What Do Bridging Students Understand by ‘Assumed Knowledge’ in Mathematics?

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Abstract
Over 100 students taking part in mathematics bridging courses were asked in a survey about their understanding of ‘assumed knowledge’ for studying mathematics units at university. Further data were obtained by email from 16 students who agreed to further participation. A phenomenographic analysis was carried out on all responses to obtain categories for students’ conceptions of ‘assumed knowledge’. A two dimensional outcome space was proposed, with the categories increasing in complexity and expansiveness on each dimension. One dimension related to students’ understandings about the purpose of ‘assumed knowledge’, while the other pertained to the content or substance of the ‘assumed knowledge’. We termed these aspects the ‘why’ and ‘what’ of ‘assumed knowledge’ conceptions. The results show the diversity of student awareness about ‘assumed knowledge’ ranging from perceiving it as vague and pointless ‘stuff’ to a cohesive body of foundational knowledge for tertiary study. The study provides qualitative data relevant to the debate on prerequisites versus ‘assumed knowledge’ for university entry.

Introduction

The topic of prerequisites for university entrance is a controversial one in Australia. On the one hand universities are competing for students, thereby making it difficult to insist on the completion of advanced mathematics units for entry, even into science or engineering courses. On the other hand Australia’s Chief Scientist, Professor Ian Chubb, calls on universities to insist on mathematics as a prerequisite for particular pathways of study (King, 2014).

At our university, admission into most Science courses is on the basis of a secondary school leaving qualification, such as the Higher School Certificate (HSC) in New South Wales, or other approved program or entry path. While some universities in NSW prescribe prerequisites for students enrolling in mathematics, our university, in common with nine others, specifies the level of ‘assumed knowledge’ or ‘recommended studies’ for most junior units (first year or first level) in mathematics. Since these are advised rather than prerequisites for entry into the unit, it is up to the student to ensure their preparedness. Studying a bridging course is one way that students can improve their backgrounds in mathematics. But what does ‘assumed knowledge’ mean to these students? We describe an empirical study where we surveyed students taking part in mathematics bridging courses at our university and focus on responses about students’ understandings of ‘assumed knowledge’.

Research on the secondary-tertiary transition indicates many factors underpinning entering students’ difficulties with mathematics. Cox (2001) compared expectations of various departments with the probable preparedness of incoming students on a range of mathematical topics, including topics in algebra, calculus, trigonometry and indices and logarithms.
Students’ capabilities were estimated quantitatively and drew on departmental requirements for a topic as ‘required’, ‘preferred’ or ‘not specified’ as described by Sutherland and Dewhurst (1999). Findings showed that there were mismatches ranging from modest to significant between departmental expectations and student preparedness, and, in some cases, students lacked proficiency even in ‘required’ topics. Cox (2001) notes that while students may be able to catch up in some topics during their first year studies this may be an unreasonable expectation for other topics.

Beyond differences in specific mathematical content, there is a change in culture from secondary level mathematics to tertiary level: from a concentration on technical problem solving skills towards a focus on understanding more abstract concepts and rigorous methods (Leviatan, 2008). In addition, there are aspects that influence how students transition to higher education more generally, including how students cope with anxiety, their levels of motivation and their study skills (Gibney, Moore, Murphy and O’Sullivan, 2011). Factors cited as contributing to mathematics students being ‘at risk’ in engineering (Steyn and Du Plessis, 2007) include under-preparedness in mathematics, including a lack of understanding fundamental mathematical concepts, a changed teaching environment and an inability to cope with the demands of tertiary education. More specifically, Gueudet (2008) describes individual, social and institutional phenomena that cause ‘ruptures’ (p. 238) in the secondary-tertiary transition in mathematics. These include different modes of thinking, less time to practise what is learned, more diversification of topics, increased integration of knowledge, and a focus on meaning, rather than routines. Gueudet (2008, p. 249) sums up this change in learning mathematics as follows: ‘At secondary school, students just have to produce results. At university, they seem to have an increasing responsibility towards the knowledge taught’.

Research has shown too that students’ performance in tertiary mathematics units is related to the mathematics units they studied at secondary school (Jennings, 2009) and their performance in mathematics at secondary school (Wilson & MacGillivray, 2007). A further issue is how much of their senior secondary mathematics students actually remember and can use (Jennings, 2011). The declining numbers of students studying mathematics at the more advanced levels of secondary school in Australia (Barrington, 2012) and overseas (Hoyles, Newman and Noss, 2001; Hourigan & O’Donoghue, 2007) adds to the concern about students’ preparedness for a considerable number of university courses.

The lack of awareness of the importance of mathematics for the future study of engineering and science may also play a part in students’ preparedness. This was discussed at the National Forum on Assumed Knowledge in Maths: Its Broad Impact on Tertiary STEM Programs, held in February 2014 at the University of Sydney. The Forum participants, who were tertiary educators, considered that the lack of mathematics prerequisites ‘has resulted in a perception by students that mathematics is not necessary for engineering or science’ and that the confusion about the meaning of ‘assumed knowledge’ by students, parents and possibly secondary school teachers is ‘due to the wide variety of policies and terminology used by institutions’ (King & Cattlin 2014, p.6). A recommendation of the Forum developed for immediate action calls for statements about assumed knowledge requirements ‘to be clear and unambiguous allowing students to identify the essential skills needed’ (King & Cattlin, 2014, p.7).

In previous research we investigated the experiences and perceptions of students and teachers in mathematics bridging courses (Gordon & Nicholas, 2010, 2011, 2013a & 2013b). Key
challenges for teachers (Gordon & Nicholas, 2010) included the range of backgrounds of students, the complexity of the concepts and students’ perceptions about mathematics and themselves as learners of it, while students reported that learning new notations and mathematical concepts was daunting and reported difficulty adjusting to independent study and learning in the ‘university way’ (Gordon & Nicholas, 2011, p. 115). Students’ reasons for taking lower levels of mathematics in senior year(s), or dropping mathematics, documented in Gordon & Nicholas (2013a), included finding enough time for non-mathematics subjects, lack of confidence in mathematical capability, advice and perceived strategies for maximising potential ranking for university admission. We now focus on an important part of the bridging course students’ prior decision-making: their understandings of ‘assumed knowledge’ for their mathematics units at university.

Methodology

Context of the study
In NSW, there are four levels of mathematics available for study at senior secondary level: an elementary pre-calculus level called General Mathematics, an intermediate level called HSC Mathematics and two advanced levels called HSC Mathematics Extension 1 and HSC Mathematics Extension 2. Students may also decide not to study mathematics at all at senior secondary level.

Students enrolling at university have the option of studying a mathematics bridging course if they are mathematically under-prepared for their studies. At our university, there are two mathematics bridging courses; a 2 unit course which introduces the ideas of differential calculus for students who have not studied calculus before, and an Extension 1 bridging course for students who have previously studied the intermediate level of mathematics but are enrolling in degrees such as engineering or science degrees with an intended major in physics, computer science or mathematics. Since HSC Mathematics Extension 2 is usually taken only by mathematically very able students there is no bridging course to this level. The mathematics bridging courses are fee-paying, open to students from other universities and consist of 24 hours of class time held over a 12 day period prior to the beginning of the first semester.

Data collection
All students (n = 380) enrolled in the mathematics bridging courses at our university in 2012 were invited by email to take part in an anonymous online survey. The participation information statement was emailed to all bridging course students giving details about the study as approved by our ethics committee and providing the link to the survey on SurveyMonkey. One hundred and nine students took part in the survey. The last question asked if students were willing to provide further information to us by email. Sixteen students agreed to take part in this second round and provided us with their email address.

The online survey asked for some optional demographic information and then explored the influences on students’ decisions to study the level of mathematics they did in years 11 and 12 of senior secondary school by a series of open and closed questions. Survey results about students’ prior decisions and experiences about mathematics are reported in Gordon & Nicholas (2013a).

In this paper, we report on the responses of 86 students who answered the open-ended question: What do you understand by ‘assumed knowledge’? Of the 86 students who
answered this question, 62% (n=53) were female and 62% (n=53) were 20 years old or less. The majority of students, 60% (n=52), were enrolled in the 2 unit mathematics bridging course. The responses to this question together with 16 email responses to the question: ‘Could you elaborate on your understanding of assumed knowledge of maths for your university study?’ provided us with a data set of about 3000 words.

Analysis
In this qualitative study, we adopted a phenomenographic approach for our analysis to reveal our students’ awareness of the phenomenon — their collective understandings of ‘assumed knowledge’ — in context.

The outcome space is a system of categories of description that describe qualitatively different ways of experiencing a phenomenon and the relationships between them. They represent the total variation of awareness in the group at the collective level. The set of categories are usually hierarchical in the sense of increasing or expanding scope or complexity (Marton & Booth, 1997) from the narrowest conceptions to the most expansive. Thus, the ‘highest’ categories may include awareness of previous categories but not vice versa. In this approach to data analysis, the researcher explicitly adopts a second-order perspective by focusing on other peoples’ experiences ‘trying to see the phenomenon and the situation through [his or] her eyes, and living [his or] her experience vicariously’ (Marton and Booth, 1997, p. 121). Thus, in taking a second-order perspective, the researcher must consciously step back from his or her experience or understanding of the phenomenon and ‘use it only to illuminate the ways in which others are talking of it, handling it, experiencing it, and understanding it’ (Marton & Booth, 1997, p.121).

The data analysis proceeded with readings of the data set as a whole, searching for similarities, differences and complementarities between students’ statements and selecting themes or ideas that delineated the categories. This was an iterative process with each of us reading the students’ responses and discussing and reviewing the categories until a clear set of statements defining each category were agreed.

We illustrate each category in the outcome space — the set of categories of description — by a collection of quotations from the student responses. In doing so, we are not attempting to classify conceptions of individual students. We concur with Marton (1981, p. 180) that the aim of phenomenographic research is ‘not to classify people, nor is it to compare groups, to explain, to predict, nor to make fair or unfair judgments of people. It is to find and systemize forms of thought in terms of which people interpret aspects of reality …’.

Thus, our aim in this qualitative study, in common with diverse other phenomenographic studies (Bliuc, Ellis, Goodyear & Piggott, 2011; Gordon, Reid & Petocz, 2010; Prosser & Millar, 1989) is to capture the full range of variation of understandings in the student responses as they appeared. It is the ‘very identification of the different ways of experiencing a phenomenon and the variation thereby constituted [that] are a legitimate outcome and worthwhile outcome’ of a phenomenographic research study (Marton & Booth, 1997, p. 128).

Results

Descriptions of the outcome space
In this section we put forward phenomenographic categories for students’ understandings of ‘assumed knowledge’. Two dimensions of students’ awareness or understandings of ‘assumed knowledge’ are proposed. The first aspect refers to how students’ conceive of the purpose of the ‘assumed knowledge’, that is, the student’s focus is on ‘why’ there is ‘assumed knowledge’. The second dimension concerns the content of the ‘assumed knowledge’, that is, the student’s focus is on ‘what’ is ‘assumed knowledge’. Below we first describe the categories along each of the two dimensions. These are summarised in Figure 1, which shows the total outcome space. We then provide illustrative quotes from the students’ emails and surveys for the outcome space in three tables.

**Conceptions: the ‘why’ dimension**

On this dimension, five categories are proposed. These are hierarchical in the sense of increasing complexity and expansiveness.

1) **Knowledge with no purpose**
   - The first category indicates little engagement with the concept of ‘assumed knowledge’ and no real awareness of its purpose.

2) **Beneficial knowledge**
   - In this category the student is aware that having the ‘assumed knowledge’ will be of benefit in some way, but this benefit is not specified or articulated.

3) **Expected knowledge**
   - This category shows an awareness that the student is expected to have this knowledge. Here the focus is external to the student – the expectation of some third party who, in many cases, would be the lecturer. However, there is little or no awareness of its utility.

4) **Required knowledge**
   - This category includes the previous one but is more expansive in that there is an awareness that not only is the student expected to have studied the ‘assumed knowledge’ but that it has utility to the student. The student knows that he or she will need to use that knowledge in the future and there will be consequences for not having it.

5) **Foundational knowledge**
   - This is the highest category. The main awareness unique to this category is that the stated ‘assumed knowledge’ must be well understood because it forms the foundation for learning mathematics at university. The student is aware that not only will the ‘assumed knowledge’ will be used and there will be consequences for not having it, but that the ‘assumed knowledge’ forms the very foundation – the building blocks – for future study. This is an expanded awareness of the previous category.

**Conceptions: the ‘what’ dimension**

On this dimension three categories are proposed. Again these are hierarchical in the sense of increasing sophistication.

A) **An unspecified body of knowledge**
   - In this category the students is vague or confused about what constitutes ‘assumed knowledge’.

B) **Concepts, skills or topics**
   - This category presents a fragmented view of ‘assumed knowledge’, listing unrelated concepts, skills or topics.

C) **Level of mathematics completed at school (or elsewhere)**
In this category the student may mention a specific topic such as calculus but also specifies a unit of mathematics in senior secondary school or elsewhere, such as 2 Unit mathematics (referring to a previous unit in the Higher School Certificate in New South Wales). Here, the conception of ‘assumed knowledge’ is a formal ‘qualification’. Hence category C is more holistic or comprehensive than B.

<table>
<thead>
<tr>
<th>Why ⇒ What ↓</th>
<th>1 Knowledge with no stated purpose</th>
<th>2 Beneficial knowledge</th>
<th>3 Expected knowledge</th>
<th>4 Required knowledge</th>
<th>5 Foundational knowledge</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>An unspecified body of knowledge</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>Concepts, skills or topics</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>Level completed at school</td>
<td></td>
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</table>

**Figure 1: Outcome space for conceptions of ‘assumed knowledge’**

**Illustrations of categories**

Tables 1, 2 and 3 below illustrate each cell of the outcome space with exemplary excerpts from the students’ surveys and emails. Each table represents one level of the ‘what’ dimension of students’ conceptions of ‘assumed knowledge’. Where names are given, these are pseudonyms chosen by students themselves, for example, Moby or Mr X. In cases where no pseudonym was chosen, we indicate gender by F or M.

**Table 1. ‘What’ Dimension: An Unspecified Body of Knowledge**

<table>
<thead>
<tr>
<th>Category</th>
<th>‘Why’ dimension</th>
<th>Illustrative quote</th>
</tr>
</thead>
</table>
| A1       | Knowledge with no stated purpose | **Chris:** A lot of stuff that I have a very short time to learn  
**F:** A general knowledge of mathematics |
| A2       | Beneficial knowledge | **M:** The degree will be much easier with the ‘assumed knowledge’ than without  
**Moby:** My understanding is that assumed knowledge is not a university requirement to do the course, but understanding of the course content would be affected without the assumed knowledge. |
| A3       | Expected knowledge | **F:** That it’s expected that you have already covered the content before you start the course  
**F:** Knowledge that is assumed to be known |
In these examples we see that students have not defined ‘assumed knowledge’ in mathematics and conceptions of its purpose range from pointless “stuff” to information that will be used as “building blocks” in future study.

Table 2. ‘What’ Dimension: Concepts, skills or topics

<table>
<thead>
<tr>
<th>Category</th>
<th>‘Why’ dimension</th>
<th>Illustrative quote</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>Knowledge with no stated purpose</td>
<td>F: Mathematics literacy, familiarity with mathematical concepts and proficiency in the topics studied in HSC Mathematics</td>
</tr>
<tr>
<td>B2</td>
<td>Beneficial knowledge</td>
<td>June: 1. For microeconomics and macroeconomics, I think we need calculus and algebra. 2. For econometrics, I think it is based on linear algebra and statistics, not so much on calculus.</td>
</tr>
<tr>
<td>B3</td>
<td>Expected knowledge</td>
<td>Xavier Phillip: Assumed knowledge means the Mathematical skill sets that a student is expected to have in order to take up further studies in university.</td>
</tr>
</tbody>
</table>
| B4       | Required knowledge | M: Understanding of a particular topic that is required to understand the material of a particular course  
M: A prerequisite for the topics covered during the course |
| B5       | Foundational knowledge | Mr X: Assumed knowledge will be content that is either directly or indirectly referred to and is expected by the tutor/lecturer that the student has knowledge and a firm grasp of. This content will not be covered and will be used to extend the knowledge of a topic to a higher level.  
M: That the overall concepts and theories of this knowledge are grasped and understood in a way that can propel the student to effectively learn their degree requirements. This does not mean the student must have a mastery of this knowledge, but readily apply it when necessary. |

In Table 2, while ‘assumed knowledge’ in mathematics is not perceived as a cohesive body of information, there is a range in the conceived purpose of this knowledge with growing acknowledgement, through the categories, of the importance of specific topics or concepts for further study.
<table>
<thead>
<tr>
<th>Category</th>
<th>‘Why’ dimension</th>
<th>Illustrative quote</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>Knowledge with no stated purpose</td>
<td>Max Straubinger: I believe that ‘assumed knowledge’ means that one is competent at the core skills taught in the relevant HSC course. Grace: My understanding of assumed knowledge of maths in my Bachelor of Education Early Childhood at Macquarie University, is that it is assumed that I have completed the Maths 2 unit course (or equivalent). However, I’m not sure how much of this they actually expect me to use in my profession, as I intend to be a Primary school teacher.</td>
</tr>
<tr>
<td>C2</td>
<td>Beneficial knowledge</td>
<td>F: That you have completed the level of Maths or feel your Maths ability is strong enough that you wouldn’t struggle. It also means that it is not compulsory that you have completed that level in order to do your degree but it is for your benefit if you have.</td>
</tr>
<tr>
<td>C3</td>
<td>Expected knowledge</td>
<td>Lucy: The assumed knowledge for Medical Science was 2-Unit mathematics. I took this to mean that it would be assumed when I began my course that I had studied at least 2-unit math in the HSC, and had understood the concepts involved in the course, and that little to no time would be spent on re-teaching these concepts to me in lectures. Mary: It means that an applicant is expected to have completed 2 unit maths during HSC. If not, a bridging course is strongly recommended.</td>
</tr>
<tr>
<td>C4</td>
<td>Required knowledge</td>
<td>M: I’m expected to know 2u maths to understand the concepts taught in BCom F: the course just involves maths that most people (who did HSC maths) would understand so if you didn't do HSC maths then you won't understand what’s going on</td>
</tr>
<tr>
<td>C5</td>
<td>Foundational knowledge</td>
<td>F: That the content taught in the previous two years to HSC maths students is essential to understand the content of the uni course and you would fall behind without these building blocks. M: the content taught in the degree course will be taught with the expectation that the student has a foundation of Extension Maths</td>
</tr>
</tbody>
</table>

The final table shows an understanding of ‘assumed knowledge’ in terms of a more formal criterion, such as the HSC unit, and a hierarchy of the purpose of this knowledge to the student in tertiary study. Lower categories indicate that the student is unsure why the ‘assumed knowledge’ is necessary but as the categories ascend, there is recognition that the ‘assumed knowledge’ is essential for studying mathematics at university.
Discussion

This paper extends our previous findings about the perceptions (Gordon & Nicholas, 2013b) and prior decisions about mathematics (Gordon & Nicholas, 2013a) of students enrolled in mathematics bridging courses to show a range of understandings about what ‘assumed knowledge’ means. The outcome space demonstrates that these conceptions are nuanced and varied with increasing awareness along the two dimensions described: the ‘what’ and the ‘why’ of ‘assumed knowledge’. Particularly interesting is the shift along the ‘why’ axis’ from ‘assumed knowledge’ as imposed – external to the student – to more personally meaningful in its purpose. Excerpts such as: “they're going to teach you new concepts from that” or the “(student can) readily apply it when necessary”, show students starting to assume responsibility for their own knowledge and future study.

The question that immediately arises is this. Why does it matter what students conceive of ‘assumed knowledge’? We contend that students’ understandings of ‘assumed knowledge’ could affect their decisions about tertiary study and even steer them to start a course they may not manage. The participants of this study were enrolled in mathematics bridging courses indicating that they all understood that they were insufficiently prepared for their mathematics units, yet some categories in the outcome space showed a limited awareness of students as to the expectations of their tertiary institution. In some cases excerpts indicate that ‘assumed knowledge’ is perceived as unspecified or is restricted to particular topics or specific skills. In early research, we found low level conceptions of university mathematics including a view of mathematics as a set of rules and procedures to be learned by rote (Crawford, Gordon, Nicholas & Prosser, 1994). Further investigation would show whether low level conceptions of assumed knowledge translate in the future to low level conceptions of university mathematics.

Academics that prefer ‘assumed knowledge’ rather than prerequisites for entry into science based subjects argue that this method gives students options that would be denied to them if they had not studied mathematics at the required level of school, and that students should be given the opportunity to enrol in their chosen course and catch up on the mathematics (McNeilage, 2013). However, this is based on a premise that students understand the foundational nature of ‘assumed knowledge’ in mathematics. Our empirical study indicates that there are many categories of awareness about ‘assumed knowledge’ that do not recognise the necessity of understanding the ‘assumed’ mathematics for future learning, and some indeed even conceive of ‘assumed knowledge’ as without purpose or importance for further study. One of our postgraduate participants explained that as an undergraduate:

_I didn't really know what I wanted to study. The courses I considered taking at university had school mathematics pre-requisites but they didn't apply if you had a UAI (university entry score) greater than 90. So I ended up enrolling in a course... even though I lacked the recommended mathematics requirement._

We agree with the recommendation of the National Forum on Assumed Knowledge in Maths that statements about assumed knowledge requirements need to be clarified and explicit (King & Cattlin, 2014) and this may need to be done for each degree. The ‘assumed knowledge’ in mathematics may be foundational for degrees in engineering and science but may not be foundational for other degrees.
Conclusion

Students’ understandings of the mathematics they need to undertake study in programs involving science, technology and engineering is a key aspect underpinning the debate of prerequisites versus ‘assumed knowledge’, yet research into this has been lacking. The empirical findings of this study provide an opportunity for conveners of mathematics bridging courses, university teachers and designers of junior mathematics units to reflect on the messages they convey to school students and teachers about ‘assumed knowledge’ and to locate the positions of these communications in the theoretical framework discussed above.

More generally, the study alerts us to the diversity of students’ views about ‘assumed knowledge’ and, by inference, about effective preparation for university mathematics units. Research is needed to determine how students’ expectations about what mathematical knowledge they will need correlates with perceptions about transitioning to learning mathematics at university and even success in early study of mathematics. A further direction of research suggested by this study is to explore what careers advisors and mathematics teachers in senior secondary school understand by ‘assumed knowledge’ in mathematics and how this affects the advice they give students.

References


