal to the rate of cutting of the transformer flux, and to produce equal e.m.f. these relations call for equal fluxes. The time phase of the two fluxes is obviously in quadrature, and is independent of the speed. The ratio of effective values is unity at synchronous speed, and directly proportional to the speed.

Giving to synchronous speed a value of unity, the transformer flux may be expressed at any speed, S, as

$$\phi t = S.\phi f.$$

Now, if at a speed S the value of the e.m.f. across the field coil be called F, requiring the flux Qf, then through the transformer coil there will be an effective flux $= S\phi f$, and a corresponding voltage induced.

$$T = S.E.N.$$

where N is the ratio of the transformer coil turns to those on the field coil. Thus the total applied e.m.f. of the motor

$$E = \sqrt{T^2 + F^2}$$

since, as seen above, the fluxes are in quadrature. This is the fundamental equation of e.m.f.

The current in the armature consists, as seen, of two components:—

1. Due to the transformer action of the transformer coil, of which the armature is a short-circuited secondary, there is a current inversely proportional to the ratio of armature turns to transformer coil turns. If we call the ratio of armature to field turns, *a*, and giving, *n*, the same value as above

$$I_1 \times a = I \times n \quad \text{or } I_1 = I \times \frac{n}{a}$$

where I is the stator current.

2. The current produced by the speed e.m.f., and being the cause of the flux threading the transformer coil $= I_2$, whose m.m.f. is equal to that of the field coil at synchronous speed, and at any other speed S is given by the relation

$$I_2 \times a = S.I.$$

These two are in quadrature, and their geometric sum is the resultant armature current,

$$I_a = \sqrt{I_1^2 + I_2^2}$$

This analytical treatment of the actions in the ideal motor seems the best way to obtain a good idea of what actually happens in the motor. The actual treatment is modified and greatly complicated by the introduction of the effect of resistance in stator and rotor and mutual leakage reactance.

Bearing in mind the actual happenings as above, we can introduce the effect by a graphical treatment, with very little complication.

First let us construct our diagram for an ideal motor—at constant current—neglecting the effect of resistance and leakage reactance and iron losses.

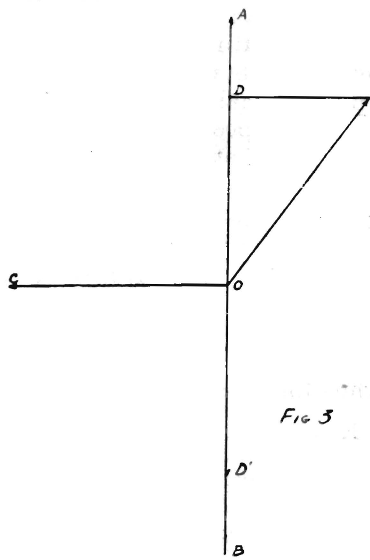

FIG 3

In figure 3 we have the m.m.f. of the transformer coil shown by vector OA. That of the armature due to the transformer action of this flux is shown by OBm, which is equal and opposite to that of the transformer coil OA. This leaves no resultant m.m.f. If we, however, introduce the effect of speed we have another m.m.f. in lagging time phase to OA by 90 degrees, which is indicated by OC, the rotation of vectors being in a clockwise direction.

The cause of this lag is obvious, since the voltage induced by rotation of the armature in the field flux is in phase with the stator current OA, and as the only limitation on the armature current is the self-induction of the armature, the current must lag 90 degrees behind its voltage.

The resultant of the three m.m.f.'s is obviously OC, resulting in a flux in the direction OC, threading the transformer coil and inducing therein a voltage OD', in phase opposition to OA, which calls for an applied voltage of OD. The self-induction of the field coil causes a drop of voltage ED, and required an applied voltage DE. The resultant voltage is thus represented by OE.

At rest the vector OD = 0, and hence OE = DE.

From this it is seen that at rest the repulsion motor acts like a choke coil.

Further, we see that the armature current at rest is proportional to the stator current, as is also the field flux, hence the torque is proportional to square of the field current, since the armature current and field flux are in phase. Thus we see that the repulsion motor has serious characteristics at starting.

In the above we have the ideal conditions, which are impossible of realisation in practice, and we have to modify our diagram considerably, to take into account other matters.

First let us take into account the effect of leakage on the ideal diagram.

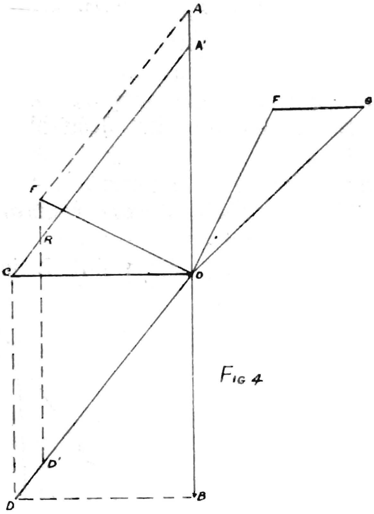

Fig 4

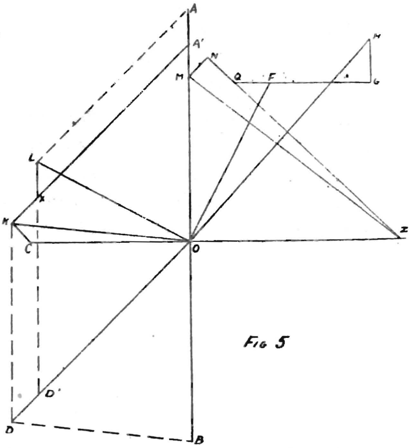

Fig 5

Referring to figure 4, OA represents the m.m.f. of the transformer coil. Only a part of this penetrates the armature, and is indicated by OA'. The counter m.m.f. of the armature is represented by OB, equal and opposite to OA'. OC is, as before, the rotation m.m.f., slightly reduced in value, since all the field flux does not pass through the armature. OC and OB have a resultant OD, which is the total armature m.m.f. Of this only a part OD' cuts the primary, and combining with OA produces the resultant primary flux OE. The voltage diagram is completed as in figure 3.

Now, in addition, let us consider the resistance of the armature. The voltage drop is in phase opposition to the current, and hence in the armature in the direction DO. To overcome this we require a voltage in the direction OD, which can only be produced by an increase in flux at right angles to OD ÷ leading quadrature.

This is represented by CK, figure 5, where the remaining vectors are as above. The total aramature flux is given by the resultant of OBm, OC, CK = OD. Only OD' threads the primary, so that the resultant flux through the transformer coil is given by OL.

The voltage diagram can now be constructed:—

OL calls for a voltage applied, represented by .. OF

The self-induction of the field coil requires .. FG

The stator ohmic drop requires .. .. .. GH

The voltage OF may be resolved into its components OM, MN, NQ, and QF, if so desired, where

OM is the e.m.f. induced by the flux OB.

MN is the ohmic voltage required by armature

NQ is the self-induction voltage of armature.

QF is the additional stator self-induction, due to leakage.

QF, FG, and GH, at a given stator current, are independent of the speed. OM is directly proportional to the speed. MN and NQ increase with the speed, but not in direct proportion.

In figure 5 produce NQ and BO to meet in Z. Join MZ.

The triangle ZMN is similar to AKC.

Since the form of the triangle does not depend on the motor speed, the angles of the triangle ZMN always remain the same.

Thus the point M moves along OA, but the point Z is *fixed* with change of speed.

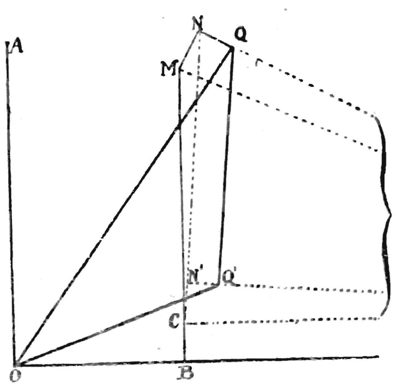

FIG. 6

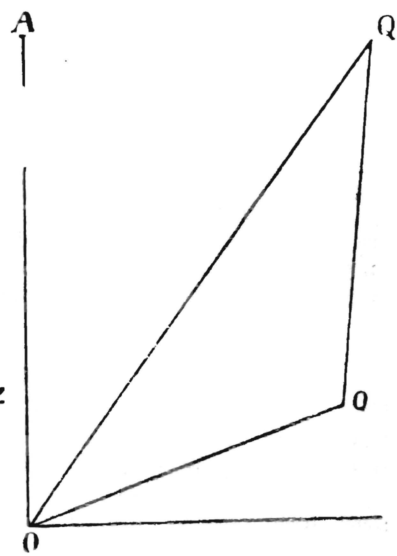

FIG. 7.—Simple diagram of the repulsion motor for a constant current, no account being taken of iron and friction losses and of the reaction of the commutating coils.

We may now construct our final diagram, as in figure 6, in which OA represents stator current and OB the self-induction of stator winding. (= QF + GF, figure 5.) And BC is the ohmic drop of voltage in the stator winding.

To complete. Through C draw a line perpendicular to OA, and make CZ = OZ, of figure 5.

Construct the figure ZCMNQ, as in figure 5, making MN = ohmic drop, and NQ the self-induction of the armature.

At standstill the triangle will have the position ZC NQ, instead of ZMNQ, so that OQ is the voltage requisite to produce the current OA in the stator.

As the machine speeds up the point M moves upwards along BM, Z remaining stationary.

It is easily seen that as the speed increases, with MN and NQ increasing, the point N moves along a straight line CN^1N, and Q along the line Q^1Q. The point Q is the one that interests us most, since OQ represents to terminal voltage at any speed.

The line CM, as mentioned above, is directly proportional to the speed, and it is easily seen that QQ^1 is also a measure of the speed for from similar triangles.

$$QQ^1 = \frac{QZ}{MZ} CM$$
$$= \text{Constant} \times CM.$$

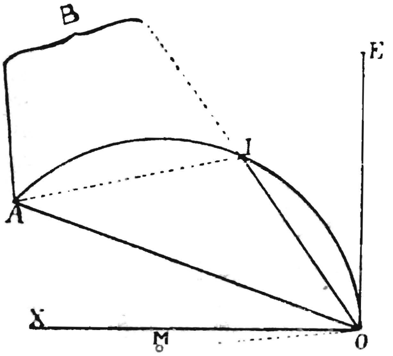

Fig. 8.—Diagram of the repulsion motor for a constant terminal voltage. This diagram takes no account of the iron and friction losses and of the reaction of the short-circuited coils.

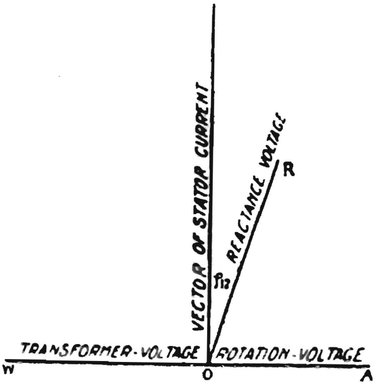

Fig. 9.—The voltages induced in the comutating coils of a repulsion motor.

From this we have the simple diagram of the repulsion motor, figure 7.

OA = Stator current.

OQ^1 = Voltage required to force OA through the motor at rest.

QQ^1 = A distance proportional to the speed U.

OQ = Voltage required to force current OA through motor at a speed U.

It is now a simple matter to derive the diagrams for a constant voltage and variable speed and current.

Let OE, figure 8, represent this constant terminal voltage, and OA represent the short-circuit current at this voltage. The difference in phase between the two is represented in both figures by the same angle,

Angle AOQ^1 figure 7 = angle AOE figure 8.

Draw the triangle AOQ similar to the triangle OQQ^1, then OE and OJ form the same angle as OQ and OA, and furthermore

$$\frac{OE}{OJ} = \frac{OQ}{OA}$$

from the similar triangles OQA and OJE.

As the angle OJA = OQ^1Q = Const., the locus of J is a circular arc whose centre a somewhere below the line OX. The angle MOX = angle between OA and QQ^1.

Speed is proportional to AB, since it is proportional to

$$\frac{QQ^1}{OQ} = \frac{AJ}{OJ} = \frac{AB}{OA} = \text{Constant} \times AB.$$

It would not be difficult to develop other diagrams from figure 8, to show the effect of varying the conditions in all kinds of manners, but such complicated diagrams are of little practical use. Even the above diagram is rather too complicated, when it is remembered that the diagram is based on the assumption that the fluxes are proportional to the m.m.f.'s, and it becomes evident that it is generally preferable to use the simple diagram of figure 4, and to calculate the current for a few points. We may now investigate the sparking voltage of a repulsion motor in its relation to the load.

We have in the coils short-circuited by the brushes three component voltages, viz.:

1. The transformer voltage, independent of the speed and only affected by the value of the flux from the field coil and its frequency. It lags 90 degrees behind the stator current. = OW, figure 9.

2. The reactance voltage, proportional to and in phase with the armature current and proportional to the speed. = OR.

3. The rotation voltage, due to the rotation of the short-circuited coils through the transformer flux. It is proportional to the field flux and the speed = OA.

On the assumption that the field and transformer flux both follow a sine law—this is not actually the case, but gives results sufficiently close for commercial purposes, the real wave shape being somewhat peculiar—we find, figure 9:—

$$OW = 4.44 \; zNM_f \; 10^{-8}$$

$$OA = 4.44 \; zN^1M_t \; 10^{-8}$$

Now, Mf gives rise in the armature winding to an effective rotation voltage

$$=;\frac{4}{1.414} sN^1M_f\ 10^{-8}$$

and Mt gives rise to a transformer voltage—effective—

$$=\frac{4.44 \times 2}{} sN\ M_t\ 10^{-8}$$

Hence very nearly

$$N^1M_f = N\ M_t$$

or

$$M_t = \frac{N^1}{N} M_f$$

Introducing this into the above questions, values for OW and OA, we get the relation

$$OA = (\frac{N_1}{N})^2\ OW$$

Hence when the speed is synchronous, we have $N_1 = N$ or

$$OA + OW = 0$$

and at this point the sparking voltage is equal to the reactance voltage. At speeds below synchronism OW is the larger, and at speeds above it is OA that predominates.

Hence the sparking voltage and its phase angle with the stator current are, therefore, different at every speed.

It is interesting to plot the sparking voltage against the load and stator current respectively, as in figures 10 and 11.

To this end figure 3 may be used by assuming various values of current, and finding the corresponding speeds. With the data of the motor given we can easily calculate OW and OA from the equations above.

The only remaining quantity to be found is the reactance voltage.

Let the angle between the stator and armature currents (= angle CA O in fig. 4) denoted by the symbol ϕ_{12} ; then

$$J_2 = J_1 \div \text{Cos}\ \phi_{12}$$

and

$$\text{Tan}\ \phi_{12} = OC \div OA\ (\text{fig. } 4.)$$

As has been mentioned previously, at a speed which we call synchronous (cp., figure 2), the rotation voltage is equal to the transformer voltage—with equal field and transformer fluxes—so that it is obvious that the ratios of

$$\frac{\text{Turns on field coil}}{\text{Turns on transformer coil}} = \frac{OC}{OA^1}$$

If we denote this ratio by Tan a
Then

$$\text{Tan } \phi_{12} = \text{Tan } a \text{ at Synchronous speed}$$

and $$\text{Tan } \phi_{12} = (\text{ N } \div \text{ N }) \text{ Tan } a \text{ at any other speed.}$$

Hence we obtain the relation between the primary and secondary currents

$$J_2 = J_1 \sqrt{1 + \left(\frac{N^1}{N} \text{Tan } a\right)^2}$$

From this the magnitude of the reactance voltage can be calculated when the slot constants are known. Its phase position is known when the angle is given. Figures 10 and 11 are obtained in this way. In the former the absolute values only are plotted, no notice being taken of their relative phase positions.

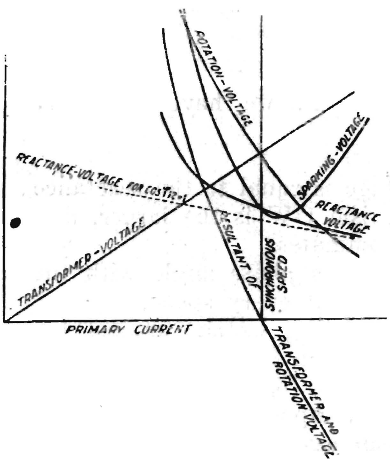

FIG. 10.—Diagram indicating the variation of the sparking voltage of a repulsion motor with the stator current.

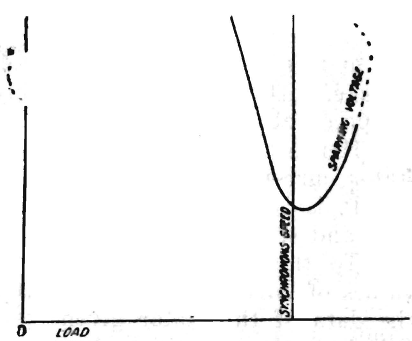

FIG. 11.—Diagram showing how the sparking voltage of a repulsion motor varies with the load.

In deriving the above equations we have neglected any effectual saturation of the magnetic paths, when which occurs the values must be calculated from the characteristic curves.

The reaction of the short-circuited coils plays a much more important part in the repulsion motor than in the series motor, since its effect on the power factor is good or bad, according to the speed. For speeds below synchronous speeds, the sparking voltage lags behind the armature current, and also behind the stator current—figure 9—and this has the effect of increasing the power factor. But when the speed exceeds synchronous the sparking voltage leads the stator current, and thus has the effect of shifting the field flux forward.

Figure 12 is a diagram of the repulsion motor at speeds below synchronism, account being taken of the reaction of the short-circuited coils upon the field. The dotted lines correspond to the vectors in figure 5—upper part only being reproduced.

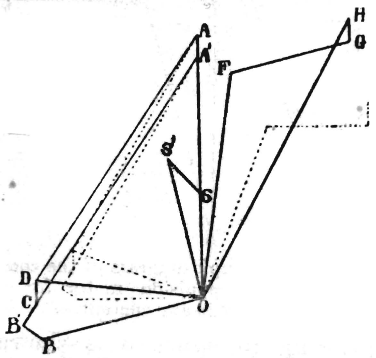

FIG. 12.—Diagram of a repulsion motor below synchronous speed, showing the effect due to the reaction of the coils undergoing commutation

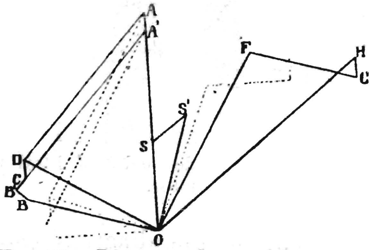

FIG. 13.—Diagram of a repulsion motor above sychronous speed, showing the effect due to the reaction of the coils undergoing commutation.

OA = ampere turns of the transformer coil, and OS, being those of the field coil, the part SS^1 being due to reaction of the short-circuited coil. The resultant field flux of the motor is thus OS^1.

As we may safely assume that the resistance of the short-circuit path is the controlling factor in the short-circuit current, the reaction of the coils on the field will be in phase with the sparking voltage.

The remainder of the diagram is identical with the previous diagram, figure 5, except for the fact that now the resultant field flux is in direction OS instead of in the direction OA.

Figure 13 is the same diagram for speeds above synchronism.

The use of these very complicated diagrams gives more accurate results than the simple diagram, but it seems that the assumptions necessary in any case preclude any possibility of very accurate results, and the simple diagram gives accurate enough values for commercial purposes, where the final test is not in the designing office, but on the test floor.

The operation of the repulsion motor is not by any means good as regards sparking, and on that account the simple motor is never used.

Its place is taken by the compensated repulsion motor, where the field winding, instead of being on the stator, is on the armature, and necessitates another pair of brushes.

This is shown diagrammatically in figure 14.

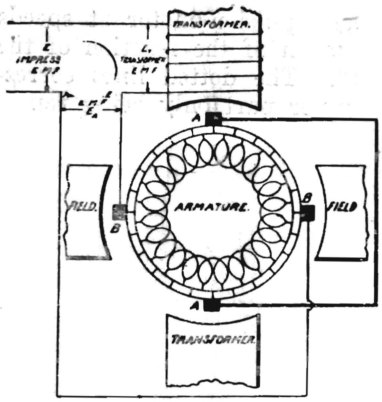

FIG. 14.—Two-pole model of ideal repulsion-series motor.

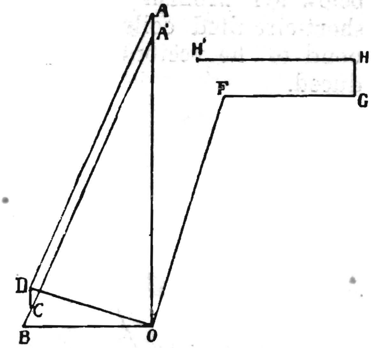

FIG. 15.—Simple diagram of the compensated repulsion motor for a constant primary current.

The simple diagram corresponding to figure 5 is shown in figure 15, where we have, in addition to figure 5, a vector HH proportional to the square of the speed. This e.m.f. is due to the rotation of the armature in the transformer flux, being proportional to the speed. As the flux is also proportional to the speed, the voltage is proportional to the square of the speed.

The derivation of the remaining diagrams would only be repetition of above, and is therefore omitted.

NOTE.

In view of the opinion expressed at the meeting at which this paper was read that some actual figures of performance would be desirable, the following results are given. In each case the motor was so designed as to cost the same to manufacture :—

1. THE SINGLE PHASE SERIES MOTOR.—

	Watts.
Iron loss in the stator	700
Iron loss in the armature	800
Copper loss in the field coil	1,920
Copper loss in the compensating coil... ...	320
Brush Friction losses	870
Resistance losses in the brush contacts ...	5,400
Bearing Friction	600
Total	11,940
Output of Motor	44,360
Input	56,300
Efficiency	79 per cent.
Power Factor	92·5 „
Motor Volts	98 volts.

2. The Simple Repulsion Motor.—

	Watts.
Iron loss in armature	200
Iron loss in stator	1,350
Copper loss in armature	1,970
Copper loss in stator	2,440
Friction loss in brushes	580
Resistance loss in brushes	2,340
Bearing friction	600
Total losses	9,410
Output	35,500
Input	44,910
Efficiency	79 per cent.
Power factor...	85 ,,

3. The Compensated Repulsion Motor.—

The losses are the same as in the above, with the following difference:—

Commutator losses are increased by	320 watts.
Armature losses are increased by	220 ,,
Total resistance losses are reduced by	195 ,,
Efficiency	78·2 per cent
Power factor	95·0 ,,