Mastery Learning to Address the Assumed Mathematics Knowledge Gap, Encourage Learning and Reflection, and Future-proof Academic Performance

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Abstract

UTS Science, Engineering and Mathematics students who have studied General Mathematics at high school are far more likely to fail their first undergraduate mathematics subject compared to their counterparts who meet the non-compulsory "Assumed Knowledge" of 2 unit Mathematics. This problem has been growing in recent years as an increasing number of students seek to improve their tertiary entrance score by taking the no-calculus General Mathematics at the Higher School Certificate. This problem is not unique to the University of Technology, Sydney - mathematical under-preparedness is a problem world-wide, with a decade, or more, long history. For some years, UTS has used diagnostic testing and pre-teaching to assist under-prepared students. Unfortunately, students who studied General Mathematics are also more likely to fail the pre-teaching subject. This suggested something more was required. Mastery Learning was chosen as a potential solution. Results to date have been promising with improvements in academic success for under-prepared students. Students have also reported increased satisfaction, confidence and retention of content. However, some students felt all Mastery Learning taught them was how to pass the Mastery Tests. Differences in student experience appear to be due to differences in how Mastery Learning was implemented.

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The challenges

First-year undergraduate Mathematics education currently faces many challenges. In Australia, outcomes, both academic and non-academic, are poor – failure rates are unacceptably high for some cohorts (for example Table 1), consequently progression is delayed, attrition is higher than desirable and the student experience is not always positive (James, Krause and Jennings, 2010). Some of these problems can be traced back to high school where around 40 per cent of Years 7 to 10 mathematics classes are taught without a qualified mathematics teacher (McKenzie, Rowley, Waldon & Murphy, 2011). Additionally, there are 13 per cent fewer calculus-trained students than in 2001 (Mathematical Association of New South Wales, 2014) - able students are selecting Mathematics subjects with no calculus to improve their tertiary entrance scores (Australian Tertiary Admissions Rank, ATAR) at the expense of meeting the assumed knowledge for their degree program (ibid). Enrolments in the Elementary Mathematics subject are trending up (Barrington & Evans, 2014). In 2013, students with these backgrounds amount to 56% of the University of Technology, Sydney (UTS) Science intake, 31% of the Engineering intake and 28% of the Mathematics intake. This gap in assumed knowledge has been reported since the late nineties (Crowther, Thompson & Cullingford, 1997) and is a problem world-wide (Hawkes &

Savage, 2000; Hoyt & Sorensen, 2001; Smith, 2004; Luk, 2005; Heck & van Gastel, 2006; Brandel, Hemmi & Thunberg, 2008; Thiel, Peterman & Brown, 2008; Rylands & Coady, 2009; Varsavsky, 2010).

As long as universities accept students without the "Assumed Knowledge" of their program, and many, though not all do this, there is a legal and moral imperative to do whatever can be done to address this lack of assumed knowledge in a way that will ensure the same chance of success for all students regardless of background (Chubb, 2014). To do otherwise would convey an expectation to students that they will be successful without necessarily possessing the assumed knowledge. This is evidenced in the following quote from a student who ignored the recommendation to undertake diagnostic testing of assumed knowledge and enrolled in their first core mathematics subject:

> *The math is too hard… it is not [at a] level that [a] general math student can do.*
> *(Subject Feedback Survey – Mathematical Modelling for Science Autumn 2013)*

When this problem first came to light, UTS and many other Australian universities introduced diagnostic testing of assumed knowledge (at UTS called the Readiness Survey) to assess the extent to which the "Assumed Knowledge" could indeed be assumed. Diagnostic tests usually work in association with a pre-teaching subject, at UTS, Foundation Mathematics. Students failing the Readiness Survey can take Foundation Mathematics prior to their first core Mathematics subject to upskill to the assumed knowledge of their program. Unfortunately, students who have studied General Mathematics are also more likely to fail Foundation Mathematics.

These observations are not specific to UTS, and like other institutions overseas something additional was suggested, something that could accommodate diverse backgrounds and support the transition to university education; improve the student learning experience; support students in their learning; offer students increased opportunities to demonstrate their understanding, build confidence and progress knowledge and skill development; and, improve assessment performance and readiness for later subjects. How can this be achieved within the constraints of the teaching and learning environments of Australian universities?

Table 1 – Synthetic sample results for Mathematical Modelling 1 for Autumn 2013
(From Groen, Coupland, Stanley, Bush, Poulton, Woodcock and Memar, 2014)

Background	Mean	Median	% fails (of)
Extension 2	78.45	81	3 (60)
Extension 1	73.93	74	0 (160)
Mathematics	55.42	59	26 (119)
General Mathematics	51.14	52	45 (22)
Tertiary study	59.29	58	25 (80)
Other	62.15	63	23 (13)
No HSC Maths	58.33	58	0 (3)

(These results were generated by random selection from the actual mark distributions.)

Data - the subjects and the students

Before going on to answer this question, a little on the subjects, programs and students concerned …

At UTS Engineering, Physics and Chemistry, and Mathematics students undertake slightly different first-year, first-semester Mathematics subjects – Mathematical Modelling 1, Mathematical Modelling for Science, Linear Dynamical Systems, respectively. However, all subjects include relations, single-variable functions, differentiation and integration. Other topics taught can include an introduction to vectors, complex numbers, introductory differential equations and, series and sequences. Students of these programs go on to take at least one other mathematics subject in their second semester of their first year. Information Technology students have no core first year mathematics in their programs. This is also true of students taking biological, medical and environmental science programs, though statistics is studied by these cohorts.

Prerequisites for entry do not exist, though Extension 1 Mathematics is recommended and Mathematics (2 unit) is assumed. (Using the classification system of Barrington and Brown (2005) for the New South Wales Mathematics subjects, Mathematics Extensions 1 and 2 are “Advanced” mathematics subjects, Mathematics (2 unit) is “Intermediate” and General Mathematics is “Elementary”.) This applies to all Engineering, Physics, Chemistry and Mathematics programs. These recommendations are often ignored, and “Assumed Knowledge” cannot always be assumed. General Mathematics and even no Higher School Certificate mathematics backgrounds occur. ATARs for Science and Mathematics programs are generally above 72, while Engineering requires an ATAR of at least 86. A recent trend in lower ATARs has occurred where students use other forms of tertiary study to meet admission requirements.

The subject Foundation Mathematics is offered to any UTS student, but primarily targets students who fail our Readiness Survey. This survey is a quick, online diagnostic test. This diagnostic pre-assessment is only compulsory for Engineering students. Engineering students who fail the test are enrolled in Foundation Mathematics. For other students failing the Readiness Survey, enrolment in Foundation Mathematics is recommended. Diagnostic pre-assessment and pre-teaching have been operating for a number of years with mixed success. For example, failure rates in Foundation Mathematics by Engineering students over the 2012-13 semesters averaged around 59%. This statistic is concerning as it follows previous changes to content, delivery and support, though it does bear some semblance to statistics reported by Rylands and Coady (2009).

Students of Foundation Mathematics and Mathematical Modelling 2 were selected for this study. Foundations Mathematics was selected because it is the first subject in the prerequisite chain for students who do not have the assumed knowledge of their program. Mathematical Modelling 2 is the second mathematics subject offered to Engineering students, and was selected as the numbers of students taking the subject in Autumn semesters are not as large as those taking it in Spring. This reflected the fact that a final decision on the continuation of Mastery Learning had not yet been taken. The 2014 cohorts from these two subjects were only dissimilar from previous years in an increase in the number of students who, on paper at least, did not meet the assumed knowledge of their program.

Mathematical Modelling 2 was (and is) offered in a blended form – weekly lectures are supported by weekly computer laboratories and tutorials. In 2014, in Foundation Mathematics, computer laboratories and tutorials were not part of the subject’s learning design. (This has since changed.) Learning management systems were (and are) employed in these subjects, though the degree of instructional software integration was limited prior to the start of 2014.

Unlike the study of Rylands and Coady (2009), both recent and non-recent school leavers were included, and unlike the study by Varsavsky (2010), students required to take the subject as well as students electing to do so were included. These data enabled a broader range of relationships to be studied. Data was collected concerning ATAR, Mathematics subject(s) studied in high school and mark(s) in the Higher School Certificate (HSC), as well as final mark and grade in Foundation Mathematics, Mathematical Modelling 1 and Mathematical Modelling 2. Data were also broken down by program. Five semesters of data was collected for the purposes of comparison. Information about the sample of Foundation Mathematics can be found in Table 2. The class sizes of the five semesters of Mathematical Modelling 2 were 286, 560, 235, 574 and 274 respectively.

Table 2: Size of the database for Foundation Mathematics

Subject		Background						
		Advanced		Intermediate	Elementary			
	Semester	Ext 2	Ext 1	Math	Gen Math	No senior maths	Tertiary studies	Other
Foundation Mathemati	Autumn 2012	3	9	41	21	1	217	22
	Spring 2012	2	5	7	9	0	128	18
	Autumn 2013	6	18	31	34	3	146	23
	Spring 2013	0	8	8	13	0	92	19
	Autumn 2014	9	24	50	63	7	141	27

The content, subject objectives and delivery mode were unchanged in both subjects with the implementation of Mastery Learning.

Methodology

Summative measures

One-sided *t*-tests comparing sample mean final marks, under the assumption of unequal variances, were used to determine whether any improvements across semesters were statistically significant. Where normality couldn't be assumed, a chi-square test was used. Autumn cohorts were compared with Autumn cohorts, and Spring with Spring (representing the two primary admission times of the programs).

Where normality could bas assumed, *z*-tests were used for pairwise comparisons of failure rates. Where normality couldn't be assumed, chi-square tests were used. Fisher Exact tests (McDonald, 2014), as implemented at Preacher and Briggs (2001) were used for cohorts where the expected numbers of failures or passing grades were too small (<5).

For pairwise comparison of medians, *z*-tests or Wilcoxon Rank Sum Tests were used (where the normality assumption held), otherwise a Mann-Whitney U test was used. These were implemented using the Mann-Whitney U-test Calculator of Stangroom (2015) (for the *z*-test and the Mann-Whitney U-test) and the implementation of the Wilcoxon Rank Sum Test was undertaken in Excel using Zaiontz (2015).

All statistical tests were initially conducted at the 5% level of significance.

Qualitative measures

Comments from student responses to the UTS Subject Feedback Survey (SFS) were examined to assess student reactions to the innovation. This survey is conducted in the final weeks of all subjects, and includes Likert questions as well as open-ended questions. The students' responses to the open-ended questions were examined to assess student reactions to Mastery Learning. Representative responses to these open-ended questions were selected for inclusion here.

Focus groups were also conducted for each subject to drill down into survey responses as well as to allow students the opportunity to provide additional feedback on their experiences of Mastery Learning. Students were invited to participate (in line with ethics approval). Small groups of no more than 10 students were assembled. The focus groups were conducted by a member of staff not involved in the teaching of the subjects. Responses were recorded and transcribed, then forwarded to the academic staff for analysis. Responses to six set questions were elicited, as well as one open-ended question. The set questions covered attitudes towards mathematics and learning, confidence and (test and/or mathematics) anxiety, as well as assessment structure, nature and impact.

Ethics approval was granted for the collection and publication of these measures (UTS HREC ref. no. 2014000277).

Upscaling our response – Mastery Learning

Mastery Learning is both a philosophy of instruction and a set of methods for teaching and assessing. As a philosophy, it endorses the belief that all students can learn and achieve the same level of content mastery when provided with the appropriate learning conditions and time. As a set of teaching and assessing methods, it requires that each student be assessed in a criterion-referenced manner based on the required instructional objectives, and achieve a prescribed level of performance in assessment tasks. Rather than 50% for a Pass, mastery assessments set the bar at (usually) 75 or 80%. Marking and feedback are immediate, or nearly so, and students failing to meet the mastery level are provided with remedial activities and then provided with additional opportunities to demonstrate mastery. The process may be repeated. Mastery Learning in first year Mathematics subjects was suggested as a possible answer to the challenges previously listed.

Mastery Learning seems to work best with a curriculum based on well-defined learning objectives organized into smaller, more specific outcomes. Mastery Learning is then ideally suited to undergraduate mathematics education. The implementation of Mastery Learning is also well-suited to online learning systems where immediate marking and feedback are available, and remedial activities can be flagged and supported. Guskey (2010) provides an overview of the elements of Mastery Learning and includes a list of useful references. The research literature indicates positive effects of mastery learning on students in the areas of achievement, attitudes toward learning, and the retention of content (Guskey & Pigott, 1988; Kulik, Kulik, & Bangert-Drowns, 1990; Anderson, 1994).

A trial of a form of Mastery Learning was conducted in Mathematical Modelling for Science in the Autumn semester of 2013 (Groen, 2013). After subdividing the subject curriculum into learning units and further into a logical sequence of smaller objectives, learning materials, instructional strategies and activities (including assessments) were identified, sequenced and executed over the teaching period. Criterion-referenced (paper) tests were developed and administered to assess the degree to which students had mastered subject learning outcomes. Typically, these were brief tests administered a week or two after instruction, and followed a piece of formative assessment. Marking was undertaken immediately and feedback provided within a week. The tests and feedback reinforced precisely what students were expected to learn, identified what they did well and what required improvement. Students could use the timely feedback to feed forward, engaging in remedial activities individually or collaboratively (including formalised peer learning) or with an educator in the Mathematics Study Centre. Students were then given a second formative assessment before being given a second opportunity to sit the test - same concepts, different questions (Bloom 1971). The best mark of the attempts was used in determining the final result for the subject. Students who had already demonstrated mastery could improve their test mark if they wished. Some students used the second chance test to challenge their exam or mathematics anxiety.

Given time constraints and self-contained learning objectives, and unlike true mastery learning, students proceeded to the next instructional unit, initially using out-of-class time to remediate if required. Though partial results only, student performance and satisfaction were promising enough to suggest that Mastery Learning could be an avenue by which the challenges identified previously could be effectively addressed. Further, current technologies could be used to address the resource requirements of Mastery Learning.

> *… fundamentally the point of the subject (any subject, really) is to help people understand the material and help them study it, and I think these second chances do both of these things. Showing exactly where people aren't doing so well and (importantly) letting them see the results of their study soon after ... I think is not only a really good way to help people target their study most effectively and understand their own learning process, but also just a great way to reinforce the value of study in people's minds – you can see your marks improving almost immediately. (SFS Mathematical Modelling for Science Autumn 2013)*

In Autumn semester 2014, Mastery Learning was implemented in Foundation Mathematics and Mathematical Modelling 2 - in Foundation Mathematics with paper tests (due to room constraints), and in Mathematical Modelling 2 with online (supervised) tests. Mastery Learning was implemented in such a way as to have the first assessment before the HECS census date, with the 'second chance' test shortly after. Four Mastery Tests were set across the teaching period. The exact weightings of the four mastery tests varied between the two subjects. In Foundation Mathematics each test was worth 16% of the final mark, with a mastery level of 13/16. In Mathematical Modelling 2, the first mastery test assessed prerequisite knowledge and skills and hence was worth a lower percentage, with the rest of the weight shared equally to add to a mark out of 64. 'Second chance' tests interspersed with formative assessment. The formative and summative assessments work hand in glove to systematically monitor student progress both for the student and the teaching staff – they integrate assessment for learning with assessment of learning.

An 80% mastery level was set. The Mastery Tests assessed the fundamental knowledge and skills objectives of the subject, those knowledge and skills that would meet the current and future academic needs of students. Passing at mastery level in all four mastery tests ensures that students achieve a Pass (>=50%) for the subject. Perfect scores ensure an upper range Pass is achieved.

The Mastery Tests were created from the software provided by the textbooks' publishers. In addition to a testing environment with varying levels of immediate scoring and feedback (from single solution, to working, to textbook reference and supplementary problems), the publishers' software included an optional online textbook, practice problems, worked solutions, and one of the pieces of software also included videos explaining concepts and working. The formative tests were also created from this software. All attempts of all Mastery Tests were supervised, timed and scheduled. Details appeared in the subject outline as well as in the university's learning management system.

Students who did not demonstrate mastery in the first attempt were given another opportunity usually two weeks later. While the assessment regime was structured, the response to remediation was varied and flexible, from online videos and interactive websites, to peer support and one-to-one tutoring in the Mathematics Study Centre.

Students achieving mastery in all tests could sit the final examination to improve their result beyond the Pass grade (Credit, Distinction, High Distinction), that is, the final exam became optional. The final exam assessed the more technically demanding aspects of the subject, and did not assessing anything already assessed in the Mastery Tests. As a consequence, the final exam became more demanding for students. However, students knew going into the final exam that they had already passed, so that one source of stress was balanced by the removal of a different source of stress. Mastery testing implemented in this way has the advantage of ensuring students can't pass a subject without meeting knowledge and skills subject learning outcomes.
Student who did not achieve mastery in the second offering of a Mastery Test (called the second-chance test) were given a final opportunity to sit the last-chance test in the final examination period at the end of semester. These students were not offered an opportunity to sit the final exam in an effort to ensure that they achieved the fundamental knowledge and skills outcomes of the subject.

The implementation of Mastery Learning at UTS bears some semblance to the implementation of the Roadmap to Redesign program described in Thiel et al (2008) and shares many of the same design principles. The implementation also bears some semblance to the "replacement" model of Twigg (2011) in that rather than moving all classes to a lab setting (the Emporium Model (NCAT, 2005)), the current blended approach was extended to make effective use of software provided by textbook publishers for assessment purposes. That is, current assessment emphasis has been placed on supervised tests rather than the homework and assignments described by Thiel. This choice was primarily of function of concerns over academic misconduct. The implementation of Mastery Learning at UTS is also similar to learning designs implemented at Cleveland State Community College and Jackson State Community College (reported in Twigg, 2011), and by thirty-two institutions under the NCAT Changing the Equation program (reported in Twigg 2013). However, again marks for "attendance, notebooks and homework" (ibid, p32) are not included in the final mark for a subject.

The outcomes

Summative Measures

Figure 1 presents the distributions of results for Foundation Mathematics for the last five semesters. For Foundation Maths we can see that the area under the curve from 0% to 50%, the proportion of students failing, is less for Autumn 2014 (Mastery Learning) than all previous semesters. Correspondingly, the proportion of students passing is greater with Mastery Learning with a bimodal distribution that is more pronounced than in previous semesters. Bimodality is perhaps not surprising given the fact that the Mastery Tests focussed on ensuring the fundamental knowledge and skills objectives of the subject. Student who woul dhave then previously failed (possibly due to a lack of assumed knowledge), now 'mastered' the fundamental requisite knowledge and skills enabling them to earn a Pass.

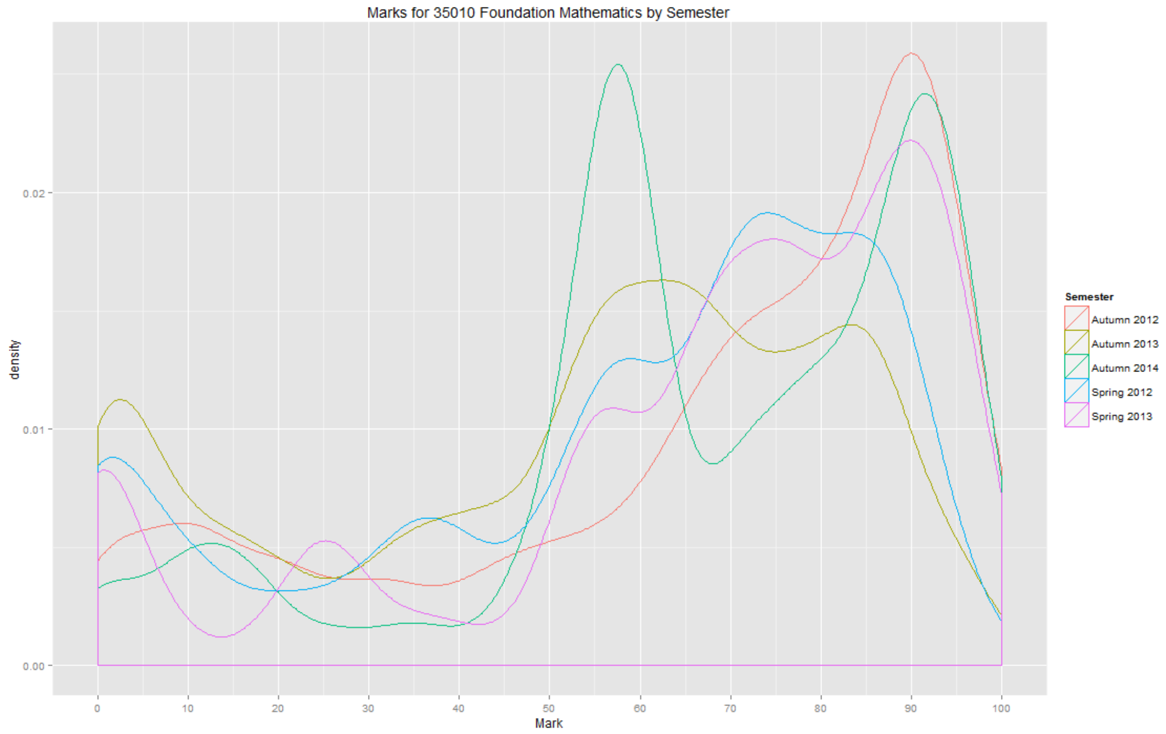

Figure 1: Distributions of final marks for Foundation Mathematics

This begs the question whether the 'bar' for a pass has been shifted up. To some extent the answer to this question depends on how you define 'up'. If 'up' is described by the number of learning outcomes students now demonstrate, the answer is yes – students are no longer in a position to decide to not learn a topic or topics, and focus on others to pass a subject. If on the other hand 'up' is defined in terms of an equally difficult to define concept of 'amount', then it may be that this has not changed. Support for this is evidenced by the grade distributions (Figures 1 and 2) – they simply have not shifted to the right.

There is not obvious reason to explain the fact that numerically fewer students achieved Credits and Distinctions. This is something requiring further examination.

The question then arises as to whether Mastery Learning in Foundation Mathematics is assisting the students who have been identified as being ill-prepared for their programs – those students who studied General Mathematics (or no senior mathematics). Table 3 lists the mean and median final marks, and the failure rate by mathematical background for the three Autumn semesters ending with Autumn 2014.

In Table 3 we can see an overall reduction in failure rates for the Autumn semester cohorts – the improvement between Autumn 2012 and Autumn 2014 is significant at the 10% level, while the improvement between Autumn 2013 and Autumn 2014 is significant at the 1% level. There is also a reduction in the failure rate for the target background cohort, General Mathematics students. Using Fisher's Exact Tests to compare Autumn 2012 with Autumn 2014, the reduction in failure rate is significant at the 10% level, while the reduction between Autumn 2013 and Autumn 2014 is not significant. One contributing factor to this is the large increase in size of the General Mathematics cohort in 2014. There are also reductions in the failure rates for students coming from a Mathematics background, though these are not significant at the 5% level.

Table 3: Foundation Mathematics results by background

Background	Autumn 2012			Autumn 2013			Autumn 2014		
	mean	median	%fail	mean	median	%fail	mean	median	%fail
Extension 2	97.00	98	0% (0/3)	77.50	83.5	0% (0/7)	87.00	90	0% (0/8)
Extension 1	85.11	87	0% (0/9)	66.78	78	0% (0/15)	80.71	92.5	8% (2/24)
Mathematics	73.59	81	33% (4/12)	56.00	57	25% (6/24)	72.88 -†	81-‡	14%-- (7/50)
General Mathematics	49.57	50	48% (10/21)	30.97	19	45% (5/11)	52.97 -†	57*‡	28%*- (18/63)
No HSC maths	-	-	-	-	-	-	55.5	55.5	0% (0/2)
All	66	76	21%	53	60	29%	66-†	70--	19%*‡

Key: - = improvement not significant; * = 2014 improvement significant at the 10% level; † = 2014 improvement significant at the 5% level; ‡ = 2014 improvement significant at the 1% level. First symbol for 2012 comparison, second symbol for 2013 comparison.

An alternative explanation for the improvements in failure rate in Autumn 2014 is that more capable students taking General Mathematics (as described previously). However, if we use ATAR as a proxy for ability, when the distributions of ATARs were examined for the three, year cohorts, approximately 60% of the Autumn 2014 cohort achieved ATARs of 80 or greater, with the proportions for the Autumn 2012 and Autumn 2013 cohorts, 83% and 93% respectively. The mean ATARs for Autumn 2012, 2013 and 2014 are 74, 80 and 67 with the reduction between Autumn 2012 and Autumn 2014 being significant at the 10% level and the reduction between Autumn 2013 and Autumn 2013 being significant at the 1% level. It would then appear that an explanation based on more able students taking General Mathematics in the 2014 intake does not hold.

A one-sided t-test with unequal variances on the mean overall final mark finds that the improvement in mark is significant at the 5% level for the Autumn 2013/Autumn 2014 comparison, but the Autumn 2012/Autumn 2014 comparison is not significant. These findings are duplicated for the mean final mark for the General Mathematics and Mathematics cohorts.

Median final marks also improve for nearly all background cohorts with Mastery Learning. For the cohorts of primary interest, the General Mathematics and Mathematics cohorts, the improvement in median for the General Mathematics cohort for the Autumn 2012/Autumn 2014 comparison is significant at the 10% level, and the increase in median when Autumn 2013 and Autumn 2014 is significant at the 1% level. For the Mathematics cohort, the

Autumn 2012 comparison is not significant, while the Autumn 2013 comparison is significant at the 1% level ($p = 0.00036$).

We can then conclude that there is evidence of improvement in most of the statistics measuring achievement in the primary target cohort (General Mathematics background), though this is not universal. There is also a significant reduction in overall failure rate. However, Mastery Learning does not seem to afford students with the "Recommended" knowledge any advantage. Nor, however, does it seem to offer them any disadvantage.

One of the significant problems faced by Engineering students and staff at UTS was the fact that students who passed Mathematical Modelling 1 were not as likely to pass Mathematical Modelling 2. It appeared that retention of knowledge and skills was poor, and/or Mathematical Modelling 2 was of a higher level of difficulty. It was then interesting to see what minor changes in content and the implementation of Mastery Learning would have on student outcomes.

Improvement in the distribution of marks is even more pronounced for Mathematical Modelling 2 (Figure 2) than for Foundation Mathematics (Figure 1) – the area under the curve from 0 to 50% is again lower than all previous semesters with a strong peak around 60%.

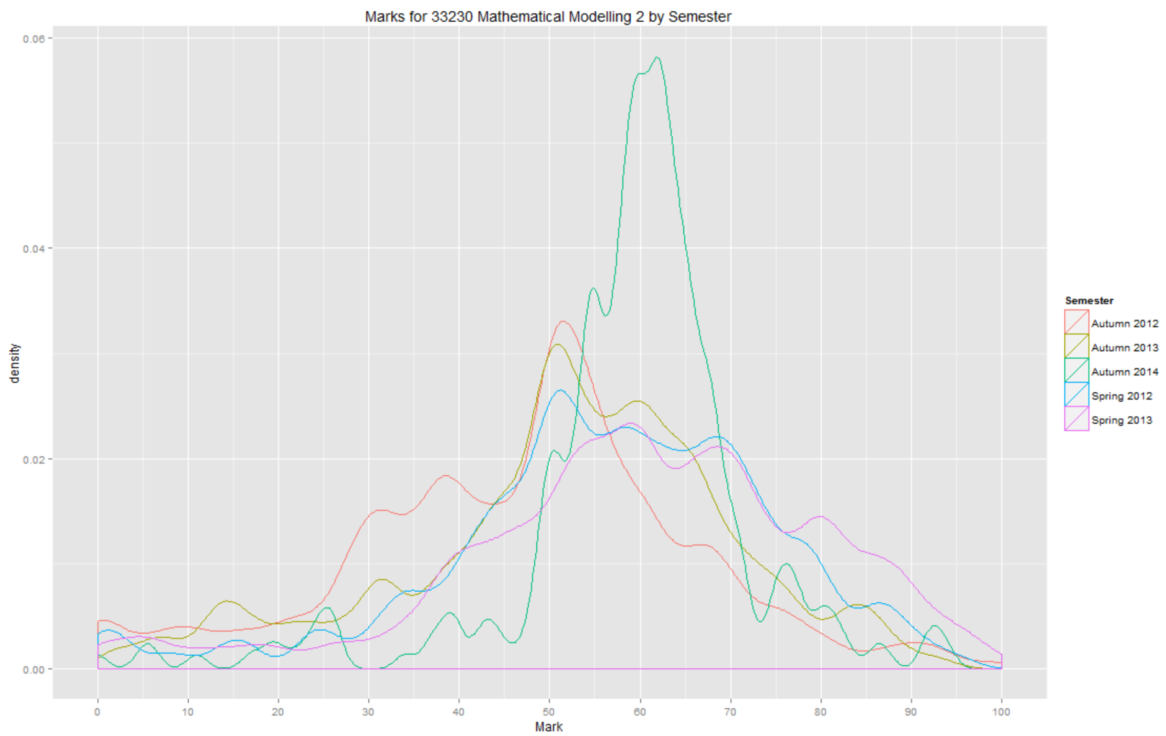

Figure 2: Distributions of final marks for Mathematical Modelling 2

From Table 4 we again see significant reductions in failure rates for all Mathematical Modelling 2 students when Mastery Learning is used – the improvements in overall pass rates (reduction in failure rates) for both Autumn comparisons are significant at the 1% level. This result is particularly pronounced when the results in Autumn 2014 are compared with those in Autumn 2012. The improvement is less large when Autumn 2014 is compared to Autumn 2013. Indeed, there is a reduction in failure rate in 2013 compared to 2012. This reduction may be attributable to change in mode and small changes in content that were undertaken in Autumn 2013. It is unclear whether these changes would have led to further reductions in failure rate without implementing another change to the learning design of the subject. It seems reasonable to conclude that at least some of the reduction is attributable to the implementation of Mastery Learning.

For students who received a Pass in Mathematical Modelling 1, both comparisons to Autumn 2014 resulted in significant reductions in failure rate (at the 1% level). For students with Credits, the Autumn 2012/Autumn 2014 comparison was significant at the 1% level, though the Autumn 2013/Autumn 2014 comparison was not. For students with Distinctions, both comparisons in failure rate were not significant at the 5% level.

However, there is a reduction in the mean and median final mark for students who obtained High Distinctions in Mathematical Modelling 1 when Autumn 2014 is compared to Autumn 2013.

Table 4: Mathematical Modelling 2 results by Mathematical Modelling 1 result

MM1 result	Autumn 2012			Autumn 2013			Autumn 2014		
	mean	median	%fail	mean	median	%fail	mean	median	%fail
Pass	38	40	72% (53/74)	51	53	35% (33/93)	58‡‡	59‡‡	20%‡‡ (18/88)
Credit	50	52	35% (16/46)	60	64	16% (5/32)	61	61‡-	0%‡- (0/20)
Distinction	53	51	32% 9/28	65	68	16% (3/19)	65	63‡-	6%-- (1/16)
High Distinction	63	64	0% (0/16)	84	87	0% (0/7)	74	77	0% (0/9)
All	47	50	46%	52	53	36%	59‡‡	60‡‡	12%‡‡

Key: as for Table 3.

The improvements in overall mean final mark for both Autumn semester comparisons are significant at the 5% level, in fact are significant at the 1% level, with the implementation of Mastery Learning. The improvements in median for both Autumn comparisons were significant at the 1% level using the Wilcoxon Rank Sum Test. The improvements in mean and median final marks are significant (at the 1% level) for both comparisons of students who obtained a Pass in Mathematical Modelling 1, and also for students achieving Credits or Distinctions for Autumn 2012 are compared with Autumn 2014. However, the mean and median final marks are not significant at the 5% level for Credits and Distinctions for the Autumn 2013/Autumn 2014 comparison.

While it is not possible to credit the significant improvements in academic achievement in Mathematical Modelling 2 solely to Mastery Learning (given the small changes in 2013 previously mentioned), it certainly cannot be discounted that Mastery Learning contributed significantly to this improvement. Further, this improvement in outcomes was demonstrated for the subject's target cohort – students earning a Pass grade in Mathematical Modelling 1.

These encouraging results for Foundation Mathematics and Mathematical Modelling 2 are broadly in line with improvements in pass rates and mean final marks reported by a number of US colleges participating in the Changing the Equation program (Twigg, 2013). However, there changes to learning design were in line with the Emporium Model, so direct comparison is not entirely valid.

Student Perspectives

The linchpin of Mastery Learning is the Mastery Test. Unlike traditional tests, the '50% is a pass' concept no longer holds, 80% is required to demonstrate mastery.

> *(Facilitator) Was it scary having to get 80%? (Student) At first it was but eventually I thought 80% was achievable.*
> *(Focus group – Foundation Maths Autumn 2014)*
> *I found the testing methods a bit different and that they took a lot of the pressure out of the tests. (SFS Foundation Mathematics Autumn 2014).*

Students not reaching mastery on the first attempt are given additional attempts (up to two further attempts) to demonstrate mastery of knowledge and skills. Smaller, targetted tests with multiple opportunities facilitate continuous engagement, foster time management skill development and are less stressful and more accommodating than traditional, one-chance testing.

> *What did you particularly like about this subject?*
> *Mastery tests throughout the semester made me keep up to date with my work (SFS Foundation Mathematics Autumn 2014)*
> *The mastery tests made you learn continuously instead of cramming at the end of the semester. (SFS Foundation Mathematics Autumn 2014)*
> *Mastery tests … have taken off stress for my final exam.*
> *(SFS Foundation Mathematics Autumn 2014)*

Students appreciate the immediate detailed feedback available through online testing, enhanced opportunities and support to reflect and remediate, and multiple opportunities to demonstrate their mastery. Simultaneously, achievement, attitudes toward mathematics, learning, confidence and the retention of content improve.

> *What did you particularly like about this subject?*
> *The option of a second chance for the class tests allowed us to gain an understanding of our areas for improvement and demonstrate our ability to correct our errors. (SFS Mathematical Modelling for Science Autumn 2013)*
> *… the Mastery Tests which sort of helped me ease into [the subject and] … I have a greater appreciation for maths now. (Focus group – Foundation Maths Autumn 2014)*
> *The '2nd chance' class test. If a student has not done too well in the class test, a second seating of a test is available. It is not the same test but similar and this does wonders to a student's confidence in a subject that leaves very few standing tall. (SFS Mathematical Modelling for Science Autumn 2013)*
> *Having the mastery tests on topics. It helped retain the concepts longer.*
> *(SFS Foundation Mathematics Autumn 2014)*

Not all student experiences were as hoped for. Some students of Mathematical Modelling 2 found the online tests stressful, partly because of the testing environment and the lack of facility to show working in this environment. (This is true of the platform used for Mathematical Modelling 2 but is not true of the other online platform used.) We can see this in the comments below.

> *Although the questions were basic, the pressure behind each of these Mastery Tests was huge! It was as if I was doing a final exam every single week and because of that and I*

> *was studying part-time, and so my other subject was hampered. (Focus group – Mathematical Modelling 2 Autumn 2014)*
> *For the purposes of passing it was sufficient. But I don't feel confident moving forward. (Focus group – Mathematical Modelling 2 Autumn 2014)*
> *(Facilitator) Mastery of the test or the course?*
> *(Student) Mastery only of the test and not the subject.*
> *(Focus group – Mathematical Modelling 2 Autumn 2014)*

This last comment is concerning and suggests the design of the implementation of Mastery Learning in Mathematical Modelling 2 requires revisiting.

Staff Perspectives

The experiences of members of staff with Mastery Testing varied with the platform used. Staff using paper tests found by setting well-structured tests, marking was quite quick, but a large number of tests needed to be set for each asynchronous tutorial class and each offering of a test. Mark management was also burdensome, as was physical management of the test papers.

Staff using proprietary online testing platforms had other issues. When traffic was high on the website, the response time slowed. On rare occasions connections to the test website would drop out. Of the two online platforms used, one of the products proved to be slightly more unreliable in this regard. While the marking was almost entirely automated, the testing platform was not linked with the learning management platform. Marks then had to be downloaded from the testing platform and uploaded to the learning management platform. This was cumbersome given the number of students involved. An associated problem was the need to manage the log-ins for students on the testing platform. Another issue was the fact that time needed to be allocated to online entry of algebraic expressions in the tests because of the nature of the algebra interface. It was for this reason that some students expressed a preference for paper tests over online tests.

On a more positive note, staff reported an increase in student interactions (with each other and with staff), particularly in the areas formative feedback, and utilisation of the Mathematics Study Centre and online resources. Greater use of the textbooks were also reported, with the use of the ebook version of the textbooks increasing.

Conclusions and future directions

As can be seen in Tables 3 and 4 preliminary results have been very promising and additional first-year mathematics subjects have moved towards the hybrid form of Mastery Learning employed in Foundation Mathematics and Mathematical Modelling 2. In Spring semester 2014, online mastery testing was introduced into Foundation Mathematics, and the mastery-style learning design was introduced into Mathematical Modelling 1 and Introduction to Analysis and Multivariable Calculus (as well as the existing subjects).

The UTS experience suggests that Mastery Learning is a very promising learning design, not just for UTS, but for other tertiary institutions experiencing problems with a lack of student preparedness and poor student experience and performance.

However, student feedback from Mathematical Modelling 2 shows that there is room for improvement in the execution of Mastery Learning in this subject:

- the Mastery Tests were too easy

- the online tests produced pressure on students
- the online platforms allowed students to cheat
- marking should allow for correct working

The first two points are very likely a consequence of confusion about the purpose of the Mastery Tests, and this perhaps was not made as clear as it could have been. Familiarity with the use of online testing should go a long way to reducing pressure. Teaching staff are now aware of the strategies employed to cheat online and have taken steps to reduce the risk of cheating. The need for working to be undertaken has been included in some of the subjects, and this has proven useful for students on the borderline of mastery.

With more subjects now utilising Mastery Learning, and with some of these subjects being sequential, a study of the impact of Mastery Learning on the performance in later subjects will be able to be undertaken in the near future.

> *An education in STEM also fosters a range of generic and quantitative skills and ways of thinking that enable individuals to see and grasp opportunities. These capabilities—including deep knowledge of a subject, creativity, problem solving, critical thinking and communication skills—are relevant to an increasingly wide range of occupations. They will be part of the foundation of adaptive and nimble workplaces of the future.*
> *(Office of the Chief Scientist, 2014)*

As we see in this quote the significance of sound, long-lasting foundations in mathematics for not only Science, Technology, Engineering and Mathematics (STEM) students but all students cannot be understated. Improving outcomes in disciplinary knowledge and quantitative skills for first year students is an important step in ensuring students are work-ready.

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