The method here proposed is to have the outer curve as above, a simple circular curve, of radius R + 10 + d, but to connect the inner to the straight with a circular curve of 1,000 feet radius instead of a transition.

In fig. 3 let R be the radius of the inner curve and L its centre;
R^1 the radius of the throw-in curve and M its centre;
d = increase in distance between tracks.
$\propto$ = deflection angle = SAT.
ϕ = throw-in angle = HOB.

Let PQ be parallel to BR and at a distance d from it, and let HKO be the common tangent to the two curves at K.

Then angle LMN = angle KOB = ϕ

$$MN = MF - LD = R^1 - (R + d)$$

$$\text{and } MN = LM \cos\phi = (R^1 - R)\cos\phi$$

therefore

$$\cos\phi = \frac{R^1 - R - d}{R^1 - R} = 1 - \frac{d}{R^1 - R}$$

$$\text{or versin } \phi = \frac{d}{R^1 - R}$$

$DF = LN = LM \sin\phi = (R^1 - R)\sin\phi$

Having found ϕ and DF we proceed as follows:—

$$\text{Total tangent} = BD + DF$$

$$= (R + d)\tan\frac{\propto}{2} + DF$$

$$\text{Secant} = BW = (R + d)\sec\frac{\propto}{2} - R$$

$$\tfrac{1}{2}\text{ circular arc} = R \times \text{arc}\left(\frac{\propto}{2} - \phi\right)$$

and for the curve KF

$$\text{Tangent OK} = OF = R^1\tan\frac{\phi}{2}$$

$$\text{Secant} = R^1\sec\frac{\phi}{2} - R^1$$

$$\text{Arc KF} = R^1\text{ arc }\phi$$

Taking the same example as above we have, making R^1 = 1,000 ft.

$$O = \text{versin}^{-1}\frac{0{\cdot}3}{500} = \text{versin}^{-1}\, 0{\cdot}0006$$

$$= 1^\circ\ 59'\ 05''$$

$$DF = 500\sin\phi = 500 \times {\cdot}0346352$$

$$= 17{\cdot}3176$$

$$\text{Total Tangent} = 0{\cdot}0874887 \times 500{\cdot}3 + 17{\cdot}3176$$

$$= 61{\cdot}09$$

$$\text{Secant} = 1{\cdot}0038198 \times 500{\cdot}3 - 500$$

$$= 2{\cdot}21$$

$$\tfrac{1}{2}\text{ circular arc} = 500 \times \text{arc } 3^\circ\ 00'\ 55'' = 500 \times {\cdot}0526460$$

$$= 26{\cdot}323$$

Chord of $\frac{1}{2}$ circular arc $= 500 \times \cdot 0526398$
$= 26 \cdot 3199$

Throw-in curve :—

Tangent $= 1000 \tan 59' \; 32 \cdot 5''$
$= 17 \cdot 3434$

Secant $= 1000 \sec 59' \; 32 \cdot 5'' - 1000$
$= 0 \cdot 15$

Arc KF $= 1000 \text{ arc } 1^\circ \; 59' \; 05''$
$= 34 \cdot 64$

The lead of the inner tangent (DF) in this case $= 17 \cdot 32$ – as against 29·89 by the other method, a saving of 12·57 feet of curved track.

If we adopt 1000 feet as a standard radius for the throw-in curve; ϕ, DF, and all the details of the throw-in curve may be calculated once for all, and tabulated for the different radii of the main curve. These are given in Table II. for every half-chain and every even 50 feet from 5 chains to 7 chains, and for every chain and every 100 feet to 13 chains. In the case of curves with less than 5 chains radius the transition of the outer must be fixed before we can standardise the inner. Details are given in Table IIa. for inner curves of 250, 264, 280, 297, and 300 feet radius, with a transition on the outer of approximately 30 feet; if any other length of transition is required on the outer the inner may be calculated as above.

With regard to the limits of this method :—

The practice in N.S.W. is to apply a transition in every case where we pass from the straight to any curve sharper than 5 chains radius. If we express the curvature as a reciprocal of the radius, this is equivalent to saying that there must be a transition if the change of curvature is greater than 1/330. If instead of turning off the straight we turn off a curve of 1,000 feet radius, the sharpest curve allowable will be the reciprocal of $\frac{1}{1000} + \frac{1}{330}$ *i.e.*, 250 feet radius.

(Practically the same result is arrived at if we use the American method of expressing the curvature; a 330 ft. radius is expressed as a 17° 21′ 30″ curve and a 1000 ft. radius as a 5° 44′ 00″ curve; adding these we get a 23° 05′ 30″ curve which is a curve of radius 248 feet).

Although this principle is adhered to by the Railway Construction Branch, the Railway Commissioners evidently do not insist on it in all cases; for example, at the corner of Liverpool and Elizabeth Streets there is a curve of 165 ft. radius compounded directly with a curve of 66 ft. radius, the 165 ft. curve being transitioned, a similar procedure being adopted by them at most double track jnnctlons. In practice this is not of very great moment, as this type of curve is used only at junctions where the cars will be run at a low speed, but it is unsound in principle and, if a car were to take this curve at high speed, considerable oscillation would result. The change in this case is equivalent to that from a straight to a curve of 110 feet radius.

TABLE II.

Details of 1,000 ft. Curves which when Compounded with Inner Curves of Double Track Will Give Necessary Clearance.

Radius of Inner Curve. Feet.	Distance Between Tracks = d + 10.	Radium of Outer Curve. Feet.	φ °	φ ′	φ ″	DF Feet.	Details of Throw-in Curve. Tangent. Feet.	Details of Throw-in Curve. Secant. Feet.	Details of Throw-in Curve. Arc. Feet.
330 (5 ch.)	10·5	340·5	2	12	50	25·88	19·32	0·19	38·64
350	10·5	360·5	2	14	50	25·49	19·61	0·19	39·22
363 (5½ ch.)	10·42	373·42	2	04	50	23·13	18·16	0·16	36·31
396 (6 ch.)	10·33	406·33	1	53	38	19·96	16·53	0·14	33·05
400	10·33	410·33	1	54	00	19·89	16·58	0·14	33·16
429 (6½ ch.)	10·33	439·33	1	56	53	19·41	17·00	0·14	34·00
450	10·33	460·33	1	59	05	19·05	17·32	0·15	34·64
462 (7 ch.)	10·33	472·33	2	00	25	18·84	17·52	0·15	35·03
500	10·30	510·30	1	59	05	17·32	17·32	0·15	34·64
528 (8 ch.)	10·17	538·17	1	32	16	12·67	13·42	0·09	26·84
550	10·17	560·17	1	34	30	12·36	13·75	0·09	27·49
594 (9 ch.)	10·15	604·15	1	33	27	11·04	13·59	0·09	27·18
600	10·15	610·15	1	34	10	10·96	13·70	0·09	27·39
660 (10 ch.)	10·15	670·15	1	42	08	10·10	14·86	0·11	29·71
700	10·10	710·10	1	28	46	7·75	12·91	0·08	25·82
726 (11 ch.)	10·10	736·10	1	32	53	7·40	13·51	0·09	27·02
750	10·10	760·10	1	37	14	7·07	14·14	0·10	28·28
792 (12 ch.)	10·10	802·10	1	46	18	6·43	15·46	0·12	30·92
800	10·10	810·10	1	48	45	6·33	15·82	0·13	31·63
858 (13 ch.)	10·10	868·10	2	09	00	5·33	18·76	0·18	37·52

TABLE IIa.

Inner Radius.	DETAILS OF OUTER CURVE (TRANSITION ONLY).														d
	Radius.	ϕ'	$\frac{x_c}{R}$	x_c	x^1	$x_c - x^1$	y_c	h	S	log m	OFFSETS. 10	20	25	30	
250	260 75	3° 27′ 08·5″	0·12	31·29	15·7018	15·5882	0·6292	0·15384	31·30	5·312585	·02	·16	·32	·55	0·75
264	274·67	3° 09′ 45″	0·11	30·2137	15·1530	15·0607	0·5565	0·13816	30·22	5·304820	·02	·16	·32	·54	0·67
297	307·58	2° 52′ 23·6″	0·10	30·758	15·4178	15·3402	0·5146	0·12795	30·77	5·247571	·02	·14	·28	·48	0·58
300	310·58	2° 52′ 23·6″	0·10	31·0580	15·5681	15·4899	0·5196	0·12920	31·07	5·239140	·02	·14	·27	·47	0·58

Details of Inner Curve.

Inner Radius.	d + h	ϕ	DF	Lead.	DETAILS OF THROW-IN CURVE. Tangent.	Secant.	Arc.
250	0·90384	2° 50′ 00″	36·54	20·95	24·73	0·31	49·45
264	0·80816	2° 42′ 17″	34·23	19·17	23·61	0·28	47·21
297	0·70795	2° 35′ 28″	31·30	15·96	22·62	0·26	45·22
300	0·70920	2° 35′ 57″	31·26	15·77	22·69	0·26	45·36

In this Table ϕ' has been written for ϕ on the outer curve to avoid confusion with ϕ, the throw-in angle of the inner curve.

Where the outer curve is transitioned the lead of the inner tangent is DF − ($x_c - x^1$).

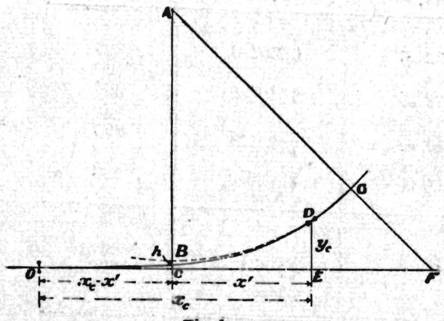

Fig.1

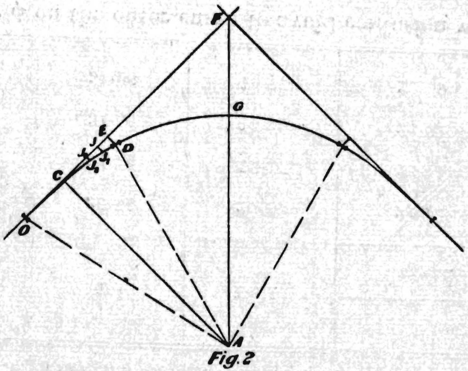

Fig.2

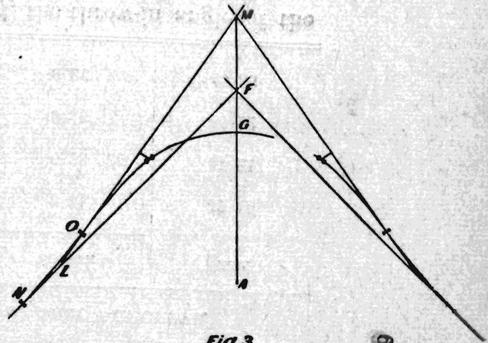

Fig.3

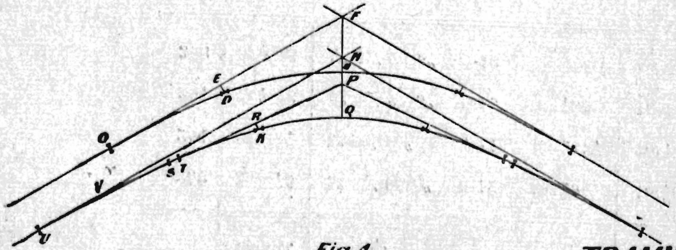

Fig 4

TRAMWAY CURVE DIAGRAMS

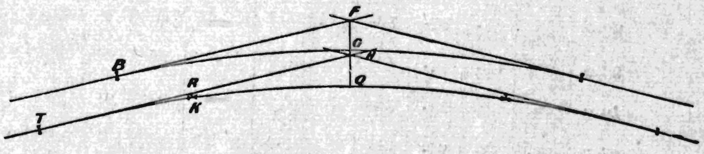

Fig.5

APPENDIX A.

TRAMWAY CURVES.

By J. C. Try, B.E.

Paper read at a meeting of the Institution of Surveyors, N.S.W., Tuesday, September 21st, 1909.

When a new tramway is to be constructed, the surveyor locates a line that will, amongst other things.

(*a*) Reduce obstruction to traffic on public roads to a minimum;
(*b*) Avoid the necessity for expensive resumption of property; and
(*c*) Keep within a ruling grade of 1 in 15.

These requirements are always borne in mind in the design and calculation of curves.

(*a*) If the tramway is to be a single line through city streets, where the cars run one way only, the track is laid 5 feet off the centre of the road on the side of the traffic. This is the case in Pitt Street and in Castlereagh Street. If the tramway is to be a single line in a suburban district, the track is laid 5 feet to the left of the centre of the road when the future duplication of the line seems probable; or along one side of the road, with a minimum clearance of 4 feet 6 inches between the rail and the footpath, when there is no provision to be made for duplication. An example of the former is the new line to Lilyfield; and, of the latter, the Manly to Brookvale Tramway now under construction. If the tramway is to be a double line, the tracks are laid equi-distant from the middle of the road and 10 feet centre to centre.

(*b*) In sharp turns it often happens that the corner property must be remodelled. In such cases the curves are designed so that the necessary resumptions shall be as limited as is consistent with a good running line. The use of a "throw-out" curve in some of these places may obviate the necessity for resumption; but as such a curve occupies a large area of the road and is not conducive to smooth running, its general use is not recommended. However, there are numerous examples of this form of curve, both in single and double track, to be found in the Sydney tramway system; as, for instance, at the corner of Queen and Moncur Streets, Woollahra, and at the corner of Penshurst Street and Victoria Avenue, near the old Willoughby terminus.

(*c*) In the preliminary survey of the line, grades are compensated for curvature, and the radii of the curves must be selected in accordance with the allowance made by the surveyor. In cases where a sharper radius is subsequeutly found to be necessary, the line is regraded at that point. On a ruling grade of 1 in 15, which is the one usually adopted, the compensated grade for a curve of 100 feet radius is 1 in $20\frac{1}{2}$.

SINGLE-LINE TRAMWAYS.

On single lines, curves whose radii are 5 chains and upwards are simple circular arcs ; curves whose radii are sharper than 5 chains are transitioned. The transition curve in use on the N.S.W. tramways is the cubic parabola. As several papers on this particular curve have been read before the Royal Society of New South Wales, it is not proposed here to deal theoretically with its functions, but merely to give a few cases of its practical application in tramway construction. In working out the examples below, the author has used the Transition Curve Table compiled by Mr. C. J. Merfield, and published in the "Journal of the Royal Society of New South Wales," Volume xxxiv.

Fig. 1 shows a general form of the cubic parabola in conjunction with a circular arc. The centre of the ciscle is at A, and the arc is G D B. The transition is D O, the length of this curve being denoted by s. The length of the abscissa O E is denoted by x_c, and the ordinate D E by y_c. A C is a perpendicular drawn from the centre of the circle, A, to meet O E at C. The length of C E is denoted by x', and B C, the distance that the circular arc falls short of the straight, by h The angle that a tangent from D makes with O E is denoted by ϕ.

Example (1). Consider the case of a curve turning through an angle of 90°. In selecting the radius (which should be fairly sharp) it is not usual to go lower than 66 feet for electric cars, or 90 feet for steam motors. If the grade will permit, we may in this instance adopt 80 feet as a good working radius. The transition for a radius of 80 feet can be made any length, up to a maximum of about 55 feet, by selecting a suitable value of $\frac{x_c}{R}$ (see Merfield's Table).

Taking $\frac{x_c}{R} = 0{\cdot}50$, the transition details work out as follows, R being the radius of the arc, 80 feet :—

$x_c = 40{\cdot}00'$ ....O E		Figs. 1 and 2.
$x' = 21{\cdot}57'$ ....C E		
$\therefore x_c - x' = 18{\cdot}43'$ ....O C		
$y_c = 3{\cdot}73'$ ....D E		
$h = 0{\cdot}77'$ ....B C		
$s = 40{\cdot}31'$ ....O D		
$\phi = 15° \; 38' \; 24{\cdot}5''$ Angle D A C		

Intermediate ordinates to the transition (J, J_1, J_2, J_3, Fig. 2) are calculated from the formula of the cubic parabola.

$$y = mx^3$$

In the table it is given,

$$\log (m\,R^2) = 9{\cdot}572035.$$

$$\text{Log } R^2 = 2 \log 80 = 3{\cdot}806180.$$

$$\therefore \log m = \overline{5}{\cdot}765855.$$

Giving x values of 10, 20, 30, and 40, we find,

Ordinate at $10' = 0{\cdot}06'$
" " $20' = 0{\cdot}47'$
" " $30' = 1{\cdot}58'$
" " $40' = 3{\cdot}73' = y_c$

Other dimensions of this curve (Fig. 2) are—
Deflection Angle (a) = Central Angle = 90ϕ — Twice Angle C A F.

Total Tangent $= (R + h) \tan \frac{a}{2} + (x_c - x')$

$= 99{\cdot}20'$.........F O.

Secant $= (R + h) \sec \frac{a}{2} - R.$

$= 34{\cdot}23'$.........F G.

$\frac{1}{2}$ Circular Arc $= R \times \text{arc} \left(\frac{a}{2} - \phi\right)$

$= 40{\cdot}99'$.........D G.

It is obvious that if $\phi = \frac{a}{2}$, the circular arc will be eliminated; and if ϕ exceeds $\frac{a}{2}$, the transitions will overlap. Therefore $\frac{a}{2}$ is the limiting value for ϕ.

Example (2). An example of a "throw-out" curve is shown in Fig. 3. Taking the deflection angle at F = 90°, and the throw-out angle at L = 8°, the deflection angle at M becomes 106°. Call this angle β. Assuming the radius and transition the same as in the preceding case,

$M O = (R + h) \tan \frac{\beta}{2} + (x_c - x')$

$= 125{\cdot}62'$

$O L = R_1 \tan 4°$ (where R_1 = radius of throw-out curve). Making $R_1 = 330$ feet,

$O L = 23{\cdot}08' = N L.$

$\therefore M L = 148{\cdot}70'.$

$$L F = \frac{148{\cdot}70 \sin 37°}{\sin 45°} = 126{\cdot}56'.$$

$$\text{and } M F = \frac{148{\cdot}70 \sin 8°}{\sin 45°} = 29{\cdot}27'.$$

$\therefore$ Total Tangent (F N) $= 149{\cdot}64'.$

$\frac{1}{2}$ Circular Arc $= R \times \text{arc} \left(\frac{\beta}{2} - \phi\right)$

$= 52{\cdot}16'.$

Secant, M G $= (R + h) \sec \frac{\beta}{2} - R.$

$= 54{\cdot}21'$

Secant, F G $= (M G - M F) = 24{\cdot}94'.$

Double-Line Tramways.

In designing curves for a double line of track, an allowance for the overhang of cars must be made so that the minimum clearance between two cars passing on a curve may be always sufficient to ensure the safety of passengers standing on the footboards. For this reason the centres of tracks, which are 10 feet apart on the straight, are widened on curves of 10 chains radius or sharper, the width varying with the radius. Several devices are resorted to in order to obtain this extra width; and the cases of a "throw-in" and of a long transition on a flat inner curve may be considered.

A diagram of a double curve having a "throw-in" on the inside is shown in Fig. 4.

Example (3). Adopting the following data :—

Deflection angles at F and H = 48°.........a.

Throw-in angle at V = 2°.........γ.

Deflection angle at P = 44°.........$(a - 2\gamma)$.

Radii of main curves = 111·5 feet and 100 feet.

$\frac{x_c}{R} = 0{\cdot}40$ (for both curves).

Radius of throw-in curve = 1,500 feet, the curves are worked out thus—

Outer curve—Radius = 111·5'.........R.

Transition details—$x_c = 44{\cdot}60'$

$x' = 23{\cdot}32'$

$\therefore\ x_c - x' = 21{\cdot}28'$

$y_c = 3{\cdot}18'$

$h = 0{\cdot}71'$

$s = 44{\cdot}80'$

$\phi = 12^\circ\ 04'\ 20{\cdot}9''$

$\log m = \bar{5}{\cdot}5543772$

Ordinate at 10' = 0·04'

20' = 0·29'

30' = 0·97'

40' = 2·29'

44·6' = 3·18' = y_c

Total tangent = 112·21 tan 24° + 21·28 = 71·24'

Secant = 112·21 sec 24° − 111·5 = 11·33'

½ Circular arc = 115·5 × arc 11° 55' 39·1" = 23·19'

Inner curve—Radius = 100'.........R_1.

(By making the curves concentric, we get the track centres 11·5' apart, which is the distance required.)

Transition details — $x_c = 40{\cdot}00'$

$x' = 20{\cdot}91'$

$\therefore\ x_c - x' = 19{\cdot}09'$

$y_c = 2{\cdot}85'$

$h_1 = 0{\cdot}64'$

$s = 40{\cdot}18'$

$\phi = 12^\circ\ 04'\ 20{\cdot}9''$