Assumed Mathematics Knowledge: the Challenge of Symbols

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Abstract

Low progression rates in the mathematical sciences are of national concern. Various programs providing student support have been implemented across the higher education sector and a number of researchers have analysed the teaching of specific topics with view to making recommendations for improvement. In this position paper we suggest that insight into a potential barrier to students' choices regarding the study of mathematical sciences may be gained by adopting a complementary approach to the study of specific mathematics topics. We highlight the importance of paying attention to potential barriers to student learning as a result of discontinuity, unchartered extension and heightened complexity in the use of symbols when students progress in mathematical sciences from school to university. Symbols form the foundation of mathematical communication. We conjecture that the increase in symbol load due to unfamiliarity and increased density may cause students to lose confidence and subsequently choose a study path that minimises their need for mathematics. In this paper we propose a framework for analysing symbolic load and briefly report initial findings from a pilot study.

Rationale

Declining tertiary enrolments along with the low progression rates in mathematical sciences is a major concern worldwide including in Australia. The *Mathematics, engineering and science in the national interest* report (Chubb, Findlay, Du, Burmester and Kusa, 2012) emphasises that "the proportion of mathematics and science students in schools still goes down; and in universities (as with engineering) it is virtually flat" (p. 6) and notes that "Australia would need around 13,500 additional STEM [Science, Technology, Engineering and Mathematics] graduates per annum for a decade just to keep pace" (p. 16). For 2012, the Australian Mathematical Sciences Institute (AMSI) (2013) reported that there were 6967 first year effective full time student load (EFTSL) enrolments in its 26 Australian member universities, 3375 in second year (48% progression rate) and 1166 in third year (16.7% progression rate from first year). Looking beyond just individual subject enrolment, Taylor (2005) reported that an enrolment of over 2500 students in first year mathematics at the University of Sydney dwindled to about 200 mathematics majors (third year); numbers that are sadly typical of Australian universities. Many of the students who do not continue with mathematical sciences beyond the first year have been very successful in school mathematics. Why do these students, sufficiently keen and qualified to enroll in mathematics when they first enter university, not continue in mathematical sciences? This is a multifaceted problem but we suspect that one factor, as yet not fully investigated, is a barrier caused by the transition in the use of symbols between senior secondary school mathematics and university mathematics, and that this is heightened by variation in symbol use between the mathematical sciences.

Mathematics derives much of its power from the use of symbols (Arcavi, 2005), but research at secondary level has shown that their conciseness and abstraction can be a barrier to learning (Pierce, Stacey and Bardini, 2010; MacGregor and Stacey, 1997). In a study involving first year university physics students (Torigoe and Gladding, 2007), it was found that students' performance is highly correlated to their understanding of symbols. We anticipate that similar outcomes apply to other mathematical sciences at university. Indeed, data from University of Wollongong (Hoban, Finlayson and Nolan, 2012) shows that doing a high level of mathematics at school is the best predictor of success in their 'CHEM 101' (Chemistry 1A: Introductory Physical and General Chemistry) subject. Hoban et al. also comment that it is the understanding of mathematics (and we believe this necessarily includes the reading and writing of mathematics), rather than the ability to apply mathematics to chemistry that is important (2012). We anticipate that many students have difficulty with the new and more intense ways in which symbols are used at university (to be described below), with the consequence that they do not understand the mathematical content as well as they did before, leading to a decrease in positive affect, which in turn might discourage enrolment in further mathematical subjects.

Studies of students' mathematics learning experiences at the university level, designed to reveal reasons for low progression rates in mathematical sciences, have mainly focused on understanding mathematical *concepts*, such as the notion of tangent (Biza and Zachariades, 2010), duality in linear algebra (De Vleeschouwer, 2010) or vector space (Dorier, 1997 & 2000). These studies can lead to better instruction in particular topics. While examining the teaching of particular mathematics topics is important, we suggest that insight into a potential barrier to students' choices regarding the study of mathematical sciences may be gained by adopting a complementary approach to the study of specific mathematics topics, by examining students' experience of the very foundation of mathematical language across topics; that is mathematical *symbols*.

The issue of reading, recognising and understanding symbols underpins all mathematics topics. How can students solve problems using the tangent, for example, if the concatenation 'tan x' does not mean much to them? It is not simply about understanding that the three letters 't a n' placed together stand for 'tangent'. Rather it is about students being comfortable with the whole symbolic sentence and, for example, acknowledging that writing 'tan θ' instead of 'tan x' is not a purely subjective choice from the teacher who chooses the Greek letter arbitrarily. With its often too implicit conventions, the very writing of mathematics sets the domain in which a problem is posed, and probably the one in which the solution should be tackled. It helps (or at least should help) set the mind of its reader to a specific range of problems, and hence to the tools for solving it. In the case of 'tan x' and 'tan θ', what appears to be arbitrary in fact anticipates two different discourses. While the latter expression anticipates the problem to be very likely about the *geometry* feature of the tangent (θ being deciphered as an angle which measures between 0 and 2π), the expression 'tan x' indicates that the focus will rather be on examining the tangent *function* from $\mathbb{R}$ to $\mathbb{R}$, with all its properties. If students do not understand the 'prompt' behind these two different expressions, not surprisingly they will find it difficult to fully understand what is asked (and what direction to take for solving the problem) and what mathematical tools they are supposed to employ in order to succeed.

At university, not only does mathematics become much more symbolic, but its writing is more subtle and requires increased 'flexibility' from the reader. One cannot take for granted that a symbol supposedly mastered at the secondary level will continue to be so at university because its meaning may not remain the same. The domain(s) with which a given symbol was prominently associated at secondary level often turn(s) out to be importantly extended at university level. There are multiple examples documented in mathematics education research relating to such changes within the school years. Take the example of letters in algebra. Students may well have understood that 'n' stands for an unknown and may know well how to solve equations involving such unknowns, but when it comes to shifting their perception and seeing the same letter as standing for a variable (for which there is no need to seek a specific value), research at secondary level has shown (Bardini, 2003; Bardini, 2004; Bardini, Radford and Sabena, 2005; Bardini, Pierce and Stacey, 2004) that students can struggle immensely. If the letter has been long anchored to a specific status (that of unknown) and an associated domain (solving equations) and then it suddenly acquires a different position (that of variable), the passage from one to the other may be far from evident from the students' perspective.

A proposed conceptual framework – the notion of symbol load

Mathematical language is concisely described by Drouhard and Teppo (2004) as consisting of symbolic expressions, natural language and compound representations, such as diagrams and tables which usually also contain symbols and natural language. To assist with the analysis of elements of this language Serfati (2005) provides us with an epistemological approach specific to mathematical notations that both embraces the syntactical aspect of a symbol and also investigates the underpinning mathematical concept(s) conveyed.

Using a simplified version of Serfati (2005), mathematical symbols can be thought of in three categories: letters (including letter-like shapes from any language), other figures, and compound templates which combine letters and figures in a two-dimensional shape. All of the above can be combined to make symbolic expressions, which might be short (even one symbol) or long. Examples are given below.

Letters: a, A, α, ℝ, π, ∂,
Figures: +, %, ≤, √, ∫, =:
Compound templates: {1, 2, 3}, r^2, b_0, $\frac{3}{8}$, $\int\int f\ \ x,y\ \ dx\,dy$, E(X)
Symbolic Expressions: $y = mx + c$, $x + a\ ^{n} = \ \substack{n \\ \\ k=0}\ \genfrac{}{}{0pt}{}{n}{k}\ x^k a^{n-k}$

In our analysis we refer to students' experience of the changes in symbols, frequency of symbol use, and the various meanings of symbols that they need to deal with as they progress in mathematics as 'symbol load'. We view this symbol load as constituted by two components, 'symbol familiarity' and 'symbol density'. These are described below.

Symbol density
One simple measure of symbol load can be to look at 'the number of symbols' in a mathematical text that we define here as 'symbol density'. An increase in symbolic density between secondary school and university mathematics is recognized intuitively but has not been measured to establish the extent of this challenge. One simple approach would be to note the proportion of characters which are symbols. For example, the following two lines describe precisely the same task, although the second is clearly 'more symbolic'.

Find the positive solutions to the equation $x^3 - x^2 = 0$.

Find $\{x \in \mathbb{R} : x > 0 \text{ and } x^3 - x^2 = 0\}$.

In the first line, 7 characters out of a total of 44 are symbols, giving a symbolic density of 0.16. In the second line, there are 15 symbols in a total of 22 characters, giving a symbolic density of 0.68. The symbolic density captures part of the symbolic load students might experience.

Symbol familiarity

A simplified version of Serfati's approach provides a framework for defining and analysing symbol familiarity. According to Serfati a mathematical symbol has three attributes: materiality (what it looks like), syntax (how it is combined with other symbols), and meaning (2005). As presented in our previous work (Bardini, Oldenburg, Stacey and Pierce, 2013), consider Eq. 1 below and the familiar small dash '$-$' which appears three times, as three different signs.

Eq. 1 $$\begin{pmatrix} 1 & 0 \\ a & b \end{pmatrix} - \begin{pmatrix} 1 & -1 \\ b & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ a-b & b \end{pmatrix}$$

For all three signs, the materiality includes the straightness of the dash, its short length, and its position a little above the lower line of writing. Used in $a - b$ the sign means subtraction of (unknown) numbers. The syntax of this sign includes that it is a binary operator, that the left/right order matters, and that in an expression such as '$3\times5-2$', it does not take precedence. Another '$-$'sign in Eq. 1 with the same materiality but a different meaning indicates a negative number. The syntax of this sign includes that it operates on the number to its right. The third '$-$' sign in Eq. 1 means subtraction of matrices. This shows that even within the same equation, it can be necessary to attribute different meanings to one (material) sign. As mathematics advances, it is hypothesised that not only symbols with new materiality are introduced to students but also symbols with known materiality but with altered and/or added meanings.

Table 1 demonstrates some changes to symbols which students encounter as they learn further mathematics. For example, in the top row of Table 1 we note that the familiar school mathematics '$y=mx+c$' form for an affine function (often referred to as a linear function in school texts) commonly takes a new materiality in university statistics where the '$y = b_0 + b_1x$' symbolization is used in order to prepare for model involving several variables, for example: $y = b_0 + b_1x_1 + b_2x_2 + b_3x_3$. In the last row of the table we note that the '$\times$' symbol, indicating the standard multiplication process in school mathematics, can also be used to represent vector multiplication in university mathematics. What looks like the same symbol (same materiality) takes on a new meaning, which the student must recognise from the context. As Bardini, Oldenburg, Stacey and Pierce (2013) report, even an apparently common and straight forward mathematical word such as 'equals' may be represented by different symbols when mathematical software is employed.

Table 1. Changes to symbols from school to university

	Same materiality		New materiality	
Same meaning	School Maths: $f\ \ x\ \ = x^2$	Uni. Maths: $f\ \ x\ \ = x^2$	School Maths: m and c $y = mx + c$	Uni. Statistics: b_0 and b_1 $y = b_0 + b_1 x$
Extended meaning	School Maths: Letters stand for real numbers	Uni. Maths: Letters also stand for complex numbers	School Maths: b a	Uni Maths: Double integral
Restricted meaning	Mathematics: c used as a general constant	Physics: c restricted to speed of light	Mathematics: one equals sign with multiple meanings =	Computer Science: specific signs for specific meanings e.g. =: assign
New meaning	School Maths: × multiply numbers	Uni. Maths: × vector product	Not applicable	

As Table 1 suggests, the analysis of symbol familiarity relies on a double investigation: one that looks at symbols that are 'new to students' (i.e. symbols with new materiality) and one that examines symbols that are 'known to students' (i.e. symbols with same materiality).

'New symbols'

This first analysis should meet the need for a comprehensive list of university mathematical symbols that have not been introduced at secondary school and the domain in which they appear. For each 'new' symbol, the questions that drive such analysis may include the following: is the new symbol standing for a concept introduced at secondary school or is the new symbol representing a new concept? In the latter case, is this concept stemming from a concept introduced at secondary school? If so, two questions should be posed, regarding both the symbol's (i) materiality and (ii) meaning. The first one can be phrased as: is there a smooth transition, symbolically speaking, from the representation of the secondary school concept to the new one? In other words, is the continuity in the concept reflected in the materiality of its related symbol(s)? The second question looks at the meaning of the symbol and analyses whether it is the same, restricted, extended or a totally new meaning when compared to school practices.

'Known symbols'

Questions related to this part of the symbol familiarity analysis may include: (for each symbol) what school and university subject (linear algebra, statistics, engineering, mathematics), context (linear equations, vectors, probability) and course (mathematics, engineering, physics, etc.) does it appear in? When the symbol is used in more than one place, is the symbol used consistently across all in which it appears? What are the differences and the similarities between use in, for example, Year 12 Mathematics Methods and university subjects and within one education institution (symbols used in statistics subjects, mathematics subjects, engineering subjects)? It is well known that even within the same mathematical subject, the same concept can often be represented with different symbols. In linear algebra, for example, vectors are sometimes represented by square or round brackets '[ ]' or '( )' (matrix notation), presented vertically or horizontally, sometimes by ' ' (ordered set notation) and sometimes by an underlined letter. The choice for one representation or the other depends on what feature of the vector the problem is highlighting (cf. the discussion on how mathematics writing sets the mind of the reader to a specific range of problems, presented at the beginning of this paper). Examining to what extent these symbolic subtleties are fully understood by students is at the core of our message.

A pilot study as starting point

In a 2013 pilot study, we examined four of the most commonly used Victorian Year 12 Mathematics Methods textbooks, and the lecture notes and tutorial exercises from first year mathematics subjects in two major Victorian universities. These universities are amongst the largest Australian urban universities and the students' backgrounds from both universities are comparable. Mathematics subjects delivered in the first year at both universities can be said to be of similar content. In 2014 we observed students in two first year undergraduate mathematics tutorials for one semester at one of these universities. Following the request from the Program coordinator, interaction with students was kept to a minimum and researchers limited themselves to observation. Notes taken from these observations are the data sources for this part of the study and are reported below.

From the text book analysis we found that the complexity of mathematical notation in first year mathematical subjects compared to the common use of symbols in school textbooks varied greatly from one university to another, depending on which topic the university chose to focus on at the beginning of their first year subject. One university chose its first year subject to initially focus on calculus, because this was naturally aligned with students' prior knowledge from Year 12, and symbol familiarity appeared to be high. Conversely, the other university introduced its first year subject with linear algebra, and because some concepts were new to students and because of the specifics of the topic (higher use of set notation), the familiarity proved to be rather very low. Interestingly, symbol density seemed highly related to the familiarity, probably because of some of the specifics of these two topics (calculus and linear algebra).

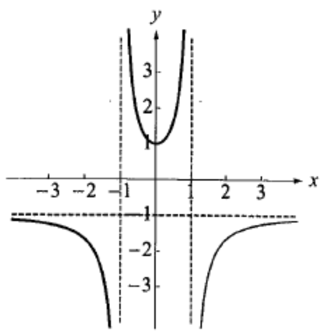
We also found important changes in symbols' meaning at University, often times leading to discontinuities with school, such as the notation for inverse functions. (The notation used is not 'f^{-1}'; the general case is written "… an inverse function of *f* is a function *g* such that …". The specific inverses discussed are the inverse circular functions, where "… we will only use the arcsin notation" as this "avoids potential confusion between $\text{Sin}^{-1}(x)$ and $1/\sin(x)$".)

It is likely that difficulties students faced and managed (at least from the perspective of correctly answering examination questions) at secondary level flourish at university, when mathematics becomes progressively more densely symbolic.

In tutorial classes working on complex numbers, for example, we observed evidence of a discontinuity due to new materiality for students who had used the notation of $cis\theta$ at school but were now required to work with $e^{i\theta}$. Students appeared to be reluctant to use $e^{i\theta}$, with one of them asking if they could continue to use the school notation $cis\theta$ because, according to the student, "it makes sense: *c* from cos, *i* and *s* from sin", suggesting the student did not grasp the purpose and advantages of the exponential notation. Later in calculus students were expected to move between both of the classic symbolisations, changing materiality: $\frac{dy}{dx}$ and $f'(x)$, with the reasons underpinning the choice of either notation remaining often opaque to students. When having to work with real intervals, a lack of rigour has also sparked our attention. In a calculus introductory problem, students were asked to find, for a given function *f* (graph provided –see Figure 1), "(i) the interval(s) on which *f* increases and (ii) the interval(s) on which *f* decreases". Typical answers from students included the following:

Student A: (i) "from (0,1) and (1,∞)" ; (ii) "concave down from (-∞, -1) to (1, ∞)"
Student B: "(-∞, -1) ∩ (-1,0)"
Student C: "[0,1) ∪ (1,∞)"

Figure 1. Graph of *f* used to determine intervals where *f* increases and decreases



All three responses reveal interesting characteristics.

By mixing words and intervals' notation in answers to both (i) and (ii), it appears that student A does not yet fully master the intervals' notation, with the writing 'from…to' redundant to the very meaning of the opening and closing parenthesis of the mathematical notation. Equally interesting is student A's usage of 'and' and 'to'. When using 'and' in (i), the student seems to be favoring the intervals' notation, yet the presence of 'from' at the beginning of the sentence, moreover incompatible with 'and', reveals the notation is still somehow vaguely understood. Inconsistently using either 'and' or 'to' further supports this.

Students B and C seem to have merged both (i) and (ii) in their answers, looking for intervals where *f* is both increasing and decreasing. From a symbolic point of view, student B's misusage of '∩' suggests his/her will to translate into symbols the word 'and'. Should the student had realized that '∩' means 'the *intersection of…and…*' (and not only 'and'), he would have seen that his/her answer is ultimately the empty set.

What we have summarized above are only a couple of examples from common topics.

Concluding remarks and discussion

The aim of this paper was to provide a complementary view point regarding transition issues from secondary school to university mathematics, by examining the use of mathematical notations at both institutions. In casual conversation with students about our project, a common response was that at university they felt that there was a lot of difference in the use of symbols.

Extensive research on students' understanding of mathematical symbols at secondary level has been carried over the past decades and it is now well established that symbols' conciseness and abstraction can be a barrier to learning.

Our pilot study has shown that discontinuities and extensions also flourish at university and our aim was to raise awareness when looking for potential teaching learning implications. Take the example of letters in algebra. Research at secondary level has shown that students can struggle immensely when it comes to shifting their perception and seeing a letter standing for an unknown to the same letter as standing for a variable. At university, students are required to flexibly navigate between letters as unknowns, variables, and constants as before, but the role of letters as parameters expands greatly. Letters are often used in these multiple ways within one equation, so this needs to be explicitly negotiated before students can begin to work with it.

From the analysis of Year 12 textbooks and University lectures notes we found that there was an increase in symbol density and we also found important changes in symbols' meaning. We do not advocate that the symbol load at University should remain at its equivalent school level. It is inevitable (and desirable, may we add) that at University more symbols are introduced and/or new meanings emerge. However, if such changes may seem trivial to the expert, they can prove to be a stumbling block to the novice.

From our observations it seems that students may not be entirely comfortable with some specific notations and may sometimes be reluctant in adopting new notations –probably because their benefits and purposes are not fully understood. For experts to explicitly address issues of symbol familiarity and symbol density in their teaching, current discontinuities need to be identified. Care needs to be taken to ease students' transition to new symbol familiarity and greater symbol density. Teaching staff need, for example, to acknowledge the diversity of symbols they use and eventually agree on the set to be used.

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