The effect of different maximum speeds is shewn on Diagram No. 9 , which gives N at any station stop up to one minute for each of the signalling systems considered, and shows also the curves for maximum trains per hour in each case. The train length assumed is 500 feet and $f a=1 \frac{1}{4}$ m.p.h.p.s. The bottom right hand curves shew the relative capacities under the three systems considered, the maximum speed being 40 m.p.h., with $f a=1 \frac{1}{4}$ m.p.h.p.s. and $\mathrm{fb}=2$ m.p.h.i.s.


Diagram No. 9.
Under the conditions usually met in a rapid transit railway an increase in maximum speed results in only a slight increase in schedule speed, expressed either as actual speed or as a proportion of the previous speed. Diagram No. 10 shews graphically the relationship between schedule speed and maximum speed for station stops of 0 to 60 seconds, distances between stops from 1,000 to 6,000 feet. The dotted examples
shew that for a 3,000 feet section an increase in maximum speed from $30 \mathrm{~m} . \mathrm{p} . \mathrm{h}$. to $45 \mathrm{~m} . \mathrm{p} . \mathrm{h}$. with a station stop of 40 seconds increases the schedule speed from $15.6 \mathrm{~m} . \mathrm{p} . \mathrm{h}$. to 17.8 m.p.h., or only 14.1 per cent. The diagram is based on assumed $\mathrm{fa}=1 \frac{1}{4} \mathrm{~m} . \mathrm{p} . \mathrm{h} . \mathrm{p} . \mathrm{s}$., coasting from V at $0.07 \mathrm{~m} . \mathrm{p} . \mathrm{h} . \mathrm{p} . \mathrm{s}$. to 0.8 V , if possible, and braking at $2 \mathrm{~m} . \mathrm{p} . \mathrm{h} . \mathrm{p} . \mathrm{s}$.


Diagram No. 10.
The effect of station stop on schedule speed is also clearly indicated on this diagram, the example shewn being for 35 m.p.h. The shortening of the station stop from 50 seconds to 30 seconds produces a greater effect on schedule speed than an increase in maximum speed from $30 \mathrm{~m} . \mathrm{p} . \mathrm{h}$. to $45 \mathrm{~m} . \mathrm{p} . \mathrm{h}$. Of course, on longer sections this effect would be less pronounced, though it will be seen that in ordinary cases the
station stop is the most important factor influencing the schedule speed.

A moment's thought will shew that with any system of signalling the distance between trains at points remote from stations is great if trains are to operate through and stop at stations. Ordinarily it will be I.V-L, which even with 40 trains per hour amounts to over 4,000 feet if the section is of sufficient length. The possibility of arranging for a system of speed control near and in stations therefore presents itself, the free track between stations being signalled in the usual manner. A neat and successful system with this object has been adopted by the Interborough Rapid Transit Co. for their express tracks in the New York subway. With continuous overlap automatic signalling a train checked when entering a station would have to stop about 900 feet from the station, assuming makimum speed of about $35 \mathrm{~m} . \mathrm{p} . \mathrm{h}$. and deceleration


Diagram No. 11.
of $2 \mathrm{~m} . \mathrm{p} . \mathrm{h} . \mathrm{p} . \mathrm{s} .$, and could not start again until the train in the station had moved the length of the station, the time element being long on account of the leaving train having to accelerate from rest. The system used on the Interborough provides for the incoming train to stop at a point only 295 feet from the platform, and it may start again almost as soon as the outgoing train commences to move. This feature materially reduces the headway in cases where incoming trains are checked, and further provides features akin to perfect speed control when free running conditions prevail.

Diagram No. 11 shews the signalling system diagramatically, and the movements of a train leaving the station and a checked train entering. Signals 1 to 7 are arranged as shewn,
each with an automatic train stop. The presence of a train in the station will raise the train stops 2,3 and 4 , signals 2 to 7 being at danger. An approaching train will be cautioned at 1 and may advance to 2. A time element relay controls 2 which will clear for the approaching train provided it takes a predetermined time in passing from 1 to 2 . The time is fixed on the assumption that signal No. 1 is passed at full speed and a service application of brakes applied. The train may now advance to 3 under similar conditions and then to 4, where a stop is compulsory if the train in station has not moved. The time element relays controlling 2 and 3 are set so that it is impossible for the incoming train to strike the train stop at 4 at greater velocity than could be destroyed between 4 and the platform end with the automatic application of brakes at 4 .

Signal 4 is arranged to clear when the leaving train passes W., 5 when it passes X., 6 when it passes Y., and 7 when it passes Z. The increase in capacity, after the installation of this system was over 21 per cent.

## Station with Bifurcated Tragk



Diagram No. 12.
It should be noted, however, that automatic protection ceases as soon as the leading train has passed Z; should it stall immediately afterwards the prevention of accident depends entirely on the driver of the following train bringing his train to rest at the station.

Another feature concerning the capacity of rapid transit tracks is brought about owing to the disparity between the space interval between trains in the vicinity of a station and the space interval on the free track between stations. By bifurcating the track in platforms, as shewn on Diagram No. 12 , it is possible to route only alternate trains to the same side of the platform, giving limiting conditions between alternate trains similar to those obtaining in previously deduced cases so far as station stop influence is concerned, and doubling the capacity of the intervening track providing the track limit-
ing condition is not met. Another controlling feature may be the distance to the points at the leaving end of the station, for it is assumed for safety conditions that a starting signal could not be given to a train on one side of the platform until the points were locked in position to suit it after the passage of the train from the other side.

In cases of automatic signalling with three or two overlaps between trains running freely, routing consecutive trains to opposite sides of a platform or alternate trains to the same one of a pair of tracks forming a bifurcation, the interval between alternate trains for free running conditions

$$
=(k \cdot n+1) \frac{V}{2 \cdot f b}+\frac{V}{2 . f a}+\frac{L}{V}+t s
$$

In general train about to leave station may be given starting sig-
V2
nal immediately previous train has cleared point $k(n-1) \frac{1}{2 \text { fb }}$ feet from station, i.e. after having travelled $k(n-1) \frac{}{2 f b}+L$ feet.

Assuming train that has left station is running at full speed, the minimum time interval between trains equals

$$
\frac{V}{2 \mathrm{fa}}+\mathrm{k}(n-1) \frac{\mathrm{V}}{2 \mathrm{fb}}+\frac{\mathrm{L}}{\mathrm{~V}}
$$

Giving a minimum time interval between alternate trains

$$
=\frac{V}{f a}+k(n-1) \frac{V}{f b}+\frac{2 L}{V}
$$

The minimum station stop, therefore, becomes

$$
\begin{gathered}
t s=\frac{V}{f a}+k(n-1) \frac{V}{f b}+\frac{2 L}{V}-(k \cdot n+1) \frac{V}{2 \cdot f b}-\frac{V}{2 f a}-\frac{L}{V} \\
=\frac{V}{2 f a}+(k n-2 k-1) \frac{V}{2 f b}+\frac{\mathrm{V}}{\mathrm{~V}}
\end{gathered}
$$

$V=35 \mathrm{~m} . \mathrm{p} . \mathrm{h} ., \mathrm{fa}=1 \frac{1}{4} \mathrm{~m} . \mathrm{p} . \mathrm{h} . \mathrm{p} . \mathrm{s} ., \mathrm{fh}=2 \mathrm{~m} . \mathrm{p} . \mathrm{h} . \mathrm{p} . \mathrm{s} ., \mathrm{L}=500 \mathrm{ft}$.
$\mathrm{ts}=14+(\mathrm{kn}-2 \mathrm{k}-1) 8.75+9.75$
$\mathrm{k}=1.5, \mathrm{ts}=14+(1.5 \mathrm{n}-4) 8.75+9.75$
$\mathrm{n}=3$, ts $=14+4.37+9.75=28.1$ seconds.
$\mathrm{n}=2$, ts $=14-8.75+9.75=15$ seconds.

With station stops longer than these, the capacity of bifurcated tracks at a station is double the capacity with a single track and an equal station stop, providing track limiting conditions are not met between stations.

Note-If distance $k(n-1) \frac{V^{2}}{2 f b}$ is less than distance
to rail joint beyond points the minimum distance used above will be increased. Also it will be apparent that this case must be analysed to see that points are set prior to giving starting signal. In general time interval is ample.

In the case where $n=2$, with an automatic speed control system similar to that previously ${ }^{\text {described, the train starting }}$ from the station will leave with the speed control device in operation, meaning that it would have to reduce speed in accordance with the normal braking curve to stop at the first signal. Normally the train in advance would accelerate to full speed before the tail-end arrived at the first signal in advance of the station, and would run through the second block in advance of the station at full speed. Under these conditions it would be possible to locate a ramp in the first block subdividing the block, the function of the ramp being only to cut out the auto. speed control device if the block in advance were free.

Since the only conditions enforced by the speed control device are that the speed of a train must be less than that given by a normal braking curve terminating at the first signal, the leaving train may accelerate to a certain speed, then have that speed reduced if block in advance is occupied, or continue to accelerate if the block in advance be clear. Such a subdivision of the first block will not interfere with capacity if the time taken for the rear end of train to traverse the second block is less than the time taken for the front end of a train leaving the station to reach the subdivision or releasing ramp.

Let $d=$ distance between signals in feet.
$\mathrm{s}=$ distance to subdivision ramp in feet.
Speed at point $S$ will be such that train would stop in d-s feet; also train has accelerated to this speed in $S$ feet,

$$
\begin{gathered}
\frac{S}{d-S}=\frac{f b}{f a} \\
S=\frac{f b}{f a+f b} \cdot d=\frac{k}{2} \frac{V^{2}}{(f a+f b)}
\end{gathered}
$$

Time for head end of leaving train to go from station to $S$

$$
=\sqrt{\frac{k \cdot V^{2}}{f a(f a l+f b)}}=\frac{V}{f a} \sqrt{1+\frac{k b}{f a}}
$$

Time for tail end of previous train to traverse second block

$$
=\frac{\mathrm{k} \cdot \mathrm{~V}}{2 \mathrm{fb}}
$$

Interval for ramp to get indication on minimum headway of train

$$
I_{s}=\frac{V}{f a} \sqrt{\frac{k}{1+\frac{f b}{f a}}}-\frac{k}{2} \frac{V}{\mathrm{fb}}
$$

If $\mathbf{f b}=\mathrm{m} . \mathrm{fa}$.

$$
\mathrm{Is}=\frac{\mathrm{V}}{\mathrm{fa}}\left\{\sqrt{\left.\left.\frac{\mathrm{k}}{\mathrm{l}+\mathrm{m}}-\frac{\mathrm{k}}{2 . \mathrm{m}}\right\},\right\} .}\right.
$$

This interval is positive for values of $k$ less than $\frac{4 \mathrm{~m}^{2}}{1+\mathrm{m}}$ for $\mathrm{k}=1 \frac{1}{2}, \mathrm{fa}=1.25 \mathrm{~m} . \mathrm{p} . \mathrm{h} . \mathrm{ps}$.

$$
\begin{aligned}
& \mathrm{fb}=2 \mathrm{~m} . \mathrm{p} \text { h.p.s. } ; \quad \mathrm{V}=35 \mathrm{~m} . \mathrm{p} . \mathrm{h} \\
& \mathrm{~m}=1.6 \text {, and Is }=8 \text { seconds. }
\end{aligned}
$$

Treating now the case of perfect automatic speed control, a starting signal may be given to a train immediately the points are locked in the reverse position after the passage of the previous train.

Let $l=$ distance from platform to rail joint ahead of points.
$\mathrm{tp}=$ time required for points to operate.
If $L+l$ is greater than $\frac{\mathrm{V}^{2}}{2 \mathrm{fa}}$, train clears points at a time after starting

$$
=\frac{\mathrm{V}}{2 \mathrm{fa}}+\frac{\mathrm{L}}{\mathrm{~V}}+\frac{l}{\mathrm{~V}}+\mathrm{tp}
$$

Minimum time interval between trains

$$
=\frac{\mathrm{V}}{2 \mathrm{fa}}+\frac{\mathrm{L}}{\mathrm{~V}}+\frac{l}{\mathrm{~V}}+\mathrm{tp}
$$

In case l. of perfect speed control with bifurcated tracks, interval between alternate trains

$$
=\frac{V}{2 f_{a}}+\frac{V}{f b}+\frac{L}{V}+t s
$$

Interval between consecutive trains

$$
\frac{V}{4 \mathrm{fa}}+\frac{\mathrm{V}}{2 \mathrm{fb}}+\frac{\mathrm{L}}{2 \mathrm{~V}}+\frac{\mathrm{ts}}{2}
$$

If this equals the minimum interval

$$
\begin{gathered}
\frac{\mathrm{V}}{4 \mathrm{fa}}+\frac{\mathrm{V}}{2 \mathrm{fb}}+\frac{\mathrm{L}}{2 \mathrm{~V}}+\frac{\mathrm{ts}}{2}=\frac{\mathrm{V}}{2 \mathrm{fa}}+\frac{\mathrm{L}}{\mathrm{~V}}+\frac{l}{\mathrm{~V}}+\mathrm{tp} \\
\quad \text { giving a minimum station stop of } \\
\mathrm{ts}=\frac{\mathrm{V}}{\mathrm{fa}}\left(\frac{\mathrm{~m}-2}{2 \mathrm{~m}}\right)+\frac{\mathrm{L}+2 l}{\mathrm{~V}}+2 \mathrm{tp}
\end{gathered}
$$

In case 2 (the usual case) of perfect speed control, with bifurcated tracks, interval between alternate trains

$$
=\frac{\mathrm{V}}{\mathrm{fb}}+\sqrt{\frac{2 \mathrm{~L}}{\mathrm{fa}}}+\mathrm{ts} \text { or } \frac{\mathrm{V}}{2 \mathrm{fb}}+\sqrt{\frac{2 \mathrm{~L}}{4 \mathrm{fa}}}+\frac{\mathrm{ts}}{2}
$$

between consecutive trains.
If this equals the minimum interval the minimum station stop is

$$
\begin{gathered}
\mathrm{ts}=\frac{\mathrm{V}}{\mathrm{fa}}-\frac{\mathrm{V}}{\mathrm{fb}}-\sqrt{\frac{2 \mathrm{l}}{\mathrm{fa}}}+\frac{2 \mathrm{~L}}{\mathrm{~V}}+\frac{2 l}{\mathrm{~V}}+2 \mathrm{tp} \\
\mathrm{~V}=35 \mathrm{~m} . \mathrm{p} . \mathrm{h} ., \quad \mathrm{fa}=1 \frac{1}{4} \mathrm{~m} . \mathrm{p} . \mathrm{h} . \mathrm{p} . \mathrm{s}, \quad \mathrm{fb}=2 \mathrm{~m} . \mathrm{p} . \mathrm{h} . \mathrm{p.s} . \\
\mathrm{L}=500 \text { feet. }
\end{gathered}
$$

$$
\text { Minimum ts }=6.6+\frac{27}{51.3}+2 \mathrm{tp}
$$

tp need not be greater than 5 seconds, whilst $l$ will seldom exceed 350 feet giving minimum ts under these conditions $=30$ seconds.

If $L+l$ is not greater than $\frac{V^{2}}{2 f a}$ train clears points at a time after starting


$$
\begin{aligned}
& \text { giving minimum interval of } \sqrt{\frac{61}{2(\mathrm{~L}+\mathrm{l})}}+\mathrm{fa} \\
& \text { minimum ts }=2 \sqrt{\frac{2(\mathrm{~L}+l)}{\mathrm{fa}}-\sqrt{\frac{2 \mathrm{~L}}{\mathrm{fa}}-\frac{\mathrm{f}}{\mathrm{fb}}}+2 \mathrm{tp}}
\end{aligned}
$$

For conditions previously assumed $l$ must be less than 220
With $I=220^{\prime}$, min. ts $=25$ secs.
In addition to the limits imposed on the capacity by the arrangements at the station, the conditions under free running between stations must be examined.
(1). With Three or T'wo Overlaps.


$$
\text { Time interval }=\frac{1}{2}\left(\frac{\mathrm{kn}+\mathrm{l}}{2} \frac{\mathrm{v}}{\mathrm{fb}}+\frac{\mathrm{v}}{2 \mathrm{fa}}+\frac{\mathrm{L}}{\mathrm{v}}+\mathrm{ts}\right)
$$

Distance interval between trains

$$
\frac{1}{2}\left(\frac{\mathrm{kn}+1}{2} \frac{v}{\mathrm{fb}}+\frac{\mathrm{v}}{2 \mathrm{fa}}+\frac{\mathrm{L}}{\mathrm{v}}+\mathrm{ts}\right) \mathbf{v}-\mathrm{L}
$$

If this equals shortest distance

$$
\begin{aligned}
& \mathrm{kn} \frac{\mathrm{v}^{2}}{2 \mathrm{fb}}=\frac{\mathrm{kn}+1}{4} \frac{\mathrm{v}^{2}}{\mathrm{fb}}+\frac{\mathrm{v}^{2}}{4 \mathrm{fa}}+\frac{\mathrm{L}}{2}+\frac{\mathrm{v.ts}}{2}-L \\
& \frac{\mathrm{kn}-1}{4} \quad \frac{v^{2}}{\mathrm{fb}}-\frac{\mathrm{v}^{2}}{4 \mathrm{fa}}+\frac{\mathrm{L}}{2}=\frac{\text { v.ts }}{2} \\
& \mathrm{v}=35 \mathrm{~m} . \mathrm{p} . \mathrm{h} \text {. } \\
& \frac{\mathrm{k} n-1}{4} \cdot \frac{51.3^{2}}{2.93}-\frac{51.3^{2}}{4 \times 1.83}+\frac{500}{2}=\frac{51.3}{2} \text { ts } \\
& 225(\mathrm{kn}-1)-360+250=25.6 \text { ts } \\
& 225 \mathrm{kn}-335=25.6 \mathrm{ts} \\
& \mathrm{k}=1 \frac{1}{2} \\
& 337 \mathrm{n}-335=25.6 \text { ts } \\
& \text { (1). } n=3 \\
& \mathrm{ts}=26.4 \text { (minimum) } \\
& \text { (2). } \mathrm{n}=2 \\
& \mathrm{ts}=13.3
\end{aligned}
$$

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## (2). With Perfect Speed Control.

$$
N=9 \frac{3600}{\frac{v}{\mathrm{fb}}+\sqrt{\frac{2 L}{\mathrm{fa}}}+\mathrm{ts}}
$$

Time Interval between trains $\frac{1}{2}\left\{\frac{\mathrm{v}}{\mathrm{fb}}+\sqrt{\frac{2 \mathrm{~L}}{\mathrm{fa}}}+\mathrm{ts}\right\}$
Distance interval between trains

$$
\frac{1}{2}\left\{\frac{v}{f b}+\sqrt{\frac{2 L}{f a}}+t s\right\}^{v}-L
$$

In limit this distance $=d \mathrm{~d}$.

$$
\frac{51.3}{2} \sqrt{\frac{682}{1.25}}+\frac{51.3}{2}-\mathrm{ts}={ }^{\circ} 500
$$

$$
51.3 \times 11.7+20.6 \mathrm{ts}=500
$$

$$
600+20.6 \mathrm{ts}=500
$$

$$
20.6 \text { ts }=-100
$$

(i.e., the following train cannot reach the limiting distance, whatever the value of the station stop.)

Under general practical conditions it is evident that bifurcation at stations enables double the number of trains that would be possible with a single track to pass through the station.

With $\mathrm{V}=35$ m.p.h., $\mathrm{fa}=1 \frac{1}{\frac{1}{4}} \mathrm{~m} . p . h . p . s ., \quad \mathrm{fb}=2 \mathrm{~m} . \mathrm{p} . \mathrm{h} . \mathrm{p.s}$. , $\mathrm{L}=500$ feet, the limiting maximum capacity is twice that of $\cdot \mathrm{a}$ single track with-
(a) $\mathrm{k}=1 \frac{1}{2}, \mathrm{n}=3$ station stop of 28 seconds.
(b) $\mathrm{k}=1 \frac{1}{2}, \mathrm{n}=2$ station stop of 15 seconds.

$$
\begin{aligned}
& \text { i.e. } \frac{v^{2}}{2 \mathrm{fb}}=\frac{1}{2}\left\{\frac{v}{\mathrm{fb}}+\sqrt{\frac{2 L}{f a}}+\text { ts }\right\} v-L \\
& =\frac{1^{2}}{2 \mathrm{fl}}+\frac{\mathrm{v}}{2} \sqrt{\frac{2 \mathrm{~L}}{\mathrm{fa}}}+\frac{\mathrm{v}}{2}+\mathrm{ts}-\mathrm{L} \\
& \text { or } \frac{v}{2} \sqrt{\frac{2 \mathrm{~L}}{f a}}+\frac{v}{2}+\mathrm{ts}=\mathrm{L} \\
& \mathrm{v}=35 \mathrm{~m} . \text { p.h. } \quad f a=1 . \unrhd 5
\end{aligned}
$$

(c) Perfect speed control, allowing 5 seconds for movement of points minimum station stop $=30$ seconds or less according to distance from station to switch, and if time for switch movement be eliminated minimum station stop $=20$ seconds or less.

In all cases if the station stop be greater than the critical values determined the capacity is doubled by bifurcation.

It is to be noted in connection with automatic speed control, that, with the usual arrangement of time limit relays and train stops, it is possible for a train when accelerating to pass a stop, without being tripped, at a speed higher than that at which it would be tripped if coasting or running at constant speed. Such an accelerating train would, of course, be tripped after the next time interval.

Diagram No. 13 shews the development of automatic control by which a train, whether running at constant speed, up to 40 m.p.h., coasting or accelerating, will be brought to rest at a distance not more than 125 feet beyond the normal stopping position, or derailed at a point 80 feet beyond such position at a speed not greater than 13.6 m.p.h.

It will be seen that this control is obtained by the use of four time limit relays and four additional train stops; any further reduction of possible overrun can only be obtained by an increased number of relays and a reduction of the time interval below the working limit.

The diagram shews the position of train stops, time intervals of relays, including the time elements of the train stops, and the acceleration and braking curves used in the calculations.

One of the main factors in keeping the station stop low is the orderliness or "travelling manners" of the passengers. Where, through natural inclination or training, passengers before attempting to board trains will normally stand aside to facilitate the exit of alighting passengers, the station stop is certain to be short. The London underground railways operate with shorter station stops than any other rapid transit railways, and while there are other contributing factors, the absence of apparent hurry is responsible in a large measure for the shortness of the station stop.

Dealing first with alighting passengers, it is obvious that the seating arrangements in the cars, the location and width of doors, the width of aisles and the proportion of standing passengers will ali influence the length of station stop. To


DIacram Shewing Arrancement of Relays ano Train Stops necessart to stop within 124.4 Ft bevond Platform or to
oerail at 80 Fit betond Station at 13.6 m.PH Max. Spel. acceleration Curves baseo on Motor c.e. 237 on 350 ton Train.

