

# Using Pictorial Maps to Scaffold Problem Solving in Primary-Grade Arithmetic

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## Abstract

In this study, primary-grade students learned to solve and create arithmetic word problems using a three-phase process of visual representation. The study compared an experimental group (n=32) of third graders in Thailand using pictorial maps with a control group (n=31) using text-based problems. The visual representations called pictorial maps are unique in that they focus on place (location) in order to situate math problems in authentic contexts. In phase 1, students were given a pictorial map with imprinted objects representing keywords to help them solve a word problem. In phase 2, they used a blank pictorial map on which they could place plastic chips with imprinted images representing the problem's keywords. In phase 3, they used a blank sheet with cut-outs of images representing keywords that they could use to represent their own word problems. Results revealed significantly higher post-test scores for the experimental group. Implications point to the value of mathematics' teachers working with art teachers in their school to identify ways to use drawing to support representations of keywords and of other elements in word problems.

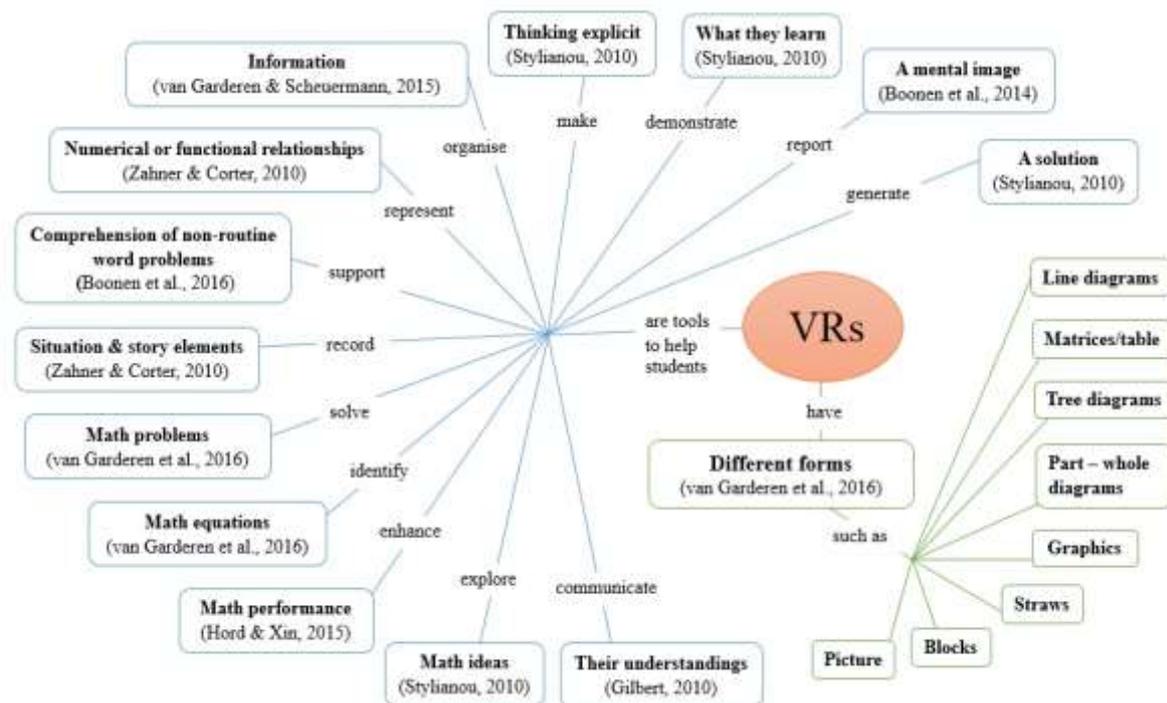
## Introduction

Performance in mathematics at the primary grades is important given that achievement at this level is a strong predictor of achievement in schooling in later years (Stern, 2009) and can affect future socio-emotional well-being (McCloskey, 2007). Yet, this performance may be of “particular concern for educators” given that scores tend to be lower than in other curriculum areas (Pearce, Bruun, Skinner, & Lopez-Mohler, 2013, p. 3). Peltier and Vannest (2017) found in their analysis of national assessment data in the USA that 60% of fourth-graders and 67% of eighth-graders failed to reach mathematical proficiency. Low scores are particularly common in the area of problem solving.

Morin, Watson, Hester and Raver (2017) argued that “math word problem solving continues to be a problem for many students [and] is especially difficult for students with mathematics difficulties” (p. 91). Lower scores may be due to the fact that assessment is “becoming increasingly a test of a student's ability to read and understand the problems” (p. 93). Vilenius-Tuohimaa, Aunola, and Nurmi's (2008) findings revealed a strong relationship between “performance on maths word problems [and] performance in reading comprehension” (p. 409). Fuchs et al. (2006) describe word problems as a complex set of elements including “working memory, long-term memory, attentive behaviour, nonverbal problem solving, language ability, reading skill, and concept formation” (p. 39). In general, therefore, “students' different reading levels can affect their problem-solving performances” (Özsoy, Kuruyer, & Cakiroğlu, 2015, p. 116).

In general, visual representations such as pictures “appear to enhance intuition and understanding in many areas of mathematics” (Blatto-Vallee, Kelly, Gaustad, Porter, & Fonzi, 2007, p. 434). Representations are important in general in terms of effective communication, in terms of problem solving and to “elicit student understandings” (Hill, Sharma, Obyrne, & Airey, 2014, p. 23). They are “an integral part of learning mathematics” particularly because of the intangible and invisible nature of math (Bobis & Way, 2018, p. 57). The National Council of Teachers of Mathematics (NCTM, 2000) emphasised the need for access to and creation of mathematical representations in order to “expand their [students’] capacity to model and interpret physical, social, and mathematical phenomena” (p. 4). The NCTM further recommended that students “create and use representations to organize, record, and communicate mathematical ideas... to model and interpret physical, social, and mathematical phenomena” (NCTM, 2000, p. 64).

Although the use of representations has played a role in mathematics education since classical Greece, symbolic reasoning and representation gradually dominated and it was not until this century with the use of computer-generated images that interest in visual representations in mathematics began to become important again (Stylianou & Silver, 2004). Figure 1 summarises visual representations in mathematics.



**Figure 1: Summary of visual representations**

Stylianou (2011) explained that representations such as “symbolic expressions, drawings, written words, graphical displays, numerals, and diagrams are all central to mathematics” (p. 265) but that both students and teachers have difficulty using them. Boonen, Reed, Schoonenboom and Jolles (2016) argued that teachers should be teaching students to create “their own visual representations than to provide them ready-made” (p. 60). Yet, teachers may lack the pedagogical knowledge needed to help students learn to do this (Stylianou, 2010). Similarly, Bobis and Way (2018) found that both researchers and teachers need to understand “the complex role that representations play in children’s learning of mathematics” and “how children’s minds create, interpret, and process representations” (p. 56). Bobis and Way

observed the lack of research on student-generated representations and recommended that researchers study representations (drawings) that students create themselves in order to gain insight into their thinking and understanding. Past research has tended not to focus on students' own visual representations but on "the impact of different types of illustrations, provided by the experimenter or another external source, on word problem solving" (Boonen et al., 2014, p. 23).

### **Purpose and objectives**

This gap in the literature provided the motivation for this study. In this study, the researchers first designed pictorial maps to support problem solving in arithmetic. These visual representations are unique in that they focus on place (location) in order to situate math problems in authentic contexts. The focus on place through pictorial maps allowed grade 3 students to 'map' out or depict the problem in relation to a location. The study used three phases whereby students were first given a pictorial map with visual representations of keywords to help them solve a problem. In the next phase, they were given a blank pictorial map on which they could place plastic chips with imprinted images related to the problem's keywords. In the final phase they used a blank sheet to represent the problem along with cut-out figures and objects representing the keywords.

The specific research question was as follows:

1. Does use of the maps improve students' ability to problem solve?

### **Review of the literature**

The review focused on studies conducted within the last five years (2013-2018). It was limited in scope to those studies conducted in mathematics within the primary grades (1-3). The review aimed to present a variety of types of visual representation.

**Table 1: Studies of visual representation in mathematics in primary**

<b>Study</b>	<b>VR type</b>	<b>Location</b>	<b>Grade</b>	<b>N</b>	<b>Types of student</b>	<b>Data collection</b>
van Lieshout & Xenidou-Dervou (2018)	Pictures (e.g., birds, children)	Nether - lands	1	60	High & low mathematics achievers	Experimental.
Liggett (2017)	Plastic unifix cubes	Canada	2	43	Regular	Experimental with pre & post-test.
Morin et al. (2017)	Bar model drawings	USA	3	6	Difficulty in Mathematics	Survey, Experimental with pre & post-test.
Driver & Powell (2015)	Symbolic equations & non-symbolic with picture	USA	2	413	With & without Mathematics difficulty	Experimental.

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Mulligan et al. (2013)	Vertical & horizontal grid lines of graphs/bars	Australia	1	21	High socio-economic backgrounds & range of cultural/ethnic groups	Observations & evaluation notes taken by the researchers.
Siew et al. (2013)	Tangrams	Malaysia	3	221	Regular	Experimental with pre & post-test.

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Results of these six studies all showed gains for participating students. Van Lieshout and Xenidou-Dervou (2018) found that a combination of pictorial representation with “auditory information” increased learners’ accuracy by decreasing cognitive load. Liggett’s (2017) study revealed significantly higher test scores for the experimental group using manipulatives versus the control group. Morin et al. (2017) combined schematic-based and cognitive strategy instruction with bar model drawing (unit bars) to solve word problems. They concluded that students’ bar model drawing in the younger grades can be built upon for use in solving more complex problems in later grades. Driver and Powell (2015) focused on use of pictures as a non-symbolic form for solving non-standard equations. They concluded that use of pictures may be particularly relevant in improving scores of students with mathematical difficulty. The authors recommended use of symbolic as well as non-symbolic representations in mathematics. Mulligan, Hodge, Mitchelmore, and English’s (2013) focus on graphing skills revealed that students took different approaches to representing elements such as gridlines and icons. The authors also noted progressive refinement of skills and that there are basic structural features that students need to learn. Siew, Chong and Abdullah (2013) combined phases of learning with tangrams to improve grade 3 students’ ability to move from visualisation to analysis in geometric thinking. Siew et al. found that low-ability students benefited from the approach more than the other students.

These studies have in common with the present study an underlying premise of a need to provide cognitive scaffolds in the form of representation to overcome the abstract and symbolic nature of mathematics. What is unique about this study is its focus on the notion of place or location as a basis for moving from the symbolic to the authentic. Also different in this study is the three-phase approach designed to ultimately allow students to create their own representations of problems. Finally, a unique approach taken in this study is the visual representation of keywords on plastic chips for use by students.

### **Context of the study**

The study was conducted in Thailand where the primary grade mathematics curriculum (see Mullis, Martin, Goh, & Cotter, 2016) focuses on problem solving as part of “The Basic Education Core Curriculum.” Learning standards are divided into six strands, one of which is mathematical Skills and Processes. This strand emphasises the following: “problem solving and reasoning; communicating and presenting mathematical concepts; ...and connecting

mathematics with other disciplines; and creative thinking.” Thailand participated in the 4<sup>th</sup> grade Trends in International Mathematics and Science Study (TIMSS) in 1995, and in 2011 (National Center for Education Statistics, n.d.). Praktipong and Nakamura (2006) noted regarding the 1995 results that Thai students were “fifth from the bottom among 26 participating countries” (p. 111). Students were particularly weak in problem solving. Testing in 2002 for grade three students (see Office of the National Education Council, 2004, p. 160) showed a mean of 15.7/30. Praktipong and Nakamura (2006) concluded that those who performed poorly “had linguistic and conceptual comprehension problems” (p. 120). Phonapichat, Wongwanich and Sujiva (2014) interviewed Thai math teachers to identify “problem solving difficulties.” Their findings revealed difficulties reading “keywords appearing in problems” (p. 3172).

## Methods

### Non-student participants

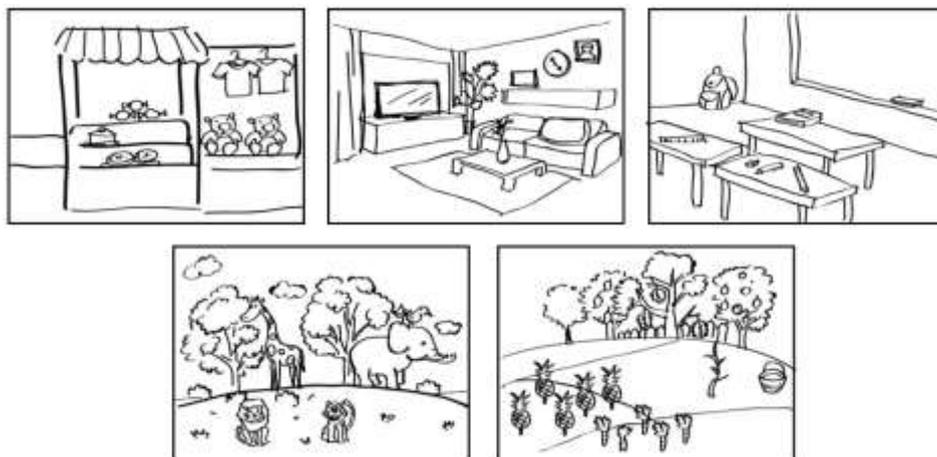
A graphic designer was responsible for transforming the storyboards created by the researcher into pictorial maps. Two participants, one with a Ph.D. in mathematics and another with a Masters in mathematics, provided input on the content and on the pre- and post-test. One participant with a Ph.D. in elementary education also provided input on the design, content and testing. There were also three participants with a Ph.D. in Educational Technology who provided input and suggestions on the design in terms of its educational effectiveness.

### Student participants for prototype testing

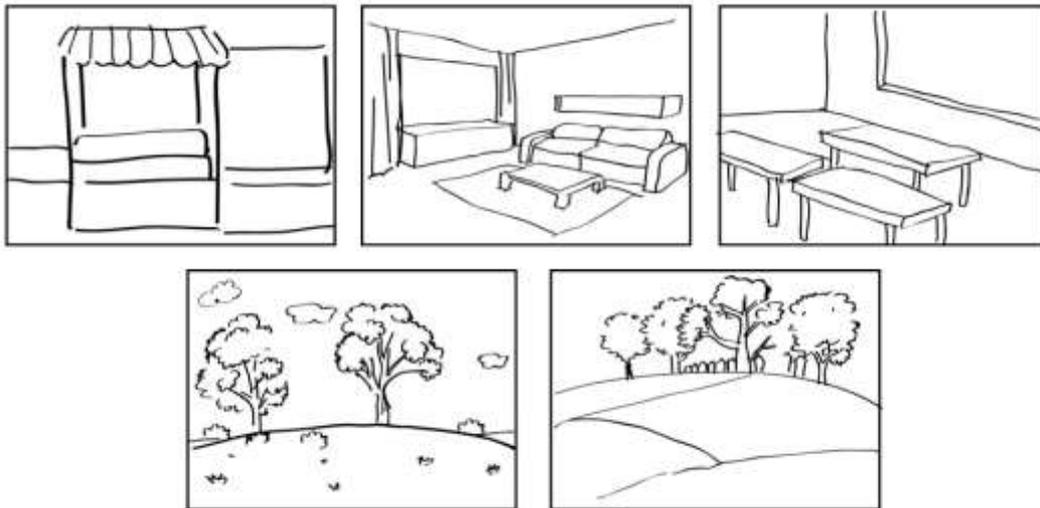
The prototype was tested twice. For the first testing, participants were 30 learners in one grade-three class from a school in north-eastern Thailand. For the second testing, the 12 learners were from a school in Bangkok.

### Prototype creation

The first step in the creation of the prototype involved drawing the storyboard. The (principal investigator) PI drew by hand a prototype that represented a place-based situation. Next, the PI used a digital design tool (smart pen & drawing pad) to redesign the same images. The PI sent the storyboard to a graphic designer who used Adobe Illustrator to create a digitised final version. Figure 2 shows a prototype of maps with objects (keywords). Figure 3 shows a prototype of maps without objects (keywords).

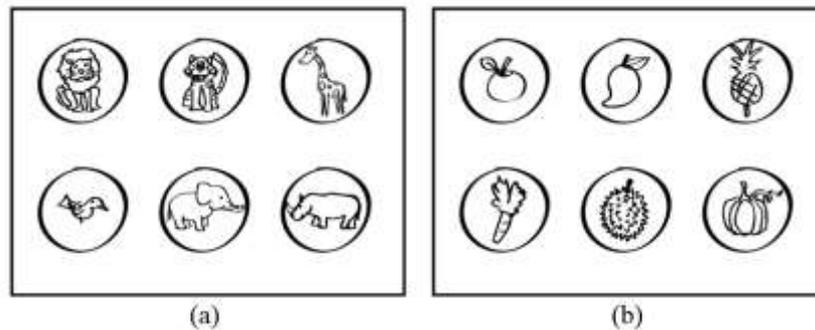


**Figure 2: Prototype of maps with objects (keywords)**



**Figure 3: Prototype of maps without objects (keywords)**

The design also included the creation of round plastic chips which represented both objects, persons, symbols and numbers that students could use to place on blank maps in order to represent keywords. Figure 4(a) 4(b) shows the initial design.



**Figure 4: Initial design of chips (keywords)**

### Procedures

After the design, the PI sent the five pictorial maps and chips to the specialists mentioned in the previous section. They assessed the match between contents and language use and gave suggestions. The first prototype testing took place at the school during the students' regular mathematics' period. Testing took place one hour per day for three days. The regular teacher and PI were both present and could observe as students were working. The first prototype test involved assessing students' ability to integrate information, understand the content and use graphical representations, appropriate font size and figures. The students were also evaluated regarding the overall time required to complete tasks.

The second prototype testing also involved three separate, one-hour sessions with the PI and mathematics' teacher. For the second prototype testing, the focus was on identifying any language difficulties, on the interpretation of graphical representations and on the types of representations that students preferred.

### **Implementation participants**

Participants for implementation were 38 boys and 25 girls (ages 8-9) (N=63) from two (one experimental and one control) grade-three classes in a public primary school in Bangkok. The students were from the same school where the second prototype testing took place. This school was chosen because the teacher had already established ethics' permission to carry out the study in this school and had established a rapport with its teachers.

### **Implementation procedures**

Implementation lasted 12 weeks. The PI worked with each of the control and experimental groups for three hours per week, one hour at a time. The PI gave both the control and experimental groups of students the same arithmetic problems (subtraction or addition) within the same length of time. The problem for both groups was situated in one of five locations: home, market, school, zoo and garden. An example is: Lin, Jane and Amy were at the market. Lin bought 19 donuts. Amy bought 40 donuts. Jane bought 12 more donuts than Amy. How many more donuts did Jane buy than Lin?

The teacher was present and the PI followed the procedures that the teacher normally followed to teach about problem solving. For the control group, students read the word problem. They were instructed as usual to find keywords in the problem. For the experimental group, there were three phases. For phase 1, the PI gave the experimental group a pictorial map related to the problem. On the map were pictures of the keywords related to the problem. The pictorial maps were a direct visual representation of the problem. The student was given the problem and could see it visually before solving it.

For phase 2, after students had been instructed in and given practice with this phase 1 approach, the PI then gave them pictorial maps on which there were no items corresponding to the keywords. Instead, the keywords were designed as images and imprinted onto plastic chips. For example, if there was an elephant in the problem, students could choose a chip with the elephant. Students could also use math symbols and number chips. They could then create visual sentences by placing chips on the maps.

For phase 3, the PI gave students large sheets of paper with an area delimited for drawing. They were also given a sheet with premade cut-outs for keywords so that they could create their own problems and depict them using pictorial maps.

### **Testing participants**

Participants for the testing were the same as those students for the implementation.

### **Testing procedures**

In the first week, all 63 students completed a pre-test on all content that would be covered in the implementation. For each of the four sub-topics (see Table 2), the PI also administered pre- and post-tests prior to and after instruction.

**Table 2: Summary of testing**

Weeks	Contents	Test
1	A, B, C, D	Pre-
2-3	A. Compare and order numbers in word problems	Pre- Post-
4-6	B. Addition word problems	Pre- Post-
7-9	C. Subtraction word problems	Pre- Post-
10-11	D. Complex word problems	Pre- Post-
12	A, B, C, D	Post-

### Instrument

The pre- and post-tests were evaluated by the mathematics' and education specialists described in the previous section. The tests were completed in the classroom by students using paper and pencil and the Thai language. The tests were adapted from heuristics in mathematics by Goh (2015). One example is: Paul and Jack go to the market. They bought 1200 marbles. Paul received 150 marbles more than Jack. How many marbles did Jack have?

### Data analysis

Analysis of the pre-test and post-test scores involved independent two-sample t-tests to evaluate the difference between the means of two independent or unrelated groups. The testing was conducted to compare the significance of scores between the experimental group (learning with pictorial maps) and the control group (learning with text).

### The design of the maps

The final design resulted in the creation of five maps. These maps were used in phase 1 of the implementation to get students used to understanding visual representations of problems. The maps include the objects representing keywords from the problems. Figure 5 shows these maps.



**Figure 5: Phase 1: Pictorial maps with objects representing keywords**

Figure 6 shows the blank maps used in phase 2. Figure 7 shows the chips to be used with the maps for representing keywords from the problems.

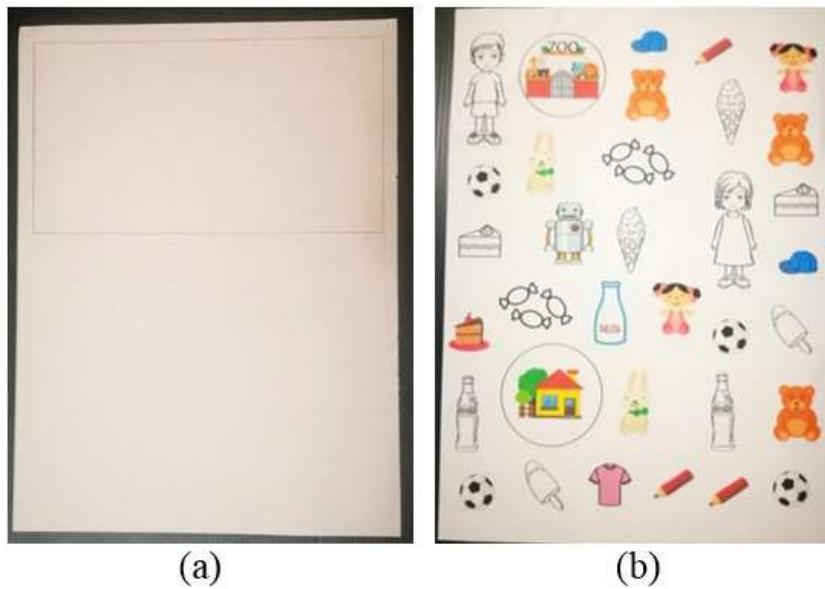


**Figure 6: Phase 2: Pictorial maps without objects representing keywords**

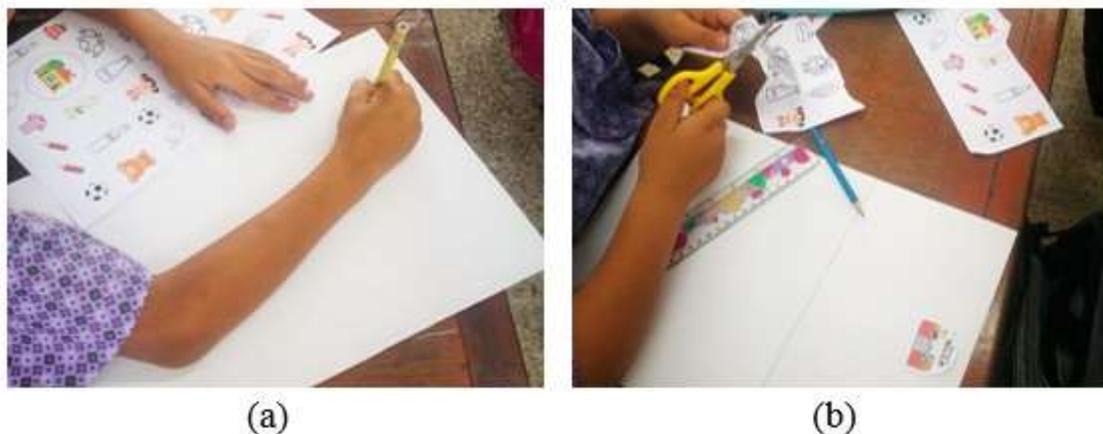


**Figure 7: Phase 2: Use of chips with blank maps**

For phase 3 of the implementation, Figure 8a shows the blank sheets and Figure 8b shows the sheets of cut-out items that students could use to create their own visual representations. Figure 9a shows one student using the blank sheets for writing his own problem. Figure 9b shows the student using the cut-out items for the keywords.



**Figure 8. Phase 3: Blank sheets and cut-out items**



**Figure 9. Phase 3: Student's use of the blank sheets with cut-out items**

## Results

An independent t-test was used to determine the difference between the means of the pre-test and post-test of the experimental group (E) (learning with pictorial maps) versus the control group (C) (learning with text) for the content related to comparing and ordering numbers in word problems. The means and standard deviations (SD) for pre-test and post-test are displayed in Table 3. The pre-test mean for the experimental group was 4.53 (SD = 0.803) and the control group was 3.97 (SD = 1.169). The post-test mean for the experimental group was 6.28 (SD = 0.888) and the control group was 5.35 (SD = 1.226). The t-test results showed a significant difference for the experimental group ( $p = 0.001$ ). These results indicate that the use of pictorial maps was more effective for comparing and ordering numbers in word problems than text alone.

**Table 3: Results of comparing and ordering numbers in word problems**

Group	Test	N	$x$	SD	$t$	p
E	Pre	32	4.53	0.803	2.237	0.029
C		31	3.97	1.169		
E	Post	32	6.28	0.888	3.442	0.001*
C		31	5.35	1.226		

\*P&lt;0.05

Table 4 shows the results for addition word problems. The mean for the pre-test with the experimental group was 5.50 (SD = 0.950) and the control group was 5.06 (SD = 0.727). The mean for the post-test with the experimental group was 7.50 (SD = 0.950) and the control group was 6.16 (SD = 0.969). The analysis of the test data indicated significant treatment effects, controlling for pre-test and post-test,  $p = 0.000$ . These results indicate that the use of pictorial maps was more effective for addition word problems than text alone.

**Table 4: Addition word problem results**

Group	Test	N	$x$	SD	$t$	p
E	Pre	32	5.50	0.950	2.038	0.046
C		31	5.06	0.727		
E	Post	32	7.50	0.950	5.535	0.000*
C		31	6.16	0.969		

\*P&lt;0.05

Table 5 shows results for subtraction word problems. The mean for the pre-test with the experimental group was 5.34 (SD = 0.827). The control group was 4.87 (SD = 0.922). The mean for the post-test with the experimental group was 7.19 (SD = 5.84) and for the control group, 5.84 (SD = 1.128). The analysis of the test data indicated significant treatment effects, controlling for pre-test and post-test,  $p = 0.000$ . These results indicate that the use of pictorial maps was more effective for subtraction word problems than text alone.

**Table 5: Subtraction word problems results**

Group	Test	N	$x$	SD	$t$	p
E	Pre	32	5.34	0.827	2.144	0.036
C		31	4.87	0.922		
E	Post	32	7.19	0.965	5.104	0.000*
C		31	5.84	1.128		

\*P&lt;0.05

Table 6 shows the t-test results for complex word problems. The mean for the students' pre-test with the experimental group was 5.13 (SD = 0.871). The control group was 4.97 (SD =

1.048). The mean for the post-test with the experimental group was 7.06 (SD = 1.014) and the control group was 6.06 (SD = 0.929). Results indicate significant treatment effects controlling for pre-test and post-test,  $p = 0.000$ . These results indicate that the use of pictorial maps was more effective for complex word problems than text alone.

**Table 6: Complex word problems results**

Group	Test	N	$\bar{x}$	SD	$t$	$p$
E	Pre	32	5.13	0.871	0.649	0.519
C		31	4.97	1.048		
E	Post	32	7.06	1.014	4.070	0.000*
C		31	6.06	0.929		

\* $P < 0.05$

Table 7 shows the results of the independent t-test used to determine the difference between the means of the pre-test and post-test of the experimental group. The overall pre-test mean for the experimental group was 13.72 (SD = 1.971) and the control group was 11.13 (SD = 2.247). The overall mean for the post-test with the experimental group was 16.41 (SD = 1.811) and the control group was 14.23 (SD = 1.854). The analysis of the data indicated significant overall treatment effects, controlling for pre-test and post-test,  $p = 0.000$ .

**Table 7: Results for initial and final pre- post-test for all areas**

Group	Test	N	$\bar{x}$	SD	$t$	$p$
E	Pre	32	13.72	1.971	4.867	0.000*
C		31	11.13	2.247		
E	Post	32	16.41	1.811	4.718	0.000*
C		31	14.23	1.854		

\* $P < 0.05$

For phase 3, results revealed that students in the experimental group could easily create pictorial maps. They could integrate information into the pictorial maps and create questions relevant to problem solving. They could also systematically arrange the story elements in chronological order. For example, Figure 10 (a, b, c & d) shows the student going to the zoo. Figure 10a shows that, at the zoo, he saw a rabbit. He bought the rabbit for 150 baht. Then, he brought it to his house. Later, he bought food for the rabbit (20 baht) and ice cream (10 baht) for himself. Underneath (Figure 10a), he wrote the math problem. In Figure 10b, another boy shows that he went to the zoo. He bought three ice creams. In the description, he explains that, before he went to the zoo, his mother gave him 100 baht. Then, his father gave him 500 baht. Each ice-cream cost 60 baht. He diagrammed the addition and subtraction problem to show that he had 420 baht remaining.

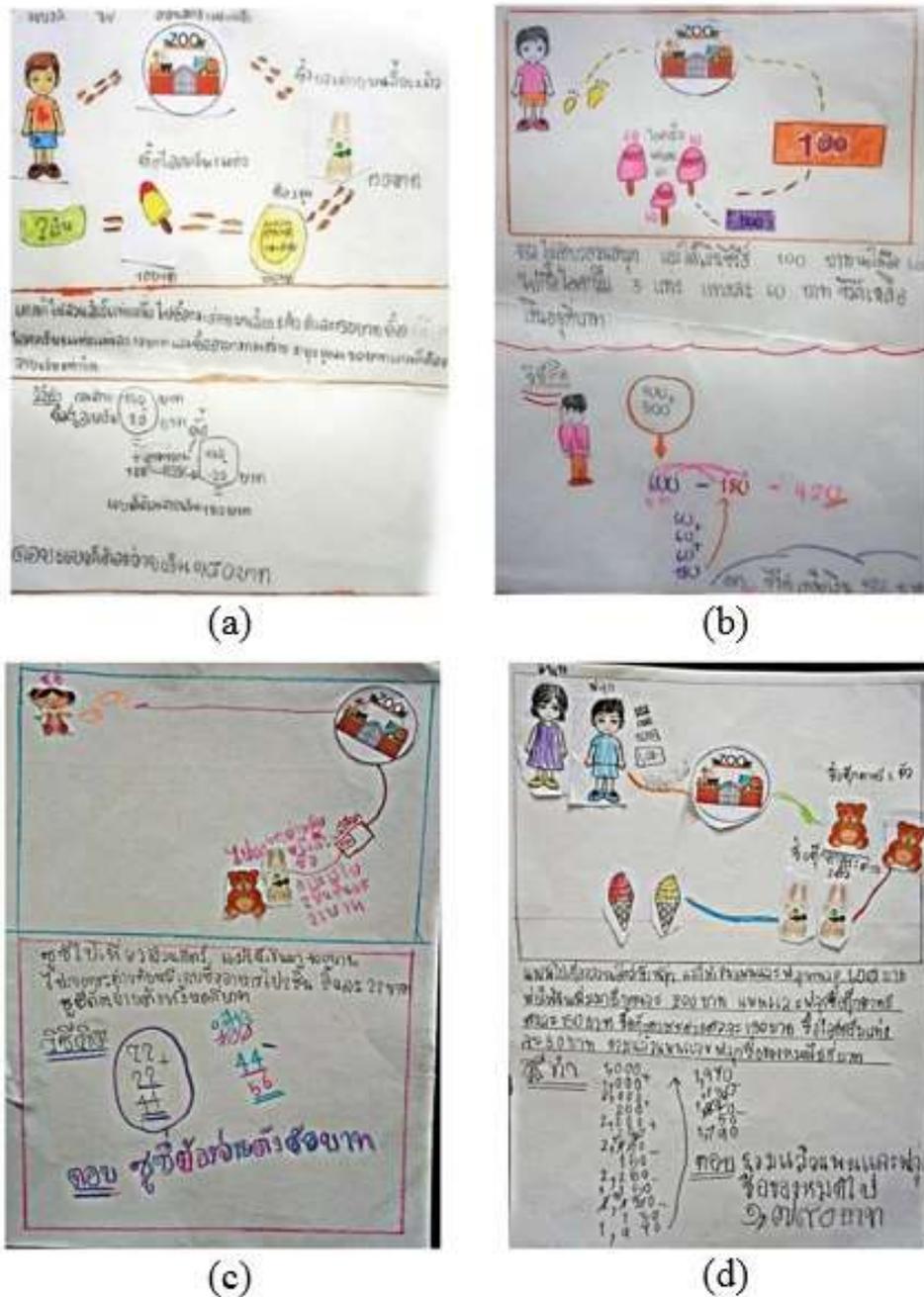


Figure 10: Students' depictions of problems using their own maps

## Discussion

This study was motivated by a need to identify ways to improve the low levels of problem solving ability in primary arithmetic. Given that primary-grade learning provides a foundation for later success, interventions at this level are key to improving performance in later years. The study was also motivated by the need to devise methods to scaffold students' ability to problem solve through visual representation. Visual representation has already been shown to help students overcome problems related to reading which may interfere with problem solving. Previous studies have relied on manipulatives and forms of pictures to help students problem

solve. This study focused on gradually building students' ability to create their own visual representations. It also focused on visual representation of keywords.

The study relied on a three-phase process to scaffold students' problem solving. The process moved from the more to the least abstract representations. In the first phase, students used a pictorial map of a place in which the problem was situated. The keywords related to the problem were already featured on the map. The map was designed to give the student a visual, concrete representation to support solving it. In phase 2, students once again used the pictorial maps. However, in this phase, there were no items corresponding to the keywords. Instead, students could choose to place on the maps, keywords, math symbols and numbers in the form of pictures on plastic chips. They could then create visual depictions of the problems by placing chips on the maps. For phase 3, students were responsible for creating their own pictorial maps using large sheets of paper with an area delimited for drawing a scene. They could use premade drawings depicting the keywords then place them on their own drawings to represent a word problem that was relevant to them and that they created themselves.

Results revealed significantly higher post-test scores for the experimental group. The success of the pictorial maps is consistent with the findings of studies of other forms of visual representation such as those reported in this paper's review of the literature. That success is likely due to the cognitive scaffolding that representation offers. It may be due as well to linguistic support for visual representation of keywords. One of the unique contributions of this study is the use of the 'chips' to represent keywords. As noted in this paper's introduction, reading has been recognised as a barrier to students' ability to solve word problems. Use of the visual keywords on chips offers scaffolds and reinforcement to the process of solving the word problems.

Other explanations may relate to the findings of Rasmussen and Bisanz (2005). In their study, pre-schoolers doing arithmetic relied on "visual-spatial working memory" (p. 137). Chu, Rittle-Johnson and Fyfe's (2017) explanation of the role of representation in mathematics is in terms of grounding content in "familiar experiences [that] connect with learners' prior knowledge" (p. 285). Hoogland and Pepin (2018) referred to the value in problem solving to depictive approaches using real-life images versus descriptive approaches for representing "the problem situation" (p. 37). The authors explained that descriptions using words tend to put students in an "answer-getting mind-set" whereas visual depiction requires more "sense-making" on the part of students so that they are "more likely [to] adopt a problem-solving attitude" (p. 38).

Another unique contribution of this study is the reliance on place and location to concretise and make real and less abstract the context of the problem. This approach provides support to the student who can link the familiar (i.e., places such as home and school) with what would normally be abstract and symbolic. It was beyond the scope of this study to determine if students' problem-solving skills might be transferred to real life. However, we can hypothesise that such transfer may be more easily accomplished given the connections made between the real world and the world of arithmetic. In general, to promote mathematical understanding, teaching must involve "multiple representations" on top of "the symbolic or abstract content" (Nordin, Tengah, Shahrill, Tan, & Leong, 2017, p. 199).

## Conclusions

This study has highlighted the value of visual approaches to representing word problems in arithmetic. Specifically, it highlights the value of supports for reading and for understanding keywords by turning them into objects that students can manipulate and use to depict problems. The significant gains made by the experimental versus the control group provide evidence for approaches that concretise and situate learning in real contexts. This study relied on pictorial maps with chips for keywords. Those interested in replicating the study's approach in other contexts may wish to use different types of pictorial maps and not necessarily chips.

In terms of implications for practice, what is important is devising a means to move the problem-solving process away from the abstract and symbolic towards more concrete, authentic forms. This study was premised on the argument that place and location provide an effective means to do this. Furthermore, the study was premised on the need to find alternate ways of allowing students to work with keywords such as with chips or other means. Mathematics' teachers working in interdisciplinary contexts or in project-based learning contexts may wish to work with Art teachers in their school to identify ways to use drawing to support representations of keywords and of other elements in word problems. Other approaches to visualization in mathematics' education involve use of digital technologies (see Birt & Cowling, 2017) that offer opportunities for "interactive, sensory-rich, experimental activities with greater opportunities for student input and creativity" (p. 1).

The study was limited to only one context in Thailand that may not be representative of other countries. It is up to the reader to identify the relevance to his or her context. However, given the poor performance of many primary students in many other contexts besides those of this study, it is likely that this approach would be relevant outside of the setting in which it was conducted. The study was conducted in the primary grades but its results may also be relevant at the elementary level through activities and visual representations that are appropriately designed for the grade level. The results reveal that the control group is a lower performing group than the experimental group since it obtained lower results for each pre-test. The results may need to be interpreted with this difference in mind. The differences obtained, although significant, cannot be completely attributed to the effect of the intervention, as there is room to think that the composition of the two groups could be the cause of a part of the difference. Given that the experimental group has higher pre-test scores than the control group, it would be of value for other researchers to try to replicate the study in other contexts with other groups of students.

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