Context, Connections and Communication: Using Journal Articles in Undergraduate Mathematics

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Abstract

Many undergraduate students do not have the opportunity to appreciate the deep connections between different areas of mathematics or the richness of links between mathematics and other disciplines. A compartmentalisation of mathematics into distinct and unrelated topics or a perceived lack of relevance to the real world or other university subjects can promote a surface approach to learning. These perceptions can be exacerbated by the different ways mathematics is communicated in different contexts or scientific fields. Units of study on mathematical modelling and applications provide opportunities to enhance transfer of mathematical concepts by showing students how mathematics is used in a variety of contexts and shared by different communities of scientists. Activities based around appropriately chosen journal articles can provide students authentic examples of context, connections and communication.

Introduction

Despite the growing demand for qualified individuals in science, technology, engineering and mathematics (STEM), many countries including Australia are experiencing a decline in the uptake of STEM related subjects in secondary and tertiary education (Chubb, 2014; Australian Industry Group, 2015). Many government policies and strategies focus on increasing recruitment into STEM subjects, but attention also needs to be given to attrition and increasing retention and progression into higher years (Chen, 2013). Furthermore, within individual disciplines students may migrate to majors that require fewer or easier mathematics units and thus limit their subsequent employment opportunities or reduce their competitiveness against others with stronger quantitative skills. The reasons for avoiding mathematics are not necessarily related to ability but may include other factors such as perceived relevance. Recently, Reengineering Australia have said that ‘conventional education has struggled to deliver subjects in a way that links the learning process to the relevant application of that learning’ (Corbett, 2014). Units of study on mathematical modelling and applications are a natural way to link learning to applications; and have been attracting serious attention in the mathematics education community (Kaiser, Blum, Borromeo Ferri & Stillman, 2011; Stillman, Kaiser, Blum & Brown, 2013). In addition, the embedding of inquiry and research into the undergraduate science curriculum is a strategy that has been gaining momentum institutionally, nationally and internationally for many years (Brew, 2010) though inquiry based activities are more common in experimental disciplines.
Incorporating authentic applications and links to other disciplines into the curriculum can promote a more balanced view of mathematics (Gravemeijer and Doorman, 1999; Poladian, 2013). Unfortunately, sometimes service courses in mathematics or statistics are too generic and not targeted to specific cohorts. In particular, units for life science students might be rehashed or truncated versions of courses originally designed for other cohorts such as physics or engineering. Choosing only one context can also be problematic. Dapueto and Parenti (1999) discuss various problems arising from ‘teaching/learning in context’ and acknowledge the difficulty in attempting to incorporate the specific language and viewpoints of the other discipline. Indeed, communication has been identified (Jones et al., 2011) as one of the threshold learning outcomes for undergraduate mathematics in Australia that needs more attention; in particular, the ‘appropriate interpretation of information communicated in mathematical and statistical form’ and the ‘appropriate presentation of information, reasoning and conclusions in a variety of modes, to diverse audiences (expert and non-expert).’ Quite often the examples of mathematical communication that students see do not resemble authentic examples from their specific scientific disciplines.

The final ingredient considered in this paper, is providing some choice and control to the student. At the tertiary level, especially in large first year classes, the choice of context and examples are controlled by the lecturer; the timing and the transitions between abstract, concrete and exemplary cases are also controlled by the lecturer. While some students notice and appreciate the transfer between different contexts, and between the real-world and the abstract, this teacher-centered approach does not suit all students (Dapueto and Parenti, 1999). In a review of research on student motivation, Pintrich (2003) identified several individual factors that can dramatically influence how students cope with different levels of choice and control. He distilled a few general design principles and these include providing content, material and tasks that are relevant and personally meaningful to the students.

We present a strategy that uses journal articles to achieve the various goals of providing a context or several contexts, illuminating connections and an opportunity to observe and also practise mathematical communication. Two cases studies are compared and contrasted: a first year service unit for life science students and a third year regular unit for mathematics majors. The units share a common pedagogical structure and similar mathematical content but have extremely different cohorts of students.

**Theoretical Background**

Kilpatrick, Swafford and Findell (2001) established five interdependent strands of mathematical proficiency and the two units of study described in this paper both have a structure based around this framework, but each has its own relevant balance of these proficiencies. The five proficiencies are conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. The first four lie mostly within the cognitive domain, are more or less self-explanatory and reasonably understood by most educators and mathematicians and thus are common elements of many curricula. Indeed, these first four have also been explicitly named and incorporated into the Australian national curriculum for primary and secondary mathematics (ACARA, 2009).

The fifth proficiency, a productive disposition, is the inclination to regard mathematics as useful and worthwhile, and to also believe in diligence and personal efficacy (Kilpatrick, 2001). Thus, it is not a cognitive proficiency but lies within the affective domain (emotions and attitudes) and within the conative domain (making personal choices and directing effort).
This fifth proficiency can be taken for granted, relegated a status different to the other four or overlooked entirely.

Appropriate use of context can be used to promote a productive disposition and Wedge (1999) raises an important distinction between two types of context. The *task context* is the ‘world’ in which the problem is set (e.g. analysing an Ebola epidemic in West Africa) and the *situational context* is the ‘learning environment’ experienced by the student (e.g. a question in a mathematics tutorial, an inquiry project in a biology course, conducting a literature review, an informed political debate). Teaching mathematical modelling and applications can easily provide interesting task contexts; incorporating the reading and discussion of journal articles of which some are chosen by the students themselves can add a valuable situational context.

In second language acquisition, the distinction between receptive skills (listening and reading) and the corresponding productive skills (speaking and writing) is well-established (Krashen, 1982) and it’s hypothesized that the transition from receptive competence to productive competence requires an intermediate skill called *noticing* (Schmidt, 1990). Mathematics is often referred to as a foreign language and, thus, inspired by this analogy, the communication activities used in the case studies in this paper concentrate first on the receptive skills (how do you read a journal article containing mathematics) and then noticing (how do you decide what is important, what is foreground, what is background and extract what is meaningful to oneself) and only embark slowly and carefully on the productive skills (writing something new oneself).

Capstone experiences are ‘a significant, culminating and assessed learning experience’ and ‘focus on providing students with an opportunity to integrate and apply prior learning, and to support the transition to professional life or post-graduate studies.’ (Lee and Loton, 2014). Normally, a capstone experience occurs in the final year of a degree, corresponding to the onset of some ‘transition’ from a formal university learning environment to a more authentic work environment. Many science students also experience a similar transition much earlier in their education if their degree has a structure where service units in mathematics finish at the end of first year. Although, hopefully, they may continue to use and learn more mathematics, this might be without the support mechanisms like tutorials and regular assessment and feedback. Thus, in such situations, the end of first year is an opportune moment for a capstone experience. By way of contrast, for students who are majoring in mathematics, the capstone experience can be presented in third year. In each case, the capstone occurs at the relevant culmination of their *formal* learning pathway in mathematics.

**Setting and institutional context**

This paper focusses on two units of study both taught at The University of Sydney. A first year unit called “MATH1013 Mathematical Modelling” and a third year unit called “MATH3063 Nonlinear Differential Equations and Applications”. The two units have a common focus on combining quantitative and qualitative methods for differential equations and showcasing both modern and historically important mathematical models used in various scientific disciplines. However, the cohort of students taking the units are extremely different. The first year unit MATH1013 is one of the compulsory service units required for all science students and this one is chosen mostly by those students intending to major in psychology, biology or go on to study in the medical sciences. These students are expected to have already seen basic calculus in high school and earlier units and this unit introduces them to differential equations in an applied context. The unit usually has about 700 students who
attend two hours of lectures and a one hour of tutorial per week for 13 weeks. For almost all of these students, it will be the last mathematics unit they do during their degree. The third year unit MATH3063 usually has about 100 students most of whom are in the final year of their degree. These students are often majoring in mathematics or physics, but the cohort also contains future mathematics teachers, and also some engineers and economics students. These students attend three hours of lectures and one tutorial per week for 13 weeks. These students have already studied linear algebra, differential equations, vector calculus and many other topics in isolation and this unit will pull these individual areas of mathematics together. Most of these students do not intend to do further or postgraduate studies. Thus the first year cohort contains many students who would rather not study mathematics but are required to do so as part of their degree, whereas the third year cohort mostly contains students who have chosen mathematics or a highly quantitative discipline.

First year unit – MATH1013 Mathematical Modelling

Learning outcomes for MATH1013

The objective of this first year service unit is imparting concepts and enabling skills in the area of mathematical models (specifically differential and difference equations) covering important applications on equilibrium, stability, resource-limited growth and interactions in population models, pharmacology, sustainable harvesting, ecology and epidemics. The high level learning outcomes are:

- Compare and discuss the results of applying different models to the same data or situation.
- Understand the limitations of models, including when and why they fail.
- Recognise information presented in different forms, and convert or transform between equivalent forms.
- Classify, interpret and construct simple mathematical models.
- Extract useful information from a model or equation without solving it exactly, including the use of graphical arguments.
- Apply simple techniques in unfamiliar situations, including generalising from simple to complex systems and verifying the generalisations.
- Combine two or more techniques or steps to complete a complex task, including using simple models as building blocks.
- Use numerical exploration to aid in the understanding of the behaviour of models, including using tools, such as calculators, efficiently to estimate and approximate.

Note that the first three outcomes listed above have a direct relation to mathematical communication.

Assessment for MATH1013

Assessment is a combination of online quizzes, in-class tests, an assignment and the final exam. The regular online quizzes are used as a combination of formative and low-stakes summative assessment of procedural fluency in basic computational techniques, fundamental conceptual understanding and also skills relevant to communication such as notation and nomenclature. Supervised tests on the same material are also held during classes for summative assessment. Strategic competence (wisely choosing between many significantly different ways to attack the same problem) and adaptive reasoning (such as using logic and proofs) are not stressed or formally assessed except in so far as these skills help with the following journal reading assignment.
For the reading assignment, students were asked to choose a journal article on a topic of interest or relevance to them. They were free to choose any article from any scientific or medical journal but were also provided with a curated list of articles from the Undergraduate Journal of Mathematical Modeling: One + Two. This is a journal published by the University of Southern Florida and contains articles written by science undergraduates who are not majoring in mathematics. In other words, the authors are very similar to the cohort enrolled in MATH1013 though usually a couple of years older and more experienced. Many of the articles use mathematical concepts and techniques that correspond precisely to those learnt in MATH1013. This type of assignment has been used twice (in 2014 and 2015). The assignment was released early in the semester and was due near the end of semester. The assignment simultaneously covered cognitive, affective and conative dimensions. Students were asked to write a short 5 pages response to the journal article they read that covered the following points:

- Context: ‘Explain how the scientific aspects of the chosen article are related to your degree, specialisation or area of interest’
- Connections: ‘Explain how the mathematical aspects are related to concepts or techniques you have studied in this unit.’
- Motivation: ‘Give your personal motivation for choosing the particular article.’
- Mathematical Communication: ‘Take a calculation, derivation or diagram from the article and explain it in your own words.’
- Reasoning and other higher skills: ‘Show your depth of understanding by discussing how to extend the article you have read. You might discuss how to simplify, generalise, improve or otherwise change some aspect of the mathematical modelling or the mathematical presentation.’

The majority of students also gave fairly generic responses to questions about context and motivation such as ‘maths is important to science’ or ‘studying epidemics is important for society.’ Most students chose only from the list of articles provided. However, some students gave very detailed statements about the context and their motivation and even linking the article to something local to Australia or NSW.

I decided to choose this article [(Courchamp, Langlais and Sugihara, 2000)] because I could recognise the maths in it ... this is the first time I’ve actually read a scientific article ... introduction of rabbits to oceanic islands, which makes it relevant to my particular interest in Australian community ecology ... since volunteering and working at the Australian Museum my interest in the way native and introduced species interact has grown. (MG, 2014)

This article’s [(Taylor, 2010)] focus on the Trap-Neuter-Return program, one of several options in the NSW government’s feral animal population control arsenal, prodded my fear of a future wherein my cat does not return home. (CB, 2015)

The author [(Hussein, 2010)] is an environmentalist and Vice President of the Student Environmental Association at USF and was also the first student to receive the prestigious Udall Scholarship ... I am studying B.Sc. and aim to specialise in Zoology and Conservation (RA, 2014)

In the commerce aspect of my degree, we study Quantitative Business Analysis, where we look at the growth and fluctuations of a business/industry over time. The modelling of oil production [(Luong, 2012)]... is similar to, and useful in some aspects of analytics such as forecasting and trend analysis. (MC, 2015)
Generally, students were very good at specifying the connection between the mathematical content of the article they read and topics in MATH1013.

The paper [(Courchamp et al., 2000)] uses sets of first-order differential equations, and uses them to find equilibrium points and assess their stability ... modified according to Verhulst equation and the Constant Effort model covered in Chapter 5 of the lecture notes. (MG, 2014)

The article [(Hussein, 2010)] ... outlines a method of approximation ... the Euler method ... takes a nonlinear model ... it uses the simple recurrence relation from Chapter 6 of the notes. (RA, 2014)

As might be expected, students were less successful at the final part of the assignment where students were asked to modify the mathematical model in some way and many students gave very generic responses such as: the model could be improved by making it more realistic; or, the data could be improved by using a larger sample. Feedback from students about this assignment is presented in a later section.

Third year unit – MATH3063 Nonlinear Differential Equations and Applications

Learning outcomes for MATH3063
The objective of this third year regular unit is to introduce students to the geometric and qualitative theory of differential equations as it is applied to nonlinear systems. Deeper and unstated outcomes are to show the usefulness of mathematics learnt in earlier years and to show how previously separate topics such as algebra and calculus are linked and interact with each other. The learning outcomes include:

- Draw and interpret phase planes and bifurcation diagrams.
- Classify and analyse the stability of steady states and limit cycles.
- Analyse systems that operate on two timescales such as relaxation oscillators.
- Understand important models of epidemics, biological, biomedical and ecological systems.

Assessment for MATH3063
Assessment is a combination of in-class tests, several assignments (each of which is based on a journal article) and the final exam. Since most of these students are majoring in mathematics (or a related highly quantitative field), they already have positive attitudes towards mathematics and believe in its general relevance. They also understand the balance between the mathematical proficiencies and which aspects of the unit target which proficiencies. At the third year level, tutorial questions provide ongoing opportunities to maintain or refresh procedural fluency and develop basic conceptual understanding; but it is strategic competence and adaptive reasoning that become much more important and become the focus of the assignments. The nature of mathematical modelling at the senior level lends itself naturally to students having to make careful decisions before attempting to construct or analyse a nonlinear model.

As with the first year unit, the assignments were to critique and dissect a journal article. However, unlike in first year, the choice of journal articles was controlled by the lecturer to ensure close synergy with the mathematical content of the unit. In 2015, all three assignments were based around the same journal article (Gray 2002) which was a mathematical analysis of a famous oscillating chemical reaction. This article was written by a senior undergraduate student and submitted to the Undergraduate Mathematics Journal published by the Rose-Hulman Institute of Technology. Thus the author can be regarded as a peer of the cohort.
The first assignment was required at the beginning of the semester and involved extracting any aspects of the paper that could be understood using previously learnt mathematics (from first and second year) and to make informed guesses about the relationship between unfamiliar mathematics and topics to appear in this unit. The second assignment was to analysis a simplified version of the model in Gray (2002) and compare and contrast the results and the effect of the simplification. The final assignment was to reproduce the main conclusions of Gray (2002) but using an alternative mathematical method from that in the paper and also applying it to an analogous problem in neurophysics. This year, one of the resource used is a review in the journal Nature about bistability in ecosystems (May, 1977). The assignments this time are about filling in the details and verifying various mathematical claims based on the curtailed information given in the review article. These third year assignments are closer to traditional mathematics assignments, and thus do involve much more mathematical effort than the journal reading assignment provided to the first year students. The feedback discussed in more detail below suggests engaging with the scientific literature was a new but valuable experience.

**Student Feedback and Evaluation**

An anonymous online unit of study survey is conducted every semester in all units of study consisting of 9 standard questions using a 5-point Likert scale and two questions where a free text comment could be provided. The responses to the Likert scale questions are summarised in Table I and the two open questions ‘What have been the best aspects of this unit of study?’ and ‘What aspects of this unit of study most need improvement?’ are discussed below.

**Table 1: Proportion of students who agreed or strongly agreed on a 5 point Likert scale with various statements in Unit of Study Surveys. Statements relevant to the focus of this paper are in bold.**

<table>
<thead>
<tr>
<th>Statement</th>
<th>First Year Nov 2015</th>
<th>Third Year July 2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size of cohort</td>
<td>721</td>
<td>120</td>
</tr>
<tr>
<td>Number of survey respondents</td>
<td>148</td>
<td>61</td>
</tr>
<tr>
<td>Overall, I was satisfied with the quality of teaching by the teacher(s).</td>
<td>76%</td>
<td>97%</td>
</tr>
<tr>
<td>The work has been intellectually rewarding.</td>
<td>71%</td>
<td>90%</td>
</tr>
<tr>
<td>I developed relevant critical and analytical skills.</td>
<td>68%</td>
<td>90%</td>
</tr>
<tr>
<td>I have had good access to valuable learning resources.</td>
<td>73%</td>
<td>92%</td>
</tr>
<tr>
<td>The assessment tasks challenged me to learn.</td>
<td>68%</td>
<td>93%</td>
</tr>
<tr>
<td>I have been guided by helpful feedback on my learning.</td>
<td>56%</td>
<td>92%</td>
</tr>
<tr>
<td>Tutorials/seminars helped me to learn.</td>
<td>76%</td>
<td>90%</td>
</tr>
<tr>
<td>Staff were responsive to students.</td>
<td>78%</td>
<td>93%</td>
</tr>
<tr>
<td>Learning outcomes were clear to me.</td>
<td>68%</td>
<td>88%</td>
</tr>
</tbody>
</table>

The survey is not compulsory and response rates for online surveys are usually extremely low. The 20% response rate is typical for first year service units. The 50% response rate from the third year students was unusually high and amongst the very highest number of respondents for regular units of study in mathematics.

**First Year Feedback**

Over 2/3 of the respondents agreed that the learning outcomes were clear, that the work was intellectually rewarding, challenging and helped them develop relevant critical and analytical
skills. Only about 100 of the first year students gave responses to the open questions. The majority of open responses positive and negative were about people, timetables or issues with technology and only a few of the comments were about the content or structure of the course. Comments about the assignment and the authentic aspects of the course were quite polarised. Some of the positive comments were

‘The unit as a whole is an integrated subject, the concepts carry on from each other and provide a deep and enriching learning experience’

‘The applications of this subject to the real world was definitely the most interesting part of this course.’

‘The practical examples were excellent as we can see what x's and y's become when the theory stops and applications begin.’

‘The assessments helped me to gain more skills, especially the assignment and online homework, as it was very challenging.’

However, several students said they would have preferred a traditional assignment and the following negative comments were typical

I'm not sure if I liked the assignment - I would have rather just done a worksheet-type assessment.

Too much writing for a mathematics assignment, and was confusing. Felt that traditional assignment would have assisted learning much better.

Assignment on reviewing-describing/discussing a mathematical model used in a journal article did little in furthering understanding in the course content - preferable to have had to complete a series of actual mathematical problems.

The assignment is really unhelpful. I'd much rather a set of hard questions that I can complete, which will actually help me in the final exam or future mathematics studies.

That students might feel uncomfortable about such a task and its relationship to more traditional assessment and expectations about the final exam was not an entirely unexpected result. Studies of Problem Based Learning (PBL) across a range of disciplines (Abrandt Dahlgren & Dahlgren, 2002) have shown that giving students ‘the autonomy as regards formulation of learning tasks and choice of texts for reading creates a dilemma for the students. This dilemma concerns the freedom to formulate learning tasks by themselves, and at the same time meet the expectations that they are studying the relevant content at the proper level.’ Although it would be difficult to quantify any correlation between mathematical ability and engagement with this type of assignment, feedback obtained from several staff at the Mathematics Learning Centre revealed that even some struggling first year students really enjoyed the activity.

**Third Year Feedback**

Over 90% of the respondents agreed that the learning outcomes were clear, that the work was intellectually rewarding, challenging and helped them develop relevant critical and analytical skills. Most of the third year students also gave responses to the open questions. As with the first year cohort the majority of open responses were about people or infrastructure. Many of the comments were also about the content, structure and assessment.

Assignment progression was helpful and guided my learning, particularly the continued use of a resource (Gray, 2002).

The assignment structure was extremely engaging. I really appreciated the focus on understanding the toolkit we developed and applying it in novel ways rather the rote, bland ‘here are a selection of problems’-style assignments that other Maths courses choose to go with.
This units focus on really understanding mathematics and how it can be applied - rather than focusing on how to tackle example problems or the derivation of theories - made it a highlight for someone who was so bored in previous Math courses that they almost failed them.

I liked how the maths was being applied so its relevance was obvious. I also liked the structure of the subject as everything built on what had been done previously in class.

There was only one negative comment about assessment.

Assessment tasks, the way it’s structured isn’t great.

Discussion

We have used journal articles in both first and third year mathematics units to provide authentic contexts and provide students with an opportunity to discover links between mathematics, their other subjects, their personal interests and the real world and to develop or improve their communication skills.

The majority of students that answered the survey in both years expressed positive views about seeing mathematics in context and the relevance of these units and links to their other units. However, there was evidence that first year students who were interested in maximising marks were not convinced of the benefits of doing the reading assignment. It would be very interesting in further work to investigate whether such a strategic/pragmatic approach is correlated with performance in mathematics, performance in their other units, whether they tend to a surface or deep approach to learning or other personality traits.

A positive and unforeseen side-effect of requiring the third year students to read and discuss new mathematical ideas, appearing in an article before they had formally learnt them in class, was that they asked more in-depth and interesting questions during lectures. Perhaps this is because the article has created a virtual road-map (or concept map) of the mathematical ideas within the unit.

Further studies should determine whether these activities increase long term retention of mathematical ideas or influence decisions about later subject choice or career pathways.

References


