THE STABILITY OF RETAINING WALLS.

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INTRODUCTION.

The main object of the following article upon the stability of retaining walls is to suggest the inclusion, in calculations for design, of an important factor of strength or resistance that most assuredly exists, but which is totally ignored in the more commonly distributed text-books anyhow. It is that of the cohesive strength of the materials of which such walls are usually constructed. A wall cannot be designed economically if one of its most important sources of strength is refused place amongst its calculated assets.

The method of consideration of Surcharged Walls here suggested differs from that of some writers, and yields different results; but the theory advocated is fully described, so the reader has full opportunity of choosing which form of reasoning he prefers, according to his judgment.

The equation obtained for the case bears a strong family resemblance to another that is to be found in at least one well-known text-book, viz., $P = \frac{k\,h^2 \cos\alpha}{2}$ the difference in value being that between $\cos\alpha$ and $\cos^2\alpha$; but upon comparing with modifications given in the same work, it is evident that the equation referred to is constructed upon some basis that is quite different to the argument herein enunciated. It may have been derived by working backwards from the hydrostatic expression $\frac{k.h^2}{2}$ with an assumption to include the angle of friction, and according with that expression, at the two extremities of α, namely, $0°$ and $90°$; but anyhow, it is quite different, notwithstanding the general similarity, and the modifications referred to indicate a substantial variation in pressure with truncated surcharge, which appears to be improbable if the argument advanced herein be logical.

In another work, the angle $\frac{90 - \alpha}{2}$ is described as the angle of maximum pressure. Herein it is argued that there is no such angle of maximum pressure. If there be such an angle, it would appear that gravity acts upon solids and fluids under quite different laws, instead of the one law modified by the element of friction.

NOMENCLATURE.

h = Height of wall.
H = Depth of material retained.
P = Total pressure of retained material.
p = Unit pressure of retained material at any depth.
P_o = Pressure acting along the direction of P, necessary to cause overturning of wall.
b = Width of base of wall.
d = Thickness (length) of base of wall.
t = Width at top of wall having back batter.
s = b — t = horizontal projection of back batter.
f = Factor of safety.
L = Lever arm of force tending to overturn wall.
l = Lever arm of force of gravity about toe of wall.
g = Specific gravity of wall.
q = Specific gravity of retained material.
w = Weight of material in wall.
W = Total weight of wall.
C = Total cohesion on base b.
k = Weight of retained material (solids) in pounds per cu. ft.
$M/_3$ = Abbreviation for middle third point.

WALLS WITHOUT COHESION.

The following equations may be evolved relating to walls without cohesion retaining any liquid (see Figs. 9 and 10):—

$$P = \frac{62{\cdot}4\, q\, h^2}{2}$$

$$PL = \frac{Ph}{3} = \frac{62\ 4\, q.h^3}{6}$$

$$f\,\frac{Ph}{3} = \frac{62.4\, q.f.h^3}{6}$$

and $b = h\sqrt{\frac{q.f}{3\,g}}$ $\qquad$ $b = \sqrt{\frac{f\,q\,h^2 + g.s^2}{3g}}$

WALLS WITH COHESION FROM MORTAR.

The foregoing equations are based solely upon the weight or mass of the material of the wall, or as it might be, in the case of a wall constructed of bricks laid dry on a smooth bottom. Such a wall, of course, would not retain water, nor could it adhere so as to be overturned in a mass; but what is meant is that no allowance has been made for the cohesion of the mortar at the base or ends of the wall; the wall is regarded as a rigid mass, loose on its foundation, and resisting overturning moment, sliding, bulging, etc., by virtue of its mass and weight alone.

But walls built for practical purposes have foundations toothed into the ground, or anyhow, more or less fastened to the ground with mortar, which has a considerable amount of cohesive strength that materially adds to the resistance to the overturning moment, hence, if the cohesive resistance amount to any important quantity, it is obvious that less weight will be required, and a smaller and less costly wall will perform the nett duty. And it thus follows that any wall designed without regard to the force of cohesion, is designed with excess of strength, and will therefore be wasteful and extravagant to construct.

It is intended to show that the cohesive resistance is no negligible quantity, but is a large and important proportion of the total resistance. So far as the writer is aware, this phase of the question is not dealt with in text-books. For effective treatment of the subject, a considerable amount of experiment under practical conditions is much to be desired, but obviously such experiments would be very expensive and beyond the means of most private enquirers. So far as the writer is aware, no such experiments upon any comprehensive scale have been carried out, hence the details in the following articles are empirical only; even so, and upon most conservative allowances, great economy is possible, and anyhow, so far as small walls are concerned, is commercially essential as compared with the results that would be derived from the above equations.

Let it be assumed that the wall to be considered is built of brickwork, set in Portland cement mortar, in the proportion of 3 parts of sand to 1 part of cement, and take the S.G. of the mass as 1·8. At a low average specification for the material the cohesive strength of such mortar may be taken at 200 pounds per square inch at the age of one month, and this represents 28,800 pounds per square foot.

Assuming the wall to be built with extreme care, as for test conditions, if it has a base one foot wide, and one foot length of the wall be taken for the argument, there will be required an upward pull of 28,800 pounds to fracture the mortar; that is to say, in opposition to a vertical pull upward, the wall has acquired what is tantamount to an increase of weight of 28,800 pounds. But the force tending to overturn the wall is not a vertical force; it is horizontal in direction.

Suppose the wall to be 2 feet wide at the base, and that it is only attached to the foundation at three points, the inside edge, the middle point, and the outside edge, with one square inch of mortar at each point; and for further simplicity, let it be assumed that these separate dabs of mortar act at *points* at the centre, and at the extreme edges respectively. The added weight is now $3 \times 200 = 600$ pounds against a vertical pull.

Take the case of a rectangular wall of breadth b, acted on by a horizonal force P, at the height $\frac{h}{3}$ above the base. Let the vertical forces of cohesion act with uniform intensity over the whole of the base. The sum of their moments about the outer edge of the base is equal to the moment of a force C, equal to their sum acting at the middle of the base. Let W be the weight of the wall if cohesion be disregarded, and w its weight if cohesion be taken into account.

Taking moments about the outer edge of the base. For equilibrium,

$$P \frac{h}{3} = (w + C) \frac{b}{2}$$

if cohesion is taken into account,

$$\text{and } P \frac{h}{3} = W \frac{b}{2}$$

if cohesion be disregarded.

If the vertical force of cohesion be taken at 200lbs. per sq. in., then on a wall of width b, C = 28,800 b lbs.

But the value of 200 pounds per square inch is a laboratory value, and it would not be practicable in general work to pay sufficient attention to the mixing of the ingredients, and to other points, or to construct an ordinary wall capable of fulfilling such strenuous conditions, so it is necessary to decide what is a reasonable working value for the force of cohesion. The cohesion of brickwork in mortar has been considered with much detail in Trautwine's "Engineering Pocket-Book," and in that publication a statement may be found to the effect that the adhesion of 3 to 1 Portland cement mortar to brickwork, may be taken as ¾ of the cohesive strength of the mortar. Upon some little accumulation of evidence the cohesive strength of mortar has been above-taken as 200 pounds per square inch, and $\frac{3}{4}$ of 200 = 150. Trautwine, however, assesses a higher value, viz., 240 pounds per square inch for 3 to 1 Portland cement mortar, and with that estimate the adhesion to brickwork is set down as 180 pounds per square inch.

Next, as Trautwine very justly represents, the complete laboratory test will always afford values considerably in excess of those derived from tests of the same materials taken from an ordinary mixing on the site of construction work, owing to greater refinement of the gauging in the former case; and he assesses a relation between the two conditions in the proportion of 240 to 175. Adhering to the lower test value of 200, as a reasonable average for cements in the open Australian markets, and adopting the same relation between laboratory work and practical construction, 146 becomes the value for the mixing box, and taking $\frac{3}{4}$ of that value as the adhesion to the bricks, 109 per square inch is afforded as a practical value for the ultimate adhesive effect of the mortar, or say 100 pounds per square inch. Under these circumstances, C = 14,400 b, is a value that might be allowed for a carefully supervised and well-built wall.

However, it is not always a good policy to calculate upon first-class supervision and workmanship, especially nowadays, when workmanship is deteriorating perceptibly every year; therefore, to provide for contingencies suggested by these conditions of the day, it may seem desirable to make a further reduction of 50 per cent., when the value becomes C = 7,200 b, which represents an ultimate cohesive strength in the cement mortar of 50 pounds per square inch.

There is yet another reason on the grounds of precaution (and it is desirable not to miss any precaution when suggesting procedure in the nature of radical change) for reducing the valuation of this very effective factor of resistance, which lies in the vague possibilities of the element "fatigue." Fatigue cannot possibly have any effect on the weight of a wall, but it is possible that it may have some upon cohesive properties. With the inclusion of an allowance for cohesion in stress calculations, the mere fact of placing the factor "f" on the P.L. side of the equation, making that side f.P.L., provides a margin common to the effects of both weight and cohesion; but since it may be possible for fatigue to affect cohesion, whilst it cannot in any way affect the action of gravity, if any factor is to be allowed upon such grounds, it must be incorporated with the co-efficient of the cohesion itself.

A dam wall may need to exert its power of resistance to overturning for 365 days per annum Will such constant strain have any effect upon the cement, equivalent to fatigue in metals? And, if so, to what extent? It seems impossible to reply to these questions without practical experiment, and obviously such experiments must occupy great length of time, and they must cost a considerable sum to carry out. The characteristic of fatigue is probably of greatest effect in materials possessing considerable elasticity, and of comparatively small degree in those which have not that attribute; in that case it might be anticipated that stone matter, which is naturally rigid in character, would be but to a small extent influenced by fatigue. Yet in laboratory experiments high tensile tests are obtained with rapid applications of load, whilst with slow application a briquette fails at an earlier stage; but whether the circumstance is due to anything in the nature of fatigue or not is difficult to determine. On the other hand, it is well known that cement increases in strength with age. This is probably accounted for by the more complete drying out of the water, and it is possible that mortar might increase in strength with age against a quick test, and yet suffer from fatigue in a prolonged one. There is no data upon which to assess an allowance upon this head, and it only remains to fix an arbitrary value. For such arbitrary value, then, suppose the co-efficient of cohesion be reduced a further 50 per cent., then $C = 3{,}600$ b, and the co-efficient now represents a cohesive effect of only 25 pounds per square inch as ultimate strength. This will be again reduced by the factor of safety, for which the diagram factor or middle third condition gives, on the average, a factor of about 2, so that with $C = 3{,}600$ b in calculations for the middle third condition, the resistance actually relied upon from the cohesion will only amount to 12½ pounds per square inch; and since it is probable that fatigue, when it occurs at all, has but little effect in stresses well within the working strength of any material, reliance to the extent of 12½ pounds per square inch as working stress for Portland cement mortar should not alarm the most cautious designer, and it seems reasonable to suppose that no one can urge objection to the inclusion of the allowance

$$C = 3{,}600 \text{ b}.$$

Hence, depending upon the conditions and upon the judgment of the designer, $C = 14{,}400$ b, $7{,}200$ b, or $3{,}600$ b, and on applying one of these values to the above equations for rectangular walls, we have:—

$$\text{PL} = \frac{62{\cdot}4\,h^3}{6} = \text{W.l.} = (w + C)\,l = \frac{(w + C)\,b}{2}$$

$$\frac{62{\cdot}4\,h^3}{6} = \frac{(62{\cdot}4\,g.h\,b + C)\,b}{2}$$

$$= \frac{(62{\cdot}4\,g.h.b + 3600\,b)\,b}{2} \quad \text{taking } C = 3600 \text{ b.}$$

$$= \frac{b^2\,(62{\cdot}4\,g.h + 3600)}{2}$$

$$b^2 (62{\cdot}4.g.h + 3600) = \frac{62{\cdot}4\ h^3}{3}$$

$$b^2 = \frac{62{\cdot}4\ h^3}{3\ (62{\cdot}4\ g.h + 3600)}$$

$$b = \sqrt{\frac{62{\cdot}4.h^3}{3\ (62{\cdot}4\ g.h + 3600)}} \quad \text{for unstable equilibrium,}$$

and

$$b^2 (62{\cdot}4\ g.h + 3600) = f\ \frac{62{\cdot}4\ h^3}{3}$$

$$b^2 = \frac{62{\cdot}4\ h^3\ f}{3\ (62{\cdot}4\ g.h + 3600)}$$

$$b = \sqrt{\frac{62{\cdot}4\ h^3\ f}{3\ (62{\cdot}4\ g.h + 3600)}} \quad \text{for stable equilibrium.}$$

It may easily be shown that when the resultant falls at one-third of the base from the toe of a rectangular wall, the factor of safety f against overturning is 3. Hence the equation for stable equilibrium may be written—

$$b = \sqrt{\frac{62{\cdot}4\ h^3}{(62{\cdot}4\ g.h + 3600)}}$$

There is no constant numerical value for f in the middle third condition with battered walls, because, putting the proportion in the same way as in the last paragraph—

$$1 : 1 - \frac{b}{3} = p_o : P$$

$$\frac{p_o}{P} = \frac{1}{1 - \frac{b}{3}}$$ and 1 varies with the slope of the batter in the wall considered as without cohesion, whilst when cohesion is included, 1 varies also with the relative values of w and C with the positions of their respective centres of action; but having once ascertained the value of b for the middle third condition, both 1 and $\frac{b}{3}$ are known; then if f is required—

$$\frac{1}{1 - \frac{b}{3}} = f.$$

The equations for rectangular walls are simple in construction, because the centre of gravity of the rectangular wall and the centre of action of the cohesive force are both on the same vertical line, which is a vertical passing through the centre of the base; but in the case of battered walls, the centre of gravity of the mass of the wall is situated on a line passing through the base, at a distance of $\frac{3\ b^2 - s^2}{6\ b - 3\ s}$ from the toe, whilst the centre of action of the cohesive force is at the centre of the base. Hence the centre of gravity of the combined resistances lies between these two points. It is, therefore, scarcely practicable to construct a general equation to suit such diverse varying conditions. With the primary equations, $PL = Wl$ or $fPL = Wl$ or rather $(w + C)\,l$, an approximation can first be obtained and then modified to suit the case.

For instance, let it be required to construct a dam wall $h = 12 = H$, with 9-inch crest and back battered, of brick in cement, $SG = 1{\cdot}8$, allowing for cohesive effect at the base to the amount of $C = 3{,}600\ b$.

$$P = \frac{62{\cdot}4\ H^2}{2} = 4492{\cdot}8 \quad L = \frac{H}{3} = 4.$$

$\therefore PL = 17{,}971{\cdot}2 = Wl$, unstable equilibrium. l is the horizontal distance between the toe of the batter and a vertical line representing the locus of the centre of action of the combined forces, gravity and cohesion. In such a small wall the cohesive effect (even at this low estimate) will be much in excess of the weight of the wall, and consequently the position of the centre of combined resistance will be nearer the centre of action of the cohesive effect, or more close to $\frac{b}{2}$ than to $\frac{3.b^2 - s^2}{6\,b - 3\,s}$; therefore, for the first approximation, assume that the leverage is $\frac{b}{2}$ though, of course, it must give a wall a little in deficit of requirements. $l = \frac{b}{2}$; the weight of the wall, w, is $\frac{62{\cdot}4.g.h\,(2b - s)}{2} = 62{\cdot}4 \times 1\ 8 \times 6 \left\{2b - (b - 0\ 75)\right\} = 673{\cdot}92$ $b + {\cdot}75)$.

$w = 673{\cdot}92\ b + 505{\cdot}44.$

$C = 3{,}600\ b,\ W = (w + C) = 4{,}273{\cdot}92\ b + 505{\cdot}44.$

But the wall is required for working conditions, and the resultant is to fall on or inside the middle third of the base, and for this condition, $Wl = fPL$, but we cannot yet assess an exact value for f. Try $f = 2$,

$$\text{then } fPL = 2 \times 17{,}971{\cdot}2 = 35{,}942{\cdot}4,$$

$$\text{and } Wl = (4{,}273{\cdot}92\,b + 505{\cdot}44)\frac{b}{2}$$

$$2{,}136{\cdot}96\ b^2 + 252{\cdot}72\ b = 35{,}942{\cdot}4,$$

$$b^2 + \frac{252{\cdot}72}{2136{\cdot}96}\,b = \frac{35942{\cdot}4}{2136{\cdot}96}$$

$$(b^2 + {\cdot}05913)^2 = 16{\cdot}819 + ({\cdot}05913)^2 = 16{\cdot}822,$$

$$b + {\cdot}05913 = 4{\cdot}1015,$$

$$b = 4 \text{ feet } (4{\cdot}04).$$

This must now be tested for the diagram condition.

The weight of the wall is $w = \{(12 \times 0{\cdot}75) + (6 \times 3{\cdot}25)\}\ 112{\cdot}32$

$= (9 + 19{\cdot}5)\ 112{\cdot}32 = 3{,}201$ pounds.

The centre of gravity line lies at $\frac{3\,b^2 - s^2}{6\,b - 3\,s}$ from the toe, and

$s = 4 - 0{\cdot}75 = 3{\cdot}25,\qquad \frac{3 \times 16 - (3{\cdot}25)^2}{24 - 9{\cdot}75} = \frac{48 - 10{\cdot}5625}{14{\cdot}25}$

$= 2{\cdot}63$ feet from the toe.

The cohesive effect is 3,600 b $= 14{,}400$ acting at $\frac{b}{2}$ or 2 feet from the toe. There is $0{\cdot}63$ between the situations. If x_1 be the distance of the combined effect centre from $\frac{b}{2}$ then the moments are $14{,}400\ x = 3{,}201\ (0{\cdot}63 - x)$,

$$17{,}601\ x = 2{,}016{\cdot}63,$$

$$x = \frac{2016{\cdot}63}{17601} = 0{\cdot}11$$

and the mean leverage is $l = \frac{b}{2} + 0{\cdot}11 = 2{\cdot}11$

Then, for the diagram $P = 4{,}492{\cdot}8$,

$W = 14{,}400 + 3{,}201 = 17{,}601$,

and $17{,}601 : 4{,}492{\cdot}8 = 4 : 1{\cdot}02$,

when $2{\cdot}11 - 1{\cdot}02 = 1{\cdot}09$,

the resultant falls $1{\cdot}09$ within the toe; but the point of the middle third is $\frac{4}{3} = 1{\cdot}3$, so that the resultant falls $0{\cdot}24$ outside the middle third. The base is a little too narrow; give it an additional 6 inches of width and try the diagram again.

$b = 4{\cdot}5\ S = 3{\cdot}75,\ P = 4{,}492{\cdot}8,\ L = 4,\ PL = 17{,}971{\cdot}2.$

$$w = \frac{62{\cdot}4\,gh\,(2b - s)}{2} = \frac{62{\cdot}4 \times 1{\cdot}8 \times 12\,(9 - 3{\cdot}75)}{2} = 673{\cdot}92 \times 5{\cdot}25 = 3538,$$

$$l_{(w)} = \frac{3\,b^2 - s^2}{6\,b - 3\,s} = \frac{\{3 \times (4{\cdot}5)^2\} - (3{\cdot}75)^2}{(6 \times 4{\cdot}5) - (3 \times 3{\cdot}75)} = \frac{60{\cdot}75 - 14{\cdot}0625}{27 - 11{\cdot}25} = 2{\cdot}96$$

$C = 4{\cdot}5 \times 3{,}600 = 16{,}200$, and $\frac{b}{2} = 2{\cdot}25,\qquad 2{\cdot}96 - 2{\cdot}25 = 0{\cdot}71,$

$$16{,}200\ x = 3{,}538\ ({\cdot}71 - x) = 2{,}511{\cdot}98 - 3{,}538\ x,$$

$$19{,}738\ x = 2{,}511{\cdot}98,$$

$$x = \frac{2511{\cdot}98}{19738} = 0{\cdot}127$$

$$l = 2{\cdot}25 + {\cdot}13 = 2{\cdot}38.$$

$$W = 3{,}538 + 16{,}200 = 19{,}738,$$

$$19{,}738 : 4{,}492{\cdot}8 = 4 : 0{\cdot}91,$$

$$2{\cdot}38 - 0{\cdot}91 = 1{\cdot}47,$$

but $\frac{4{\cdot}5}{3} = 1{\cdot}5,\qquad 1{\cdot}5 - 1{\cdot}47 = 0{\cdot}03,$